Exercise 1: Evolution of critical density (4 points)

During standard Big Bang evolution, we have seen that $\Omega_{\text{tot}}$ including all contributions moves away from one unless its initial value was precisely one. Can $\Omega_{\text{tot}}$ become infinite, and if so what does this mean?

Exercise 2: Magnetic monopoles (8 points)

Magnetic monopoles behave as non-relativistic matter. Suppose that at a temperature corresponding to the Grand Unified era, about $3 \cdot 10^{28}$ K, magnetic monopoles were created with a density of $\Omega_{\text{mon}} = 10^{-10}$. Assuming that the Universe has a critical density and is radiation dominated, what was the temperature when the density of monopoles equalled that of the radiation? In the present Universe, $T \approx 3K$. Compute the value $\Omega_{\text{mon}}/\Omega_{\text{rad}}$ would have at the present day. Is this ratio compatible with observations?

Exercise 3: Dilution of monopoles by inflation (8 points)

Consider the situation of the previous problem. If we have a period of inflation, the monopole density still reduces as $\rho_{\text{mon}} \propto 1/a^3$, but the total density, dominated by the cosmological constant, remains fixed. Since that density will be converted to radiation after inflation, we can imagine that the radiation density remains constant during inflation. How much inflationary expansion is necessary so that the present density of monopoles matches that of radiation?