KIT **SS 2016**

Higgs-Phenomenology

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Exercise Sheet 2

Discussion: Do, 19.05.16

Exercise 2: Higgs decay into gluons

Consider the decay of the Higgs boson into two gluons

$$H(p) \to g(p_1)g(p_2)\,.$$

We are mostly interested in the structure of the expressions, which is the same for all colour-charged fermions. Hence, for our example we will restrict ourselves to a single fermion mediating this effective coupling. Two Feynman diagrams exist, which can be for example drawn as keeping the position of the external particles fixed and taking both directions of the fermion loop.

(a) Show that the trace in the numerator for a single diagram can be simplified to

$$T = m\,\mathrm{Tr}\left[\gamma^\mu(\not{\ell}+\not{k}_1)(\not{\ell}-\not{k}_2)\gamma^\nu + \gamma^\mu(\not{\ell}+\not{k}_1)\gamma^\nu\not{\ell} + \gamma^\mu(\not{\ell}-\not{k}_2)\gamma^\nu\not{\ell}\right] + m^3\,\mathrm{Tr}\left[\gamma^\mu\gamma^\nu\right]$$

where ℓ denotes the (loop) momentum of the propagator connecting the two gluon lines and points from $g(k_1)$ to $g(k_2)$. Show also that the second diagram yields the same expression. For that, choose the loop momentum in the same way, i.e. now pointing against the fermion direction, and use the identity $\mathrm{Tr}[\gamma_{\mu_1}\cdots\gamma_{\mu_n}] = \mathrm{Tr}[\gamma_{\mu_n}\cdots\gamma_{\mu_1}]$.

(b) Calculate the traces and simplify the expressions by using the transversality of the gluons, $k_i\cdot\epsilon(k_i)=0$:

$$T = 4m\left(4\ell^\mu\ell^\nu + k_2^\mu k_1^\nu + g^{\mu\nu}(m^2-\ell^2-k_1\cdot k_2)\right)\,.$$

(c) Calculate the $D=4-2\epsilon$-dimensional loop integral

$$L = \mu^{4-D}\int\frac{\mathrm{d}^D\ell}{(2\pi)^D}\frac{T/4m}{(\ell^2-m^2)((\ell+k_1)^2-m^2)((\ell-k_2)^2-m^2)}\,.$$

It may be convenient to follow these steps:

- Use the Feynman parametrization

$$\frac{1}{ABC} = \Gamma(3) \int_0^1 \mathrm{d}x \int_0^{(1-x)} \mathrm{d}y \frac{1}{[A(1-x-y)+Bx+Cy]^3}\,,$$

  where $\Gamma(N+1) = N!$ for non-negative integer values of N, and $\Gamma(x+1) = x\Gamma(x)$ for arbitrary x.
- Shift the loop integration to get only quadratic terms in ℓ in the denominator.
- Observe that afterwards any terms linear in ℓ in the numerator vanish after integration due to symmetry.
- Apply again the transversality condition.
- Use

$$\int \frac{\mathrm{d}^D\ell}{(2\pi)^D} \ell^\mu \ell^\nu f(\ell^2) = \frac{g^{\mu\nu}}{D} \int \frac{\mathrm{d}^D\ell}{(2\pi)^D} \ell^2 f(\ell^2)$$

  and then afterwards

$$\int \frac{\mathrm{d}^D\ell}{(2\pi)^D} \frac{1}{(\ell^2 - M^2)^N} = i \frac{(-1)^N}{(4\pi)^{D/2}} \frac{\Gamma(N-\frac{D}{2})}{\Gamma(N)} \frac{1}{(M^2)^{N-\frac{D}{2}}}$$
$$\int \frac{\mathrm{d}^D\ell}{(2\pi)^D} \frac{\ell^2}{(\ell^2 - M^2)^N} = \frac{i}{2} \frac{(-1)^{N-1}}{(4\pi)^{D/2}} \frac{\Gamma(N-1-\frac{D}{2})}{\Gamma(N)} \frac{D}{(M^2)^{N-1-\frac{D}{2}}}\,.$$

  You may assume that the loop integration and the integrals over the Feynman parameters commute.
- Then take the limit $\epsilon \to 0$, using $\Gamma(N+\epsilon) \to (N-1)!$ for $N > 0$ and $\Gamma(\epsilon) = \frac{1}{\epsilon} + \mathcal{O}(\epsilon^0)$.

At the end, one obtains

$$L = \frac{i}{16\pi^2} \frac{M_H^2}{2m^2} \left(g^{\mu\nu} - \frac{k_2^\mu k_1^\nu}{k_1 \cdot k_2} \right) g\left(\frac{M_H^2}{4m^2} \right)$$

with

$$g(\tau) = \int_0^1 \mathrm{d}x \int_0^{(1-x)} \mathrm{d}y \frac{1-4xy}{1-4xy\tau} \equiv \frac{1}{4} A_{\frac{1}{2}}^H(\tau)\,.$$

(d) Calculate

$$A_{\frac{1}{2}}^H(\tau) = \frac{2}{\tau^2} \left(\tau + (\tau - 1) \arcsin^2 \sqrt{\tau} \right)$$

in the limit $\tau \to 0$ and show that it approaches a non-zero constant.