



Reconstructing Supersymmetry in the LHC era

Michael Rauch

(in collaboration with Rémi Lafaye, Tilman Plehn, Dirk Zerwas)

Outline

- Supersymmetry
- Current Status
- Determining SUSY Parameters
- Kinematic Edges as Experimental Input
- Possible Collider Results
- Dark Matter

Supersymmetry

Symmetry between bosons and fermions:

$$Q |\text{boson}\rangle = |\text{fermion}\rangle ;$$

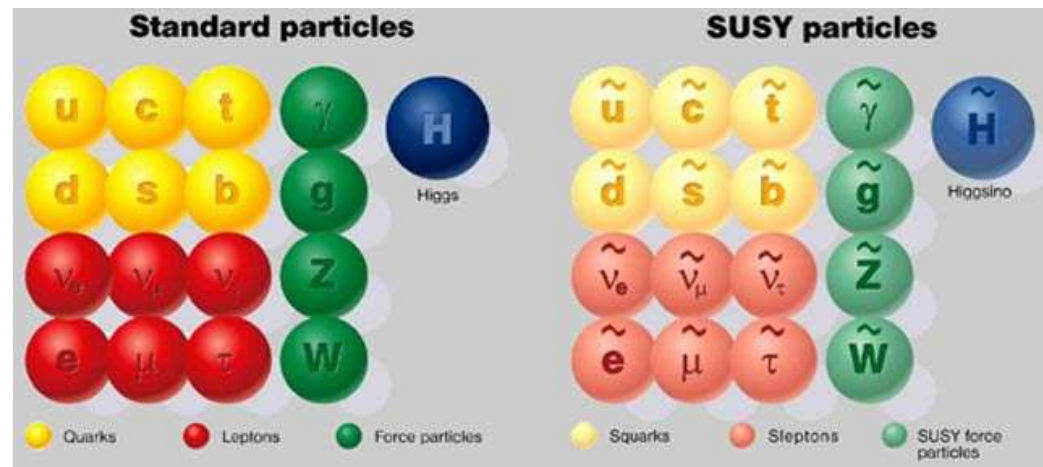
$$Q |\text{fermion}\rangle = |\text{boson}\rangle$$

Q : Supersymmetry Operator

Simplest model:

Minimal Supersymmetric Standard Model (MSSM)

- Supersymmetric partner to each Standard Model particle
- Two Higgs doublets \Rightarrow 5 Higgs bosons (h^0, H^0, A^0, H^\pm)
- Particles with same quantum numbers mix (e.g. Zino, Photino, 2 Higgsino \rightarrow 4 Neutralino)



Unification at the GUT scale ($\sim 10^{16}$ GeV):

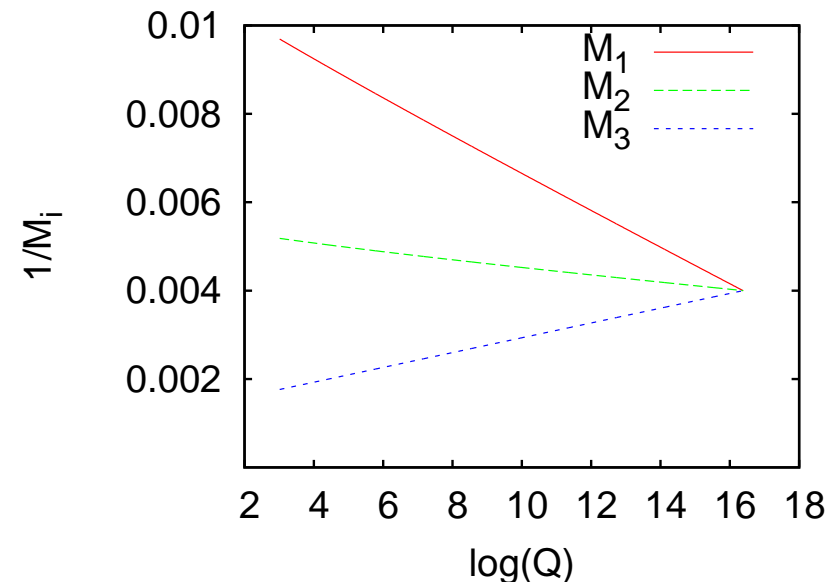
- Common scalar mass: m_0
- Common sfermion mass: $m_{1/2}$
- Common trilinear coupling: A_0

plus

- ratio of the Higgs vacuum expectation values at the electro-weak scale: $\tan \beta = \frac{v_2}{v_1}$
- one sign: $\text{sgn } \mu$

Evolve three parameters defined at the GUT scale via renormalisation group equations down to electro-weak scale:

- ⇒ Weak-scale MSSM parameters
- ⇒ Masses and couplings



Determining SUSY parameters

nowadays:

Parameters in the Lagrangian

$m_0, \mu, \tan(\beta), M_{\{1,2,3\}}, \dots$

Feynman diagrams,
RG evolution, ...

Observables:

- Masses
- Kinematic endpoints
- Cross sections
- Branching ratios
- ...

after SUSY discovery:

Observables

$m_{h^0}, \Delta m_{\tilde{g}\chi_1^0}, \text{three-particle edge}(\chi_4^0, \tilde{e}_L, \chi_1^0), \text{BR}, \dots$

?

Lagrangian parameters

M_1	<input type="text"/> \pm <input type="text"/> GeV
M_2	<input type="text"/> \pm <input type="text"/> GeV
M_3	<input type="text"/> \pm <input type="text"/> GeV
μ	<input type="text"/> \pm <input type="text"/> GeV
$\tan \beta$	<input type="text"/> \pm <input type="text"/>
...	...

⇒ Tools to reconstruct SUSY parameters

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- Standard Model experimentally very well confirmed

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 - Dark matter
 - ~ 23% Dark Matter content in the universe
 - Possible candidate in the SM: Neutrinos
 - ↔ neutrino mass limits prevent accounting for total content
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 - ~ 1σ deviation
 - $g - 2$ of the Muon
 - 3.4σ deviation

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 - Hierarchy problem
 - (higher-order corrections to the Higgs-boson mass proportional to mass of the heaviest coupling particle
 - GUT scale (?))

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- Look for possible ultra-violet completions → Supersymmetry

Current Fits

→ Fits of current data to supersymmetry (only mSUGRA)

[Allanach, Cranmer, Lester, Weber 2005-7]

[Roszkowski, Ruiz de Austra, Trotta 2006/7]

[Buchmüller, Cavanaugh, De Roeck, Heinemeyer, Isidori, Paradisi, Ronga, Weber, Weiglein 2007]

Observables:

- Dark Matter $\Omega_{\text{DM}} h^2$
- $g - 2_\mu$
- M_W
- $\sin^2 \theta_W$
- $\text{BR}(b \rightarrow s\gamma)$
- $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$
- ...

