LSZ reduction formulas

electron in in-state:

$$\langle \beta\, \text{out} |\, b^{+}_{in}(p,\lambda)\, | \alpha\, \text{in} \rangle = \text{disc. part}$$

$$- \frac{i}{\sqrt{Z_2}} \int d^4x \, \langle \beta\, \text{out} |\, \bar{\psi}(x)\, | \alpha\, \text{in} \rangle \left(-i\overleftarrow{\not\partial}_x - m\right) u(p,\lambda)\, e^{-ip.x}$$

where

$$A(x) \overleftarrow{\partial}_\mu B(x) \equiv \left(\partial_\mu A(x)\right) B(x)$$

Successive reduction leads to time-ordered product of interacting fields

electron in out-state:

$$\langle \beta\, \text{out} |\, b_{out}(q,\lambda)\, \underbrace{T(\psi_{\alpha_1}(y_1) \ldots \psi_{\alpha_n}(y_n) \bar{\psi}_{\beta_1}(z_1) \ldots \bar{\psi}_{\beta_m}(z_m))}_{T(y_1 \ldots z_m)} | \alpha\, \text{in} \rangle$$

$$= \text{disc. part}$$

$$+ \langle \beta\, \text{out} |\, b_{out}(q,\lambda)\, T(y_1 \ldots z_m) -$$

$$(-1)^{n+m}\, T(y_1 \ldots z_m)\, b_{in}(q,\lambda)\, | \alpha\, \text{in} \rangle$$

$$= \int d^3x\, \bar{u}(q,\lambda)\gamma^0 \langle \beta\, \text{out} | \overbrace{\psi_{\text{out}}(x)}^{\frac{1}{\sqrt{Z_2}}\psi(x)} e^{iq\cdot x}\Big/_{x^0=+\infty} T(y_1 \ldots z_m)$$

$$- \underbrace{(-1)^{n+m} T(y_1 .. z_m)\, \psi_{in}(x)\, e^{iq\cdot x}}_{T(\underbrace{\psi_{in}}_{\frac{1}{\sqrt{Z_2}}\psi(x)}(x)\psi(y_1)\ldots\bar{\psi}(z_m))}\Big/_{x^0=-\infty} |\alpha\, in\rangle$$

$+$ dis. part

$= $ disc. part $+$

$$\frac{1}{\sqrt{Z_2}} \int d^4x\, \bar{u}(q,\lambda)\gamma^0 \frac{\partial}{\partial x^0} \langle \beta\, \text{out} | e^{iq\cdot x} T(\psi(x)\psi(y_1) .. \bar{\psi}(z_m)) | \alpha\, in\rangle$$

$= $ disc. part

$$- \frac{i}{\sqrt{Z_2}} \int d^4x\, \bar{u}(q,\lambda) e^{iq\cdot x} (i\overrightarrow{\not\partial_x - m}) \langle \beta\, \text{out} | T(\psi(x)\psi(y_1) .. \bar{\psi}(z_m)) | \alpha\, in\rangle$$

where in the last step we have used

$$\bar{u}(q,\lambda)\gamma^0 \frac{\partial}{\partial x^0} e^{iq\cdot x} = i\,\bar{u}(q,\lambda) \underbrace{q^0\gamma^0}_{\vec{q}\cdot\vec{\gamma}+m \;=\; -\gamma^j q_j + m} e^{iq\cdot x}$$

$$= -i\,\bar{u}(q,\lambda)(-i\gamma^j\partial_j - m)\, e^{iq\cdot x}$$

and integration by parts

Summary:

e^- in: $\langle \text{out} | b^\dagger_{\text{in}}(p,\lambda) | \text{in} \rangle$ = disc. part

$$-\frac{i}{\sqrt{Z_2}} \int d^4x \, \langle \text{out} | \bar{\psi}(x) | \text{in} \rangle \left(-i\overleftarrow{\not\partial}_x - m\right) u(p,\lambda)\, e^{-ip\cdot x}$$

e^- out: $\langle \text{out} | b_{\text{out}}(p,\lambda) | \text{in} \rangle$ = disc. part

$$-\frac{i}{\sqrt{Z_2}} \int d^4x \, \bar{u}(p,\lambda)\, e^{ip\cdot x} \left(i\overrightarrow{\not\partial}_x - m\right) \langle \text{out} | \psi(x) | \text{in} \rangle$$

