Higgs-Phenomenology

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Exercise Sheet 4

Discussion: Mo, 04.07.16

Exercise 6: Fermi Theory

Fermi theory is a low-energy description of the weak interaction, integrating out the W boson propagator as a heavy field and replacing it by a four-fermion contact interaction. Taking only leptons into account, it can be written as

$$\mathcal{L} \subset \frac{G_F}{\sqrt{2}} J^\mu J^\dagger_\mu$$

with

$$J^\mu = \bar{\nu}_e \gamma^\mu (1-\gamma^5) e + \bar{\nu}_\mu \gamma^\mu (1-\gamma^5) \mu + \bar{\nu}_\tau \gamma^\mu (1-\gamma^5) \tau \,.$$

(a) Show that for muon decay, the matrix element of the full result from the SM agrees with that from Fermi theory when expanding in the momentum transfer and keeping only the leading term. Take the electron as massless, use $M_W = \frac{gv}{2}$, and identify G_F in terms of SM quantities.

(b) Estimate the size of the first correction term. Is it negligible?
Estimate the EFT validity range when assuming an $\mathcal{O}(1)$ coupling.

Exercise 7: Scalar Field theory

Consider a massless scalar field theory of fields φ, where a $\mathbb{Z}_2$ symmetry allows only operators that are even in φ. The Lagrangian up to dimension 4 is given by

$$\mathcal{L} = \frac{1}{2} \left(\partial^\mu \varphi\right) \left(\partial_\mu \varphi\right) - \frac{\lambda}{4} \varphi^4 \,.$$

(a) Derive the equation of motion.

(b) Write down the three possible operators at dimension 6.

(c) Show that all three operators are equivalent by using the equation of motion and integration by parts. The latter can also be seen as the fact that adding a total derivative to the Lagrangian does not change the equations of motion and therefore the underlying physics.

Exercise 8: Modified Field-Strength Tensors

Show the identity

$$[D^\mu, D^\nu] = \widehat{W}^{\mu\nu} + \widehat{B}^{\mu\nu}$$

with

$$\begin{aligned} D^\mu &= \partial^\mu + ig\frac{\sigma^j}{2}W^{j,\mu} + ig'\frac{1}{2}B^\mu \,, \\ \widehat{W}^{\mu\nu} &= ig\frac{\sigma^j}{2}W^{j,\mu\nu} = ig\frac{\sigma^j}{2}\left(\partial^\mu W^{j,\nu} - \partial^\nu W^{j,\mu} - g\epsilon^{jkl}W^{k,\mu}W^{l,\nu}\right) \,, \\ \widehat{B}^{\mu\nu} &= ig'\frac{1}{2}B^{\mu\nu} = ig'\frac{1}{2}\left(\partial^\mu B^\nu - \partial^\nu B^\mu\right) \,. \end{aligned}$$

Exercise 9: Modifying Higgs vertices

(a) Show that the operators

$$\begin{aligned} \mathcal{O}_{WW} &= \Phi^\dagger \widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\Phi \qquad \text{and} \\ \mathcal{O}_{\varphi W} &= \Phi^\dagger\Phi \,\mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \end{aligned}$$

are equivalent, i.e. one can rewrite the expressions to obatin the other one. What is the relative factor for the Wilson coefficients to obtain the same physics effects?

(b) Calculate the contribution to the HWW vertex from

$$\mathcal{O}_W = (D_\mu\Phi)^\dagger\,\widehat{W}^{\mu\nu}\,(D_\nu\Phi)$$

and show that it agrees with the expression given in the lecture.