⇒ Predictions for SUSY mass spectrum

Current Fits (2)

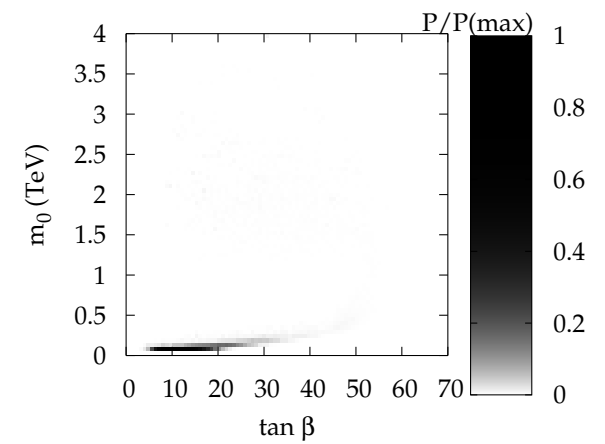
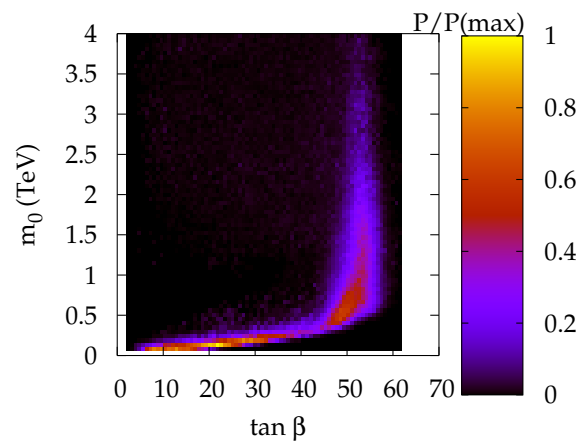
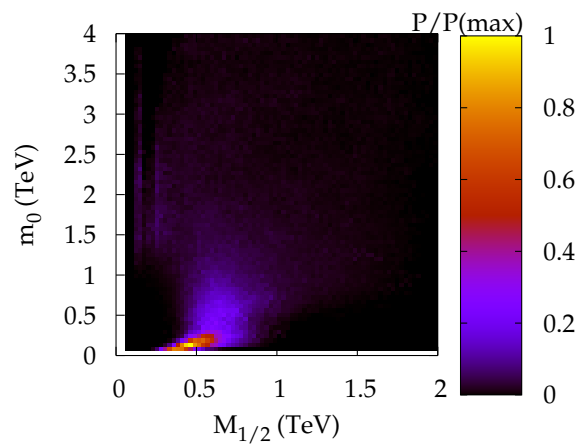
Predictions for SUSY mass spectrum

[plots by Allanach et al.]

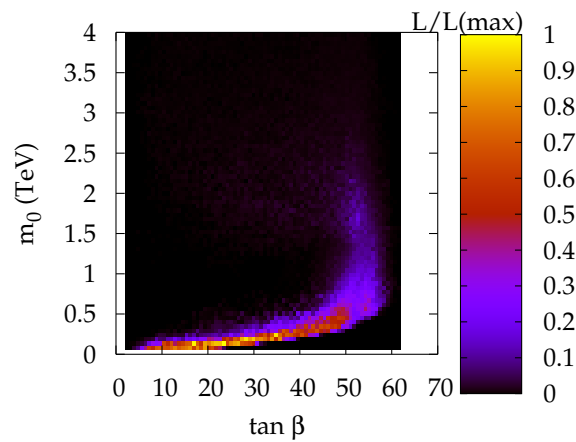
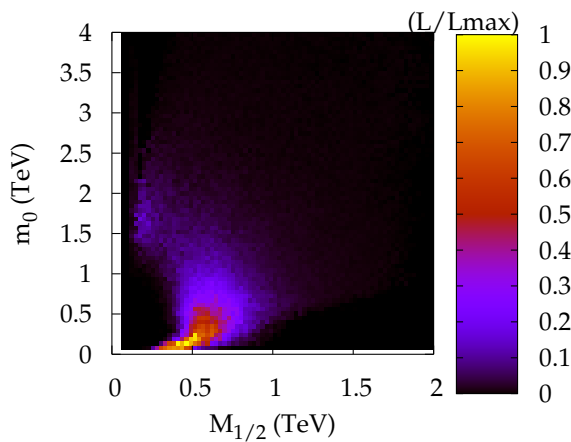
Bayesian:

(flat $\tan \beta$ prior)

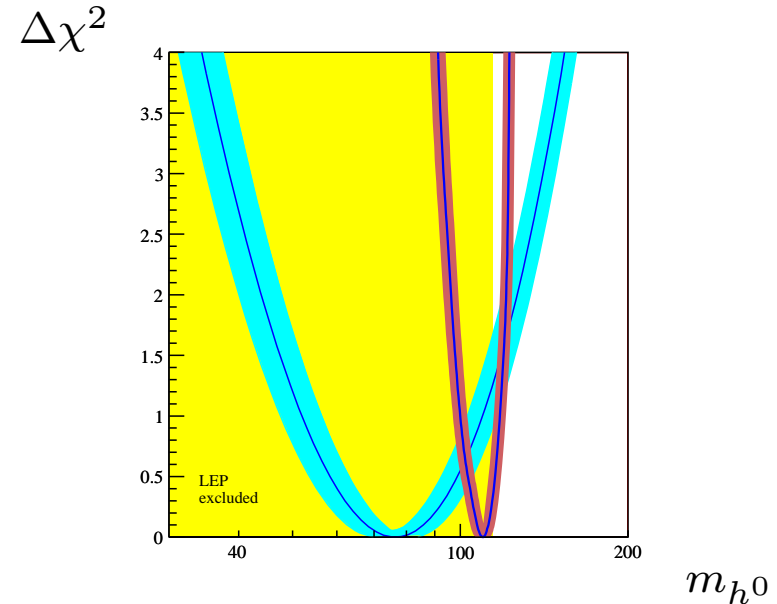
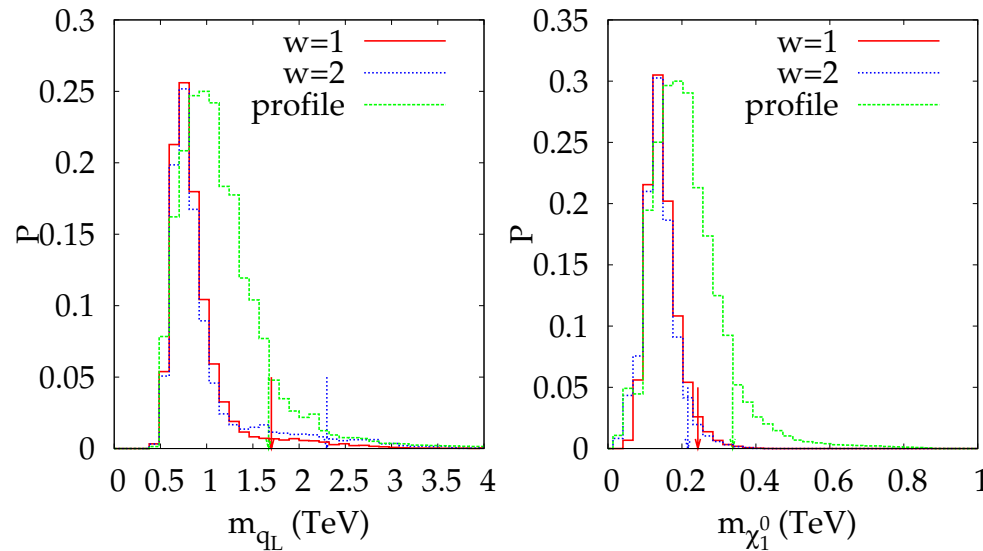
(REWSB+same order prior)



Frequentist:



Current Fits (2)



[right-hand plot by Buchmüller et al.]

- Low-energy TeV-scale SUSY fits data very well
- \Rightarrow Mass ranges for SUSY particles
- Mass of the lightest Higgs boson compatible with LEP limit

Discovery of SUSY at the LHC

\Rightarrow Additional observables from collider data

\Rightarrow SFitter

What SFitter does

- Set of measurements

- LHC measurements:

- kinematic edges, thresholds, masses, mass differences
 - cross sections, branching ratios

- ILC measurements

- Indirect Constraints

- electro-weak: $M_W, \sin^2 \theta_W$; $(g - 2)_\mu$

- flavour: $\text{BR}(b \rightarrow s\gamma), \text{BR}(B_s \rightarrow \mu^+\mu^-)$; dark matter: Ωh^2

- or even ATLAS and CMS measurements separately

- Compare to theoretical predictions

- Spectrum calculators: SoftSUSY, SuSPECT, ISASUSY

- [Allanach; Djouadi, Kneur, Moultaka; Baer, Paige, Protopopescu, Tata]

- LHC cross sections: Prospino2

- [Plehn et al.]

- LC cross sections: MsmLib

- [Ganis]

- Branching Ratios: SUSYHit (HDecay + SDecay)

- [Djouadi, Mühlleitner, Spira]

- micrOMEGAs

- [Bélanger, Boudjema, Pukhov, Semenov]

- g-2

- [Stöckinger]

Parameter Scans

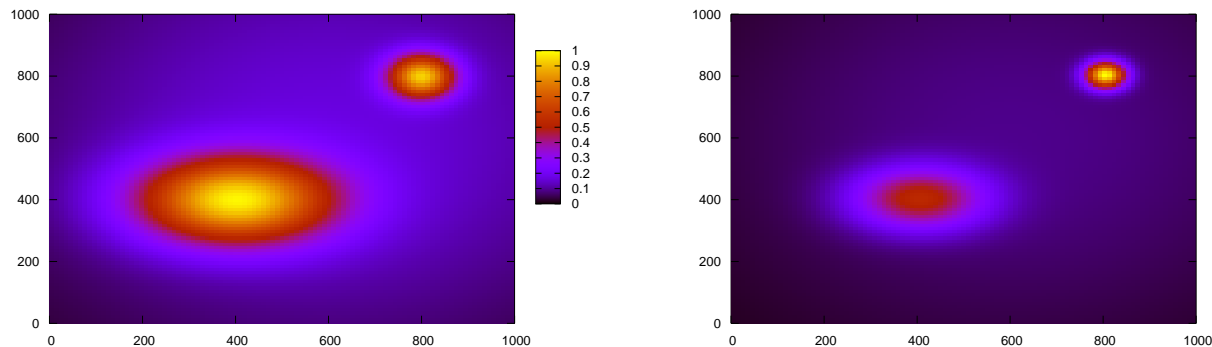
- MSSM parameter space is high-dimensional:
 - SM: 3+ parameters ($m_t, \alpha_s, \alpha, \dots$)
 - mSUGRA: 5 parameters ($m_0, m_{1/2}, A_0, \tan(\beta), \text{sgn}(\mu)$)
 - General MSSM: 105 parameters
- On loop-level observables depend on every parameter
Simple inversion of the relations not possible
⇒ Parameter scans
- Error estimates on parameters in the minimum

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Find best points (best χ^2) using different fitting techniques:

- Gradient search (Minuit) (+ Reasonably fast
- Limited convergence, only best fit)



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- Weighted Markov Chains

Markov Chains

Markov Chain (MC):

- Sequence of points, chosen by an algorithm (Metropolis-Hastings), only depending on its direct predecessor
- Picks a set of "average" points according to a potential V (e.g. inverse log-likelihood, $1/\chi^2$)
- Point density resembles the value of V (i.e. more points in region with high V)
- Scans high dimensional parameter spaces efficiently [Baltz, Gondolo 2004]
- mSUGRA MC scans with current exp. limits
[Allanach, Cranmer, Lester, Weber 2005-7; Roszkowski, Ruiz de Austra, Trotta 2006/7]

Weighted Markov Chains

Weighted Markov Chains:

Improved evaluation algorithm for binning:

[Plehn, MR]

- Weight points with value of V :

- Take care of

- Overcounting because point density is already weighted ($\frac{\text{number of points}}{\sum_{\text{points}} 1/V(\text{point})}$)

[based on Ferrenberg, Swendsen 1988]

- Correct account for regions with zero probability
(maintain additional chain which stores points rejected because $V(\text{point}) = 0$)

- + Fast scans of high-dimensional spaces $\mathcal{O}(N)$

- + Does not rely on shape of χ^2 (no derivatives used)

- + Can find secondary distinct solutions

- Exact minimum difficult to find \Rightarrow Additional gradient fit

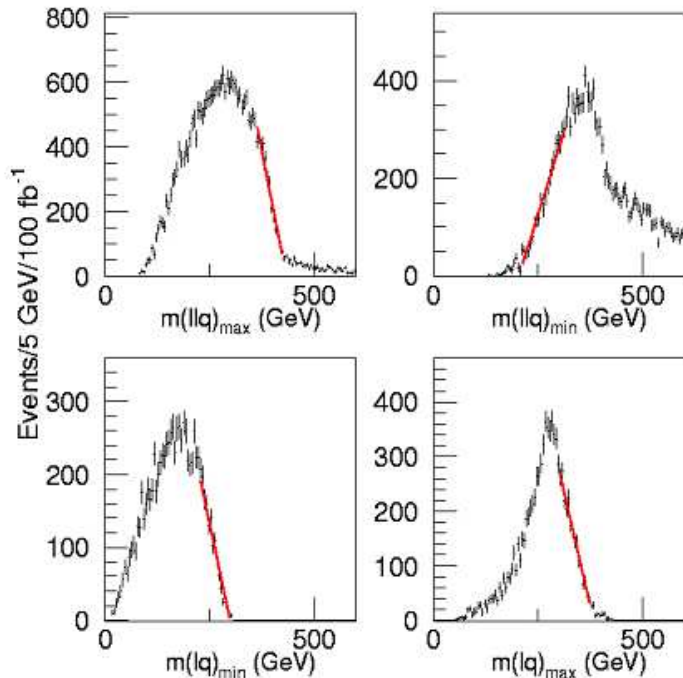
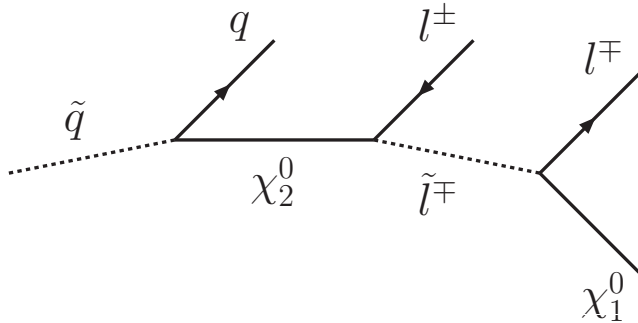
- Bad choice of proposal function for next point leads to bad coverage of the space