e^+ in: $\langle \text{out} | d^\dagger_{\text{in}}(p,\lambda) | \text{in} \rangle$ = disc. part

$$+\frac{i}{\sqrt{Z_2}} \int d^4x \, \bar{v}(p,\lambda)\, e^{-ip\cdot x} \left(i\overrightarrow{\not\partial}_x - m\right) \langle \text{out} | \psi(x) | \text{in} \rangle$$

e^+ out: $\langle \text{out} | d_{\text{out}}(p,\lambda) | \text{in} \rangle$ = disc. part

$$+\frac{i}{\sqrt{Z_2}} \int d^4x \, \langle \text{out} | \bar{\psi}(x) | \text{in} \rangle \left(-i\overleftarrow{\not\partial}_x - m\right) v(p,\lambda)\, e^{ip\cdot x}$$

Summary of summary:

Factors in reduction formula

1) time ordered product of interacting fields

$$\langle 0|T\,\psi(x_1)\ldots\bar\psi(x_n)|0\rangle$$

2) <u>Free</u> Dirac operator acting on each interacting field

$$\overrightarrow{(i\not\partial_x - m)}_{ii'}\langle 0|T\ldots\psi_{i'}(x)\ldots\bar\psi_{j'}(y)\ldots|0\rangle\overleftarrow{(-i\not\partial_y - m)}_{j'j}$$

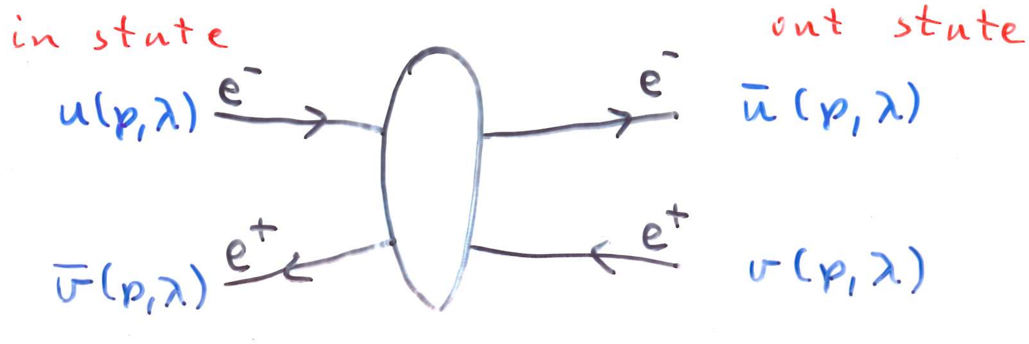
3) Each external fermion: $-\frac{i}{\sqrt{Z_2}}\int d^4x$

" " anti-fermion: $+\frac{i}{\sqrt{Z_2}}\int d^4x$

4) Each in-state particle: $e^{-ip\cdot x}$

" out-state " : $e^{+ip\cdot x}$

5) Free Dirac wave function:



Reduction formula for photons

γ in: $\langle \text{out} | a_r^\dagger(k)_{in} | \text{in} \rangle$ = disc. part

$$-\frac{i}{\sqrt{Z_3}} \int d^4x \, e^{-ik\cdot x} \, \overrightarrow{\Box}_x \langle \text{out} | \varepsilon_r(k)\cdot A(x) | \text{in} \rangle$$

γ out: $\langle \text{out} | a_r(k)_{out} | \text{in} \rangle$ = disc. part

$$-\frac{i}{\sqrt{Z_3}} \int d^4x \, e^{ik\cdot x} \, \overrightarrow{\Box}_x \langle \text{out} | \varepsilon_r^*(k)\cdot A(x) | \text{in} \rangle$$

.... and for massive scalars

ϕ in: $\langle \text{out} | a_{in}^\dagger(k) | \text{in} \rangle$ = disc. part.

$$+\frac{i}{\sqrt{Z}} \int d^4x \, e^{-ik\cdot x} \, \overrightarrow{(\Box_x + m^2)} \langle \text{out} | \phi(x) | \text{in} \rangle$$

ϕ out: $\langle \text{out} | a_{out}(k) | \text{in} \rangle$ = disc. part

$$+\frac{i}{\sqrt{Z}} \int d^4x \, e^{ik\cdot x} \, \overrightarrow{(\Box_x + m^2)} \langle \text{out} | \phi(x) | \text{in} \rangle$$