Compton scattering

$$\gamma(k_1) + e^-(p_1) \to \gamma(k_2) + e^-(p_2)$$

Eliminate disconnected part by requiring

$$k_1 \neq k_2 \ , \quad p_1 \neq p_2$$

Using LSZ reduction formula

$$S_{fi} = \langle p_2\, k_2\, \text{out} | p_1\, k_1\, \text{in} \rangle =$$

$$\left(\frac{-i}{\sqrt{Z_2}}\right)^2 \left(\frac{-i}{\sqrt{Z_3}}\right)^2 \int d^4x_1\, e^{-ip_1\cdot x_1} \int d^4x_2\, e^{ip_2\cdot x_2} \int d^4y_1\, e^{-ik_1\cdot y_1} \int d^4y_2\, e^{ik_2\cdot y_2}$$

$$\bar{u}(p_2)(i\partial\!\!\!/_{x_2} - m)\, \Box_{y_1} \langle 0 | T\, \psi_{in}(x_2)\, \bar{\psi}_{in}(x_1)\, \varepsilon_1(k_1)\cdot A^{in}(y_1)\, \varepsilon_2^*(k_2)\, A^{in}(y_2)\, e^{i\int d^4z\, \mathcal{L}_I(z)} |0\rangle$$

$$\cdot\, \overleftarrow{\Box}_{y_2} (-i\overleftarrow{\partial\!\!\!/}_{x_1} - m)\, u(p_1)$$

Interaction Lagrangian

$$\mathcal{L}_I(z) = \underbrace{-q}_{+e}\, \bar{\psi}_{in}(z)\, \gamma_\mu\, \psi_{in}(z)\, A^\mu_{in}(z)$$

Lowest non-trivial contribution from expanding $\exp\{i\int d^4z\, \mathcal{L}_I(z)\}$ to second order. Insert

$$\frac{(ie)^2}{2!} \int d^4z_1\, \bar{\psi}_{in}(z_1)\gamma^\mu \psi_{in}(z_1) A^{in}_\mu(z_1) \int d^4z_2\, \bar{\psi}_{in}(z_2)\gamma^\nu \psi_{in}(z_2) A^{in}_\nu(z_2)$$

$$S_{fi} \approx \frac{-e^2}{2} \int d^4x_1\, d^4x_2 \int d^4y_1\, d^4y_2 \int d^4z_1\, d^4z_2\; e^{i(k_2\cdot y_2 + p_2\cdot x_2 - k_1\cdot y_1 - p_1\cdot x_1)}$$

$$\cdot\ \overrightarrow{\Box}_{y_1} \overrightarrow{\Box}_{y_2} \langle 0| T A_{in}^{\varsigma_1}(y_1) A_{in}^{\varsigma_2}(y_2) A_{in}^{\mu_1}(z_1) A_{in}^{\mu_2}(z_2) |0\rangle\, \varepsilon_{\varsigma_1}(k_1)\, \varepsilon_{\varsigma_2}(k_2)$$

$$\cdot\ \bar{u}(p_2) \overrightarrow{(i\not\partial_{x_2} - m)} \langle 0| T \psi_{in}(x_2) \bar{\psi}_{in}(x_1) \bar{\psi}_{in}(z_1) \gamma_{\mu_1} \psi_{in}(z_1)$$

$$\cdot\ \bar{\psi}_{in}(z_2) \gamma_{\mu_2} \psi_{in}(z_2) |0\rangle \overleftarrow{(-i\not\partial_{x_1} - m)}\, u(p_1)$$

Time-ordered product of photon fields (supress in-label)

$$\int d^4y_1\, d^4y_2\, e^{i(k_2\cdot y_2 - k_1\cdot y_1)}\, \underbrace{\Box_{y_1} \Box_{y_2} \langle 0| T A^{\varsigma_1}(y_1) A^{\varsigma_2}(y_2) A^{\mu_1}(z_1) A^{\mu_2}(z_2) |0\rangle}\, \varepsilon_{\varsigma_1}(k_1)\, \varepsilon_{\varsigma_2}(k_2)$$

$$\underbrace{\Box_{y_1} \Box_{y_2}\, \underbrace{i D_F^{\varsigma_1 \mu_1}(y_1 - z_1)}_{-i g^{\varsigma_1 \mu_1} \int \frac{d^4 q_1}{(2\pi)^4} \frac{e^{-i q_1\cdot(y_1 - z_1)}}{q_1^2 + i\varepsilon}}\, i D_F^{\varsigma_2 \mu_2}(y_2 - z_2) \cdot 2}$$

$$+ i g^{\varsigma_1 \mu_1} \delta^4(y_1 - z_1)\; i g^{\varsigma_2 \mu_2} \delta^4(y_2 - z_2) \cdot 2$$

$$= 2\, i\varepsilon^{\mu_1}(k_1)\, i\varepsilon^{\mu_2}(k_2)\, e^{i(k_2\cdot z_2 - k_1\cdot z_1)}$$