Experimental Input (Edges)

mSUGRA SPS1a as a benchmark point:

$m_0 = 100 \text{ GeV}$, $m_{1/2} = 250 \text{ GeV}$, $A_0 = -100 \text{ GeV}$, $\tan \beta = 10$, $\text{sgn } \mu = +1$, $m_t = 171.4 \text{ GeV}$

LHC “experimental” data from cascade decays (best precision obtainable)



Measurement	Value (GeV)	Errors (GeV)	
		(stat)	(syst)
(m_{llq}^{\max}) : Edge($\tilde{q}_L, \chi_2^0, \chi_1^0$)	446.44	1.4	4.3
(m_{llq}^{\min}) : Thres($\tilde{q}_L, \chi_2^0, \tilde{\mu}_R, \chi_1^0$)	211.95	1.6	2.0
(m_{lq}^{low}) : Edge($\tilde{c}_L, \chi_2^0, \tilde{\mu}_R$)	316.51	0.9	3.0
(m_{lq}^{high}) : Edge($\tilde{c}_L, \chi_2^0, \tilde{\mu}_R, \chi_1^0$)	392.8	1.0	3.8
...

Theoretical Errors:

- mSUGRA: 3% for gluino and squark masses, 1% for all other sparticle masses
- MSSM: 1% for gluino and squark masses, 0.5% for all other sparticle masses
- m_{h^0} : 2 GeV (unknown higher order terms)

mSUGRA as a Toy Model

mSUGRA with LHC measurements (SPS1a kinematic edges):
 pick one set of "measurements", randomly smeared from the true values

Free parameters:

$m_0, m_{1/2}, \tan(\beta), A_0, \text{sgn}(\mu), m_t$

SFitter output 1:

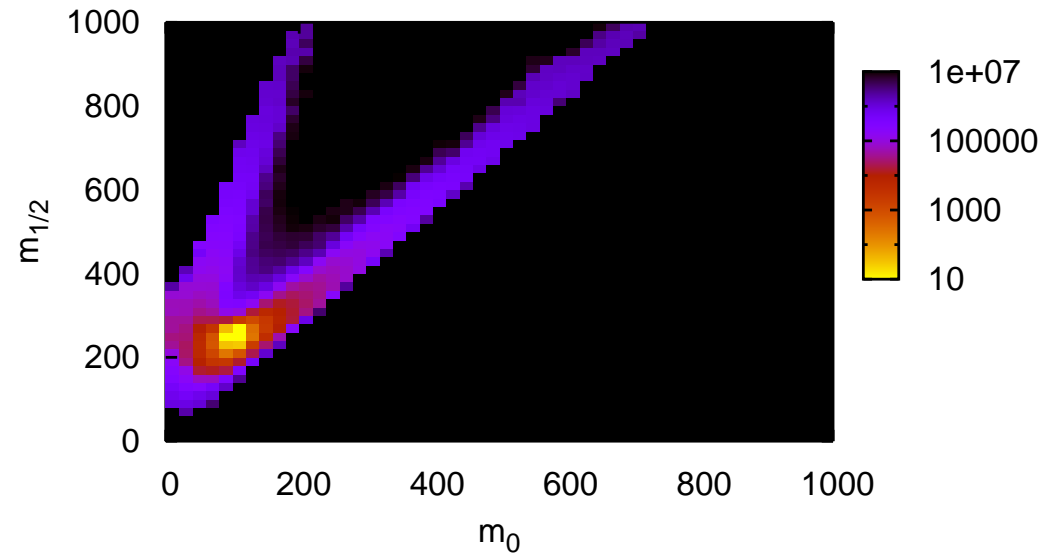
Fully-dimensional exclusive likelihood map

(colour:

minimum χ^2 over all unseen parameters)

SFitter output 2:

Ranked list of minima:

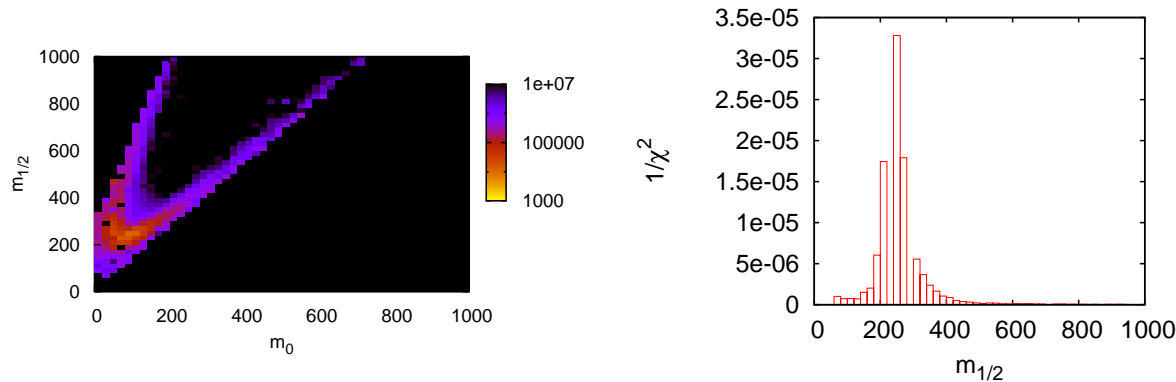


	χ^2	m_0	$m_{1/2}$	$\tan(\beta)$	A_0	μ	m_t
1)	1.32	100.4	251.2	12.7	-71.7	+	171.9
2)	7.18	106.3	243.6	14.3	-103.3	-	170.7
3)	13.9	103.5	258.2	12.2	848.4	+	174.4
4)	75.1	107.3	251.4	15.1	778.8	-	173.6
...			

Bayesian or Frequentist?

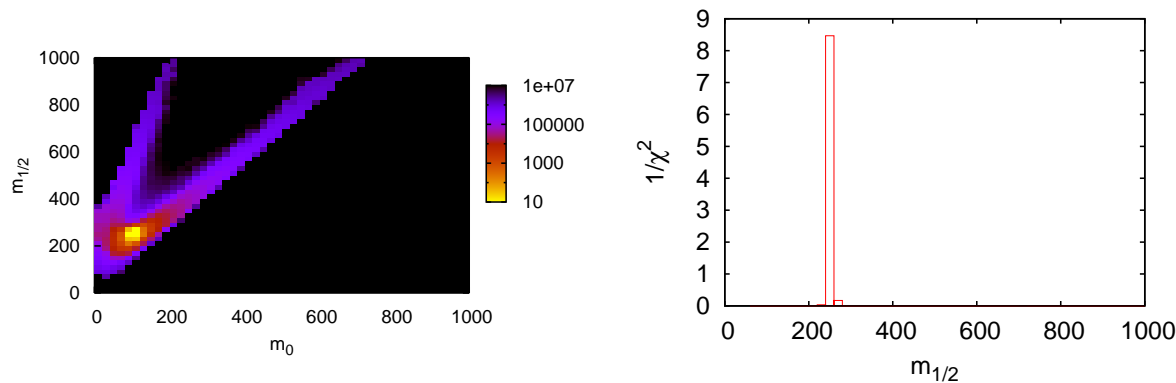
SFitter provides full-dimensional log-likelihood map
→ "project" onto plotable 1- or 2-dimensional spaces

