Higgs-Phenomenology

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Exercise Sheet 5

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Exercise 10: Exotic Higgs Multiplets

Higgs seven-plets (septets) of $SU(2)_L$ play a special role, as by coincidence these also allow to obtain the correct mass ratio for the W and Z bosons.
The general expression for the ρ parameter for several multiplets reads

$$\rho = \frac{\sum_i [4T_i(T_i+1) - Y_i^2]|v_i|^2 c_i}{\sum_i 2Y_i^2 |v_i|^2} ,$$

where T_i and Y_i are isospin and hypercharge of each multiplet, respectively, and $c_i = 1\,(\frac{1}{2})$ for complex (real) multiplets. The electromagnetic charge of the multiplet components is given by $Q = T^3 + \frac{Y}{2}$.

(a) Which hypercharge assignment is needed for the complex seven-plet to get the correct mass ratio? Which component(s) should therefore receive a vev?

(b) Work out the gauge boson mass-squared matrix in the W^i, B basis and show that it is indeed proportional to the SM case

$$M^2_{\text{SM}} = \frac{v^2_{\text{SM}}}{4} \begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix} .$$

Start with the kinetic term of the Lagrangian

$$\mathcal{L}_{\text{kin}} = (D^\mu \Phi_7)^\dagger (D_\mu \Phi_7)$$

and drop all terms not contributing to the gauge-boson masses. In the covariant derivative the usual $SU(2)$ generators $\sigma^i/2$ are replaced by different generators T^i, $i = 1, 2, 3$, which are now 7×7 matrices and whose exact form we do not need to specify.
Use the relation $T^+T^- + T^-T^+ = 2\left[T(T+1) - (T^3)^2\right]$ (why does this hold?) to simplify the expression, where $T^\pm$ denote the $SU(2)$ ladder operators.
Intermediate result: Proportionality factors are in general:
$(W^{1,2})^2$*:* $T(T+1) - (T^3)^2$*,* $(W^3)^2$*:* $2(T^3)^2$*,* B^2*:* $Y^2/2$*,* W^3B*:* $-T^3Y$

(c) If the W and Z masses were generated entirely by the seven-plet, what value of the vev v would be needed?
($M_W = g v_{SM}/2$, $M_Z = \sqrt{g^2 + g'^2} v_{SM}/2$ with $v_{SM} \simeq 246$ GeV)
Could all mass generation in the SM be accomplished by the seven-plet?

Exercise 11: Higgs triplets and the Georgi-Machacek model

One can repeat the previous exercise with Higgs triplets. Two choices are possible: real scalar Ξ with $Y = 0$ and complex scalar X with $Y = 2$.

(a) The current experimental bound on the oblique parameter T is $T = 0.09 \pm 0.13$, and is related to the ρ parameter via $\rho = 1 + \alpha T$. Assume that you have the SM Higgs doublet plus a triplet. For each of the two triplet possibilities mentioned above, what is the respective limit on the triplet vev?
What changes if there are two Higgs doublets plus a triplet?
Using both triplets, how can one preserve custodial symmetry?

(b) One can construct a bi-doublet from the SM Φ and $\tilde{\Phi}$ as a 2×2 matrix $\mathcal{H} = \left(\tilde{\Phi}, \Phi\right)$.
Rewrite the SM potential using $\mathcal{H}$ only.

(c) Analoguously, one can define a 3×3 matrix

$$\bar{X} = \begin{pmatrix} \chi^{0*} & \xi^{+} & \chi^{++} \\ -\chi^{+*} & \xi^{0} & \chi^{+} \\ -\chi^{++*} & -\xi^{+*} & \chi^{0} \end{pmatrix},$$

where it is now more natural that $\langle \bar{X} \rangle = v_X \cdot \mathbb{1}_3$.
Can one deduce anything about v_X?
How would you look for such an extension? Discuss some physical consequences of this model.