Reason for factor 2 : relabeling $z_1 \leftrightarrow z_2$ does not change remaining expression for S_{fi}

No contribution from $D_F^{\varsigma_1 \varsigma_2}(y_1 - y_2)$:

$$\int d^4y_1\, d^4y_2\, e^{i(k_2\cdot y_2 - k_1\cdot y_1)}\, \Box_{y_1} \Box_{y_2}\, i D_F^{\varsigma_1 \varsigma_2}(y_1 - y_2) \sim k_1^2\, \delta^4(k_1 - k_2) = 0$$

Feynman rule for external photons

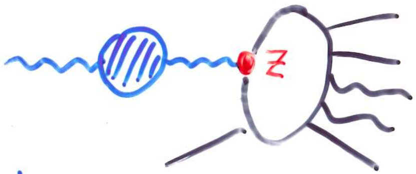
Each external photon receives a factor

$$\frac{-i}{\sqrt{Z_3}} \int d^4y \, e^{\pm i k\cdot y} \underbrace{\Box_y \langle 0 | T A^\rho(y) A^\mu(z) | 0\rangle}_{i g^{\rho\mu} \delta^4(y-z) Z_3} \varepsilon_\rho^{(*)}(k)$$

$$= \begin{cases} \sqrt{Z_3}\, \varepsilon^\mu(k) \left[e^{-ik\cdot z}\right] & \text{incoming photon} \\ \sqrt{Z_3}\, \varepsilon^\mu(k)^* \left[e^{+ik\cdot z}\right] & \text{outgoing photon} \end{cases}$$

Reason for Z_3 factor

z = label of first vertex that "really" describes interaction with external fermions



with

They sum up to full propagator. On-shell one has

$$\Box_y D^{\rho\mu}_{full}(y-z)\Big/_{k^2=0} \equiv \Box_y \langle 0|T A^\rho(y) A^\mu(z)|0\rangle \Big/_{k^2=0}$$

$$= \Box_y \langle 0 | T \sqrt{Z_3} A^\rho_{in}(y) \sqrt{Z_3} A^\mu_{in}(z) | 0\rangle \Big/_{k^2=0}$$

$$= Z_3 \Box_y D^{\rho\mu}(y-z)\Big/_{free} = i g^{\rho\mu} \delta^4(y-z) Z_3$$

$$S_{fi} = e^2 \int d^4x_1\, d^4x_2\, d^4z_1\, d^4z_2\; e^{i(k_2\cdot z_2 - k_1\cdot z_1)}\, e^{i(p_2\cdot x_2 - p_1\cdot x_1)}$$

$$\bar u(p_2)(i\overrightarrow{\not\partial}_{x_2} - m)\langle 0|T\,\psi(x_2)\,\bar\psi(x_1)\,\bar\psi(z_1)\not\epsilon_1\psi(z_1)\,\bar\psi(z_2)\not\epsilon_2\psi(z_2)|0\rangle(-i\overleftarrow{\not\partial}_{x_1} - m)\,u(p_1)$$

$$\bar u(p_2)\underbrace{(i\not\partial_{x_2} - m)\, iS_F(x_2 - z_1)}_{i\delta^4(x_2 - z_1)}\not\epsilon_1\, iS_F(z_1 - z_2)\not\epsilon_2\, \underbrace{iS_F(z_2 - x_1)(-i\overleftarrow{\not\partial}_{x_1} - m)}_{i\delta^4(z_2 - x_1)}\,u(p_1)$$

$$+\bar u(p_2)\underbrace{(i\not\partial_{x_2} - m)\, iS_F(x_2 - z_2)}_{i\delta^4(x_2 - z_2)}\not\epsilon_2\, iS_F(z_2 - z_1)\not\epsilon_1\, \underbrace{iS_F(z_1 - x_1)(-i\overleftarrow{\not\partial}_{x_1} - m)}_{i\delta^4(z_1 - x_1)}\,u(p_1)$$