Bayesian:



Marginalisation of χ^2 in all other directions

Frequentist:

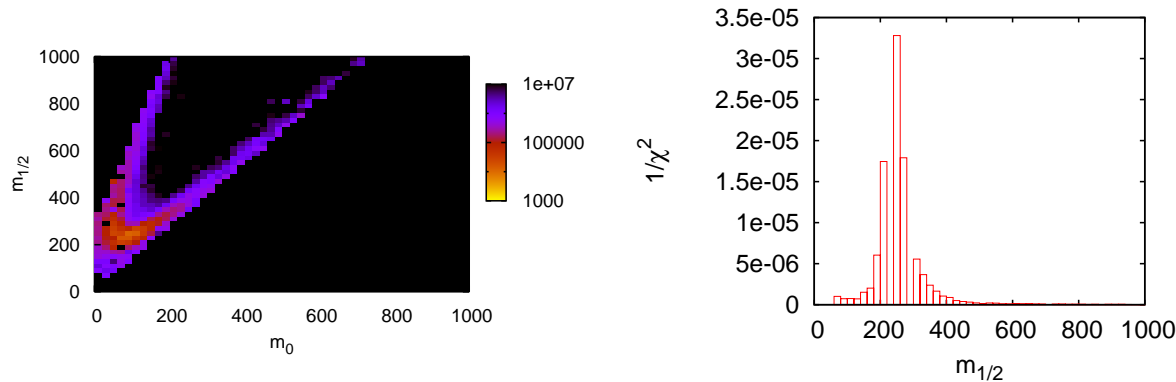


Profile likelihood: Value of bin is value of smallest χ^2 occurring in this bin

Bayesian or Frequentist?

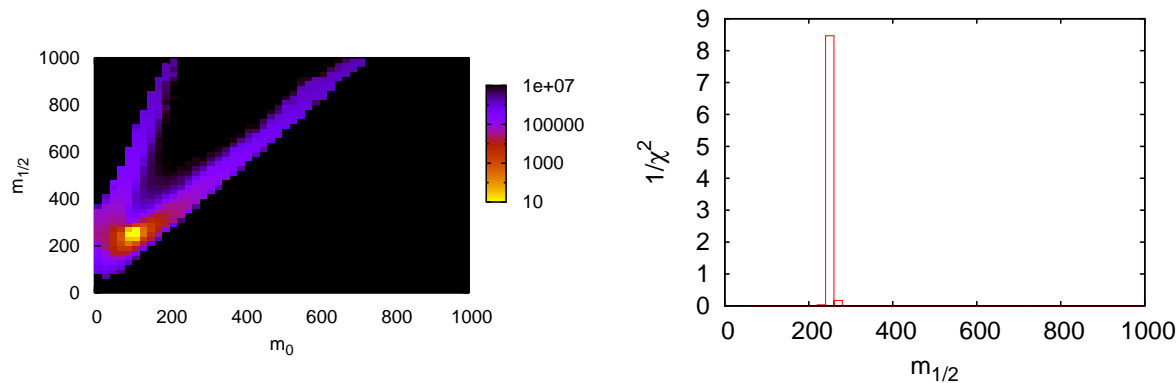
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Different methods answer different questions.

⇒ Bayesian **and** Frequentist!

Everybody can choose his/her favourite analysis ...

Purely high-scale model

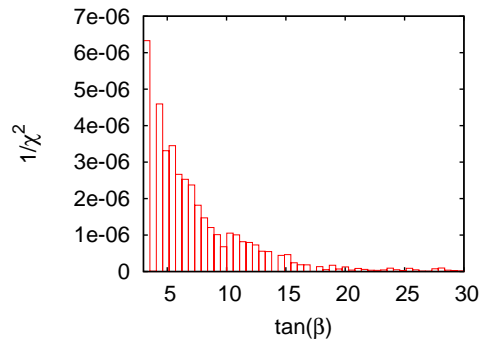
$m_0, m_{1/2}, A_0$ defined at the GUT-scale $\Leftrightarrow \tan(\beta)$ defined at the weak scale

\Rightarrow Replace $\tan(\beta)$ with high-scale quantity B

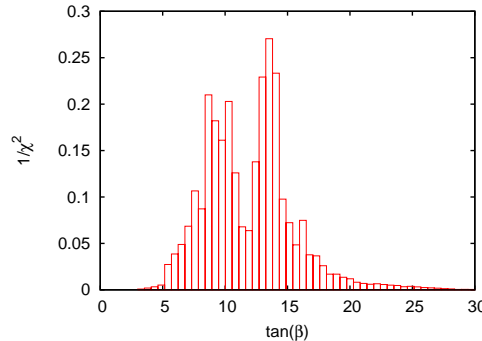
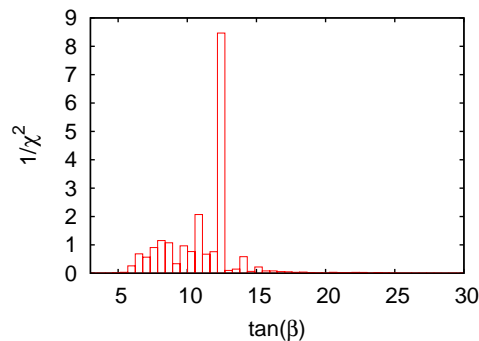
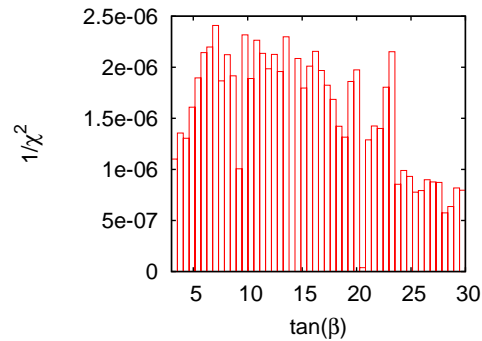
\Rightarrow Flat prior in B yields prior $\propto \frac{1}{\tan(\beta)^2}$

SPS1a with LHC kinematic edges ($\tan(\beta)$ vs. $1/\chi^2$):

flat B prior



flat $\tan(\beta)$ prior



Bayesian:

Large influence of choice of prior

Choosing flat B prior strongly favours low values of $\tan(\beta)$.

Frequentist:

Two plots should be identical (no prior in χ^2 calculation)

Indirect influence via Markov Chain proposal function

Error determination

Treatment of errors:

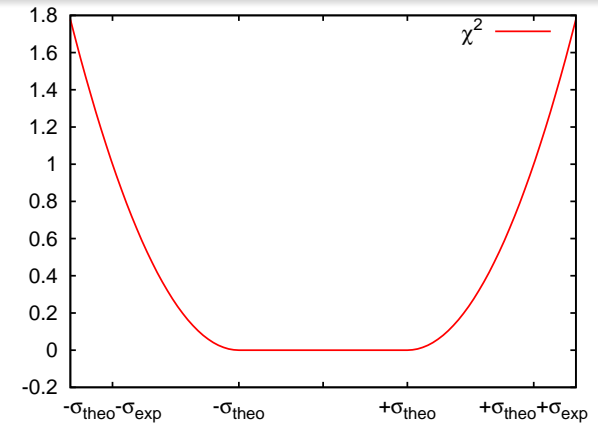
- All experimental errors are Gaussian

$$\sigma_{\text{exp}}^2 = \sigma_{\text{stat}}^2 + \sigma_{\text{syst}(j)}^2 + \sigma_{\text{syst}(l)}^2$$

- Systematic errors from jet ($\sigma_{\text{syst}(j)}$) and lepton energy scale ($\sigma_{\text{syst}(l)}$) assumed 99% correlated each

- Theory error added as box-shaped (RFit scheme [Hoecker, Lacker, Laplace, Lediberder])

$$\Rightarrow -2 \log L \equiv \chi^2 = \sum_{\text{measurements}} \begin{cases} 0 & \text{for } |x_{\text{data}} - x_{\text{pred}}| < \sigma_{\text{theo}} \\ \left(\frac{|x_{\text{data}} - x_{\text{pred}}| - \sigma_{\text{theo}}}{\sigma_{\text{exp}}} \right)^2 & \text{for } |x_{\text{data}} - x_{\text{pred}}| \geq \sigma_{\text{theo}} \end{cases}$$



⇒ Parameter errors:

	SPS1a	$\Delta_{\text{flat}}^{\text{theo-exp}}$	$\Delta_{\text{zero}}^{\text{theo-exp}}$	$\Delta_{\text{gauss}}^{\text{theo-exp}}$	$\Delta_{\text{flat}}^{\text{theo-exp}}$
		LHC masses	LHC edges		
m_0	100	4.89	0.50	2.96	2.17
$m_{1/2}$	250	3.27	0.73	2.99	2.64
$\tan \beta$	10	2.73	0.65	3.36	2.45
A_0	-100	56.4	21.2	51.5	49.6
m_t	171.4	0.98	0.26	0.89	0.97

⇒ Use kinematic edges for parameter determination instead of masses

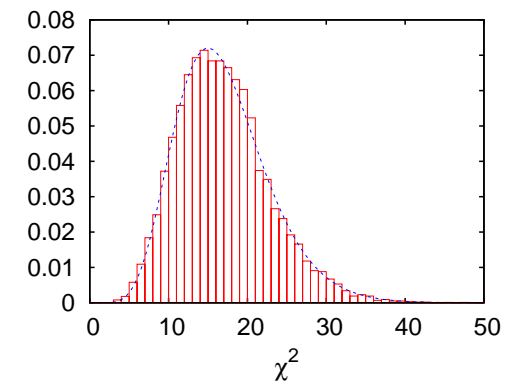
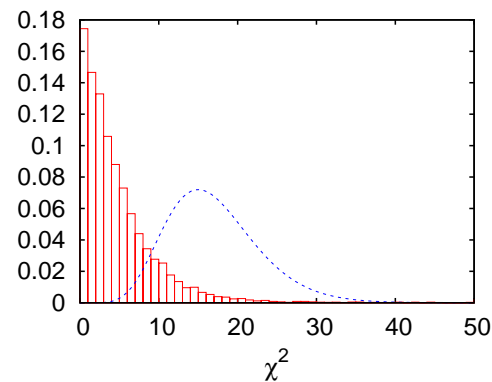
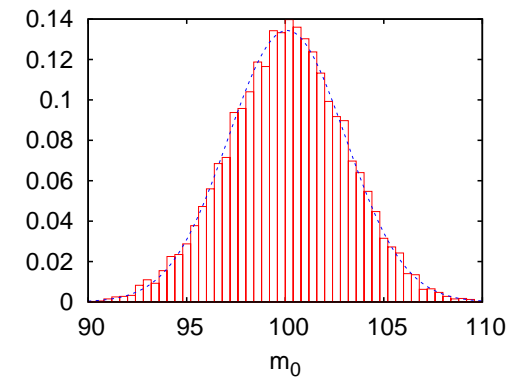
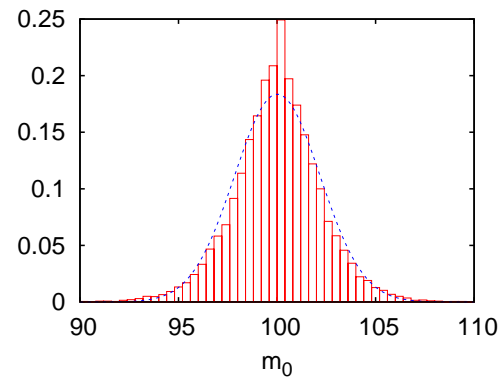
Error determination

Minuit output not usable for flat theory errors:

- Migrad function depends on parabolic approximation
- Cannot determine $\Delta\chi^2$ for Minos to yield 68% CL intervals

⇒ Need more general approach

- Perform 10,000 toy experiments with measurements smeared around correct value
- Minimise each toy experiment
- Plot resulting distribution of parameter points and fit with Gaussian

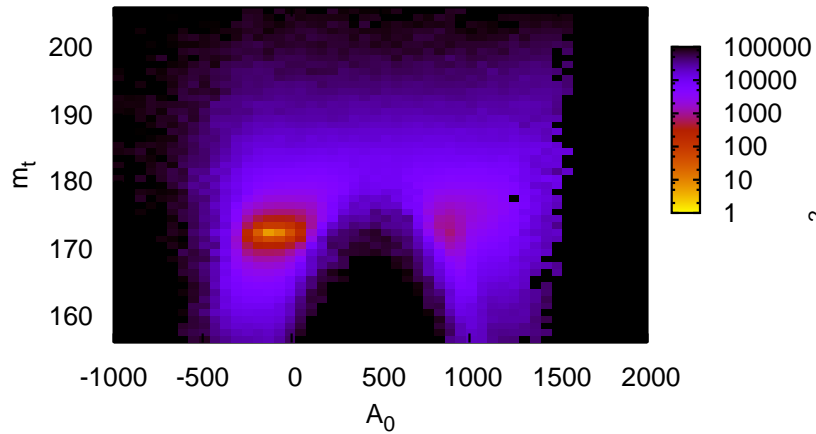


Flat theory errors

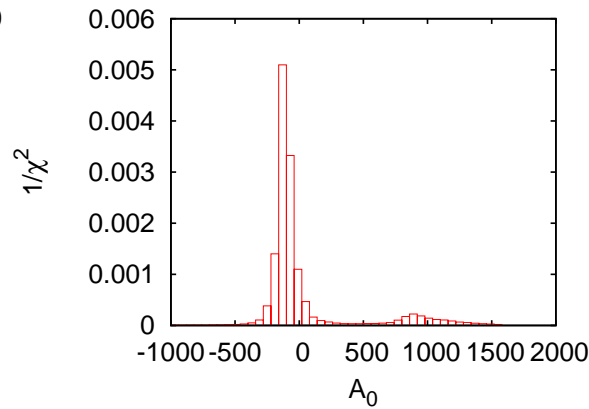
Gaussian theory errors

mSUGRA around Minima – positive μ

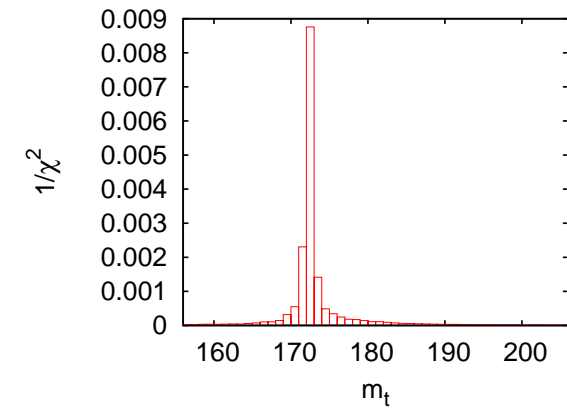
Bayesian



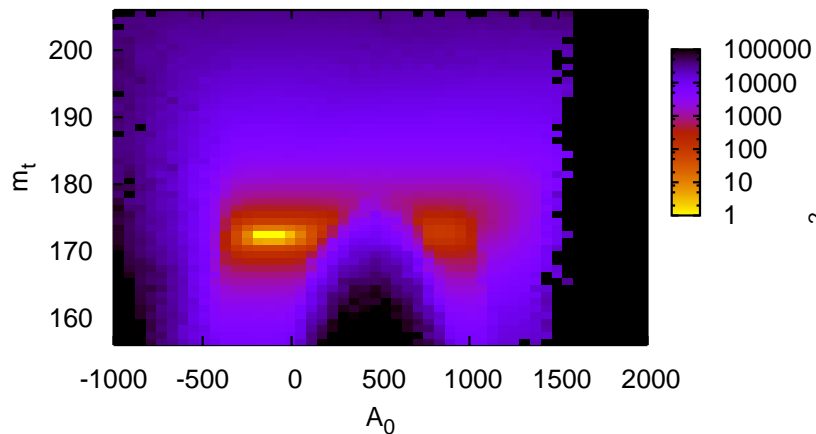
A_0



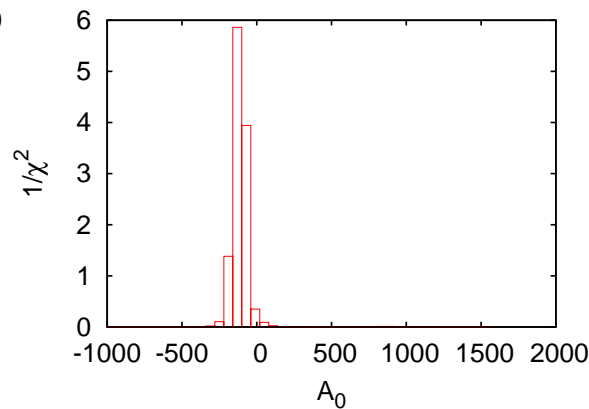
m_t



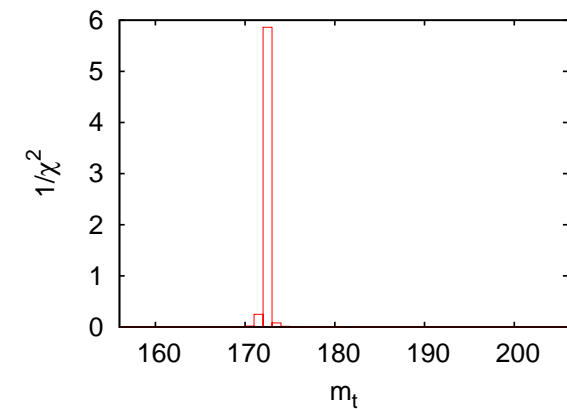
Frequentist



A_0

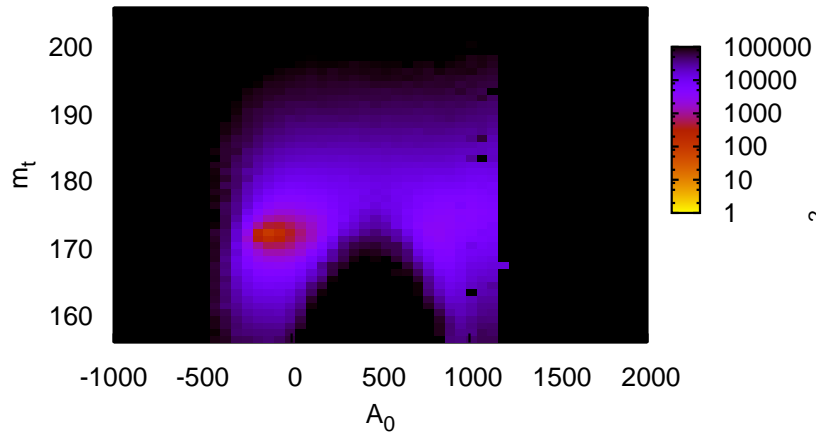


m_t

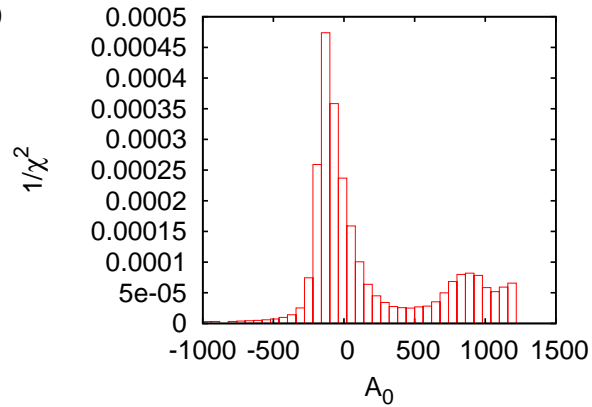


mSUGRA around Minima – negative μ

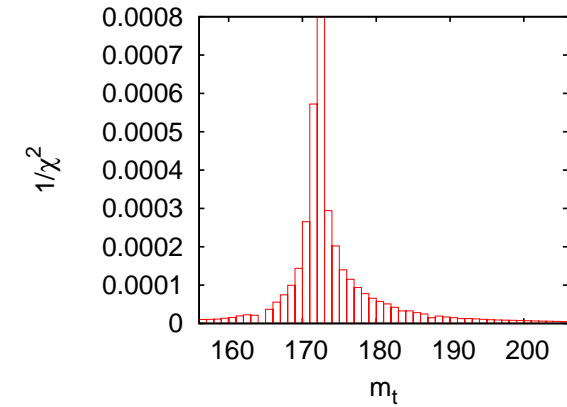
Bayesian



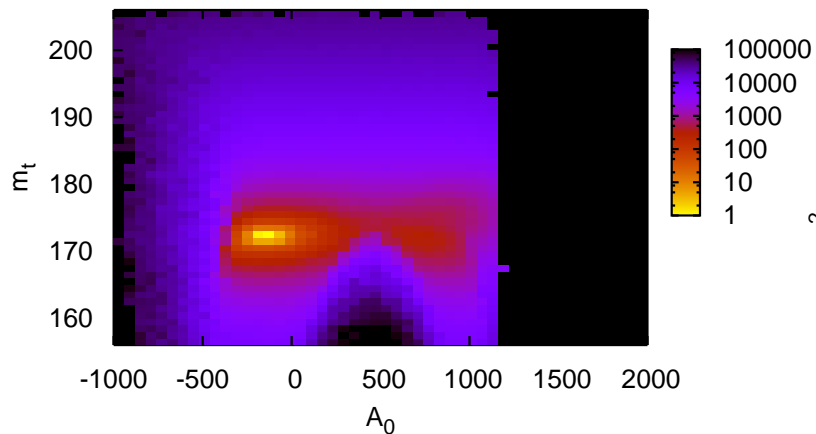
A_0



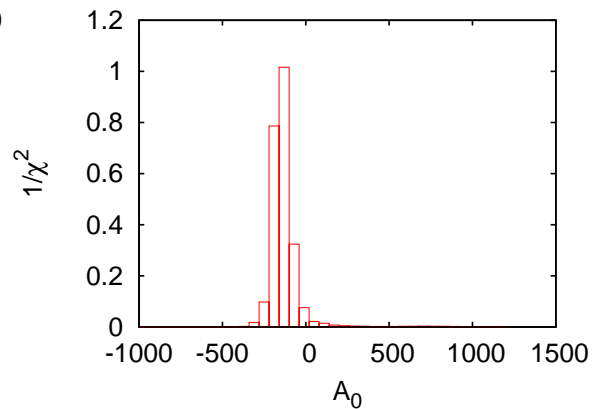
m_t



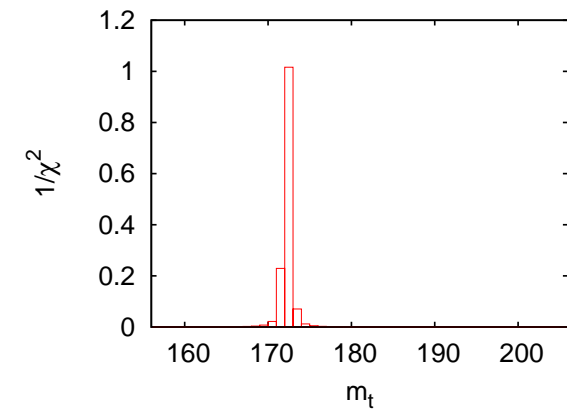
Frequentist



A_0



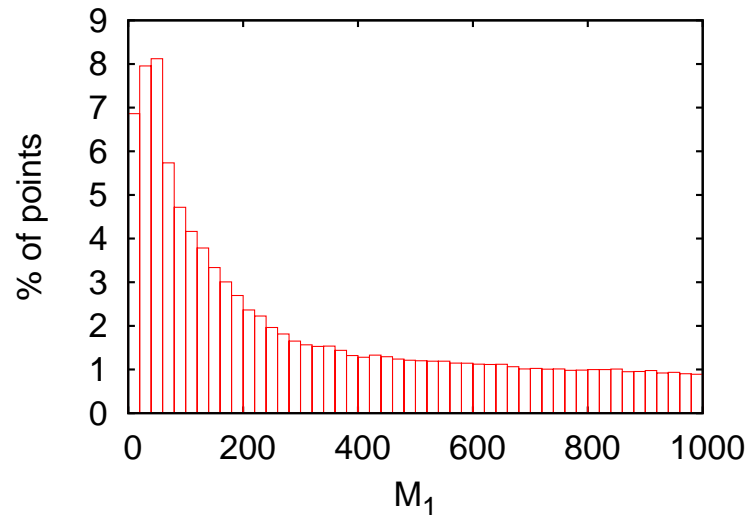
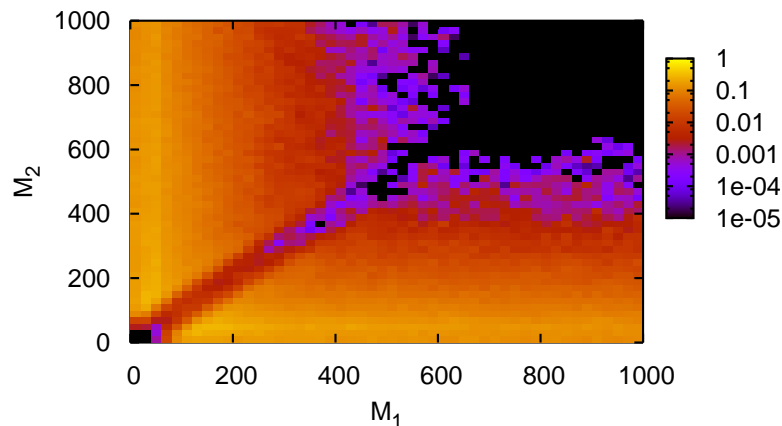
m_t



Weak-scale MSSM

- No need to assume specific SUSY-breaking scenario
⇒ SUSY-breaking mechanism should be induced from data
- Use of Markov Chains makes scanning the 19-dimensional parameter space feasible
- Lack of sensitivity on one parameter does not slow down the scan
(no need to fix parameters)
- Same SFitter output as before: Minima list and Likelihood map

MSSM using SPS1a spectrum and LHC kinematic edges:
(Bayesian, full parameter space)

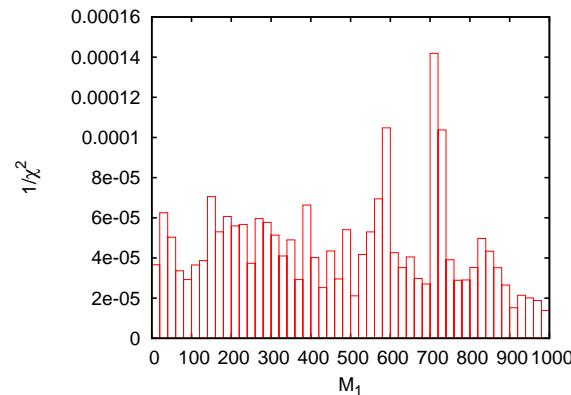
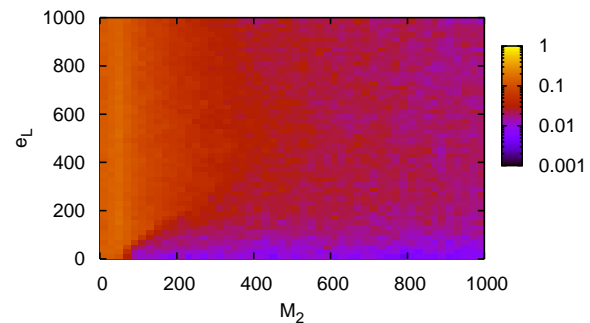
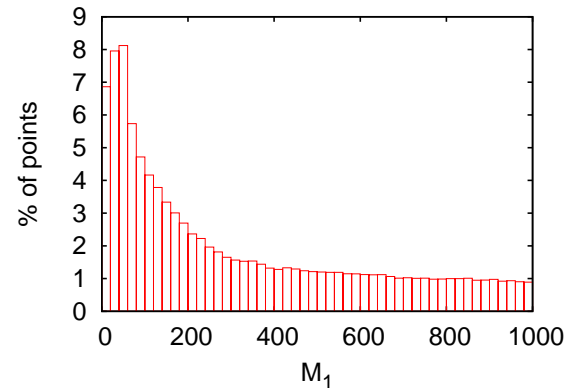