$$= -ie^2\int d^4z_1\, d^4z_2\; e^{i(k_2\cdot z_2 - k_1\cdot z_1)}$$

$$\cdot\Big\{\bar u(p_2)\not\epsilon_1 \int\frac{d^4q}{(2\pi)^4}\,\frac{e^{-iq\cdot(z_1 - z_2)}\, e^{i(p_2\cdot z_1 - p_1\cdot z_2)}}{\not q - m + i\varepsilon}\not\epsilon_2\, u(p_1) \qquad \Big|\; \begin{aligned} q &= p_2 - k_1 \\ &= p_1 - k_2 \end{aligned}$$

$$+\bar u(p_2)\not\epsilon_2 \int\frac{d^4q}{(2\pi)^4}\,\frac{e^{-iq\cdot(z_2 - z_1)}\, e^{i(p_2\cdot z_2 - p_1\cdot z_1)}}{\not q - m + i\varepsilon}\not\epsilon_1\, u(p_1)\Big\} \qquad \Big|\; \begin{aligned} q &= p_2 + k_2 \\ &= p_1 + k_1 \end{aligned}$$

$$S_{fi} = i(2\pi)^4\,\delta^4(p_1 + k_1 - p_2 - k_2)$$

$$\cdot(-e^2)\Big\{\bar u(p_2)\not\epsilon(k_1)\frac{1}{\not p_2 - \not k_1 - m + i\varepsilon}\not\epsilon(k_2)\,u(p_1)$$

$$+\bar u(p_2)\not\epsilon(k_2)\frac{1}{\not p_2 + \not k_2 - m + i\varepsilon}\not\epsilon(k_1)\,u(p_1)\Big\}$$

$$= i(2\pi)^4\,\delta^4(p_1 + k_1 - p_2 - k_2)\,\langle p_2 k_2|\mathcal{T}|p_1 k_1\rangle$$

Feynman rules for external (anti)-fermions

e^- in: $$-\frac{i}{\sqrt{Z_2}}\int d^4x\, \underbrace{\underbrace{\langle 0|T\,\psi(y)\,\bar\psi(x)|0\rangle\,(-i\overleftarrow{\not\partial}_x - m)}_{i S_F^{full}(y-x)\,(-i\overleftarrow{\not\partial}_x - m)\,/_{p^2=m^2}}}_{i Z_2\,\delta^4(y-x)}\, u(p,\lambda)\, e^{-ip\cdot x}$$

$$= \sqrt{Z_2}\, u(p,\lambda)\left[e^{-ip\cdot y}\right]$$

e^- out: $$\sqrt{Z_2}\,\bar u(p,\lambda)\left[e^{+ip\cdot y}\right]$$

e^+ in: $$\sqrt{Z_2}\,\bar v(p,\lambda)\left[e^{-ip\cdot y}\right]$$

e^+ out: $$\sqrt{Z_2}\, v(p,\lambda)\left[e^{+ip\cdot y}\right]$$

Overall sign (e.g. e^+ out vs. e^- out)

e^+ out: $$\langle 0|\, d\,\psi\,(\bar\psi\psi)^n\,\bar\psi\gamma_\mu\psi\,|0\rangle \qquad d \sim \bar\psi v$$

$$= -\langle 0|\,\psi\,(\bar\psi\psi)^n\,\bar\psi\gamma_\mu\psi\,\bar\psi v\,|0\rangle$$

e^- out: $$\langle 0|\, b\,\bar\psi\,(\bar\psi\psi)^n\,\bar\psi\gamma_\mu\psi\,|0\rangle \qquad b \sim \bar u\psi$$

$$= +\langle 0|\,\bar u\psi\,\bar\psi\gamma_\mu\psi\,(\bar\psi\psi)^n\,\bar\psi\,|0\rangle$$

Hence for e^+ out we get the factor

$$+\frac{i}{\sqrt{Z_2}}\int d^4x\,(-1)\,\langle 0|T\,\psi(y)\,\bar\psi(x)|0\rangle(-i\overleftarrow{\not\partial}_x - m)\,v(p,\lambda)\,e^{ip\cdot x}$$

$$= \sqrt{Z_2}\, v(p,\lambda)\, e^{ip\cdot y}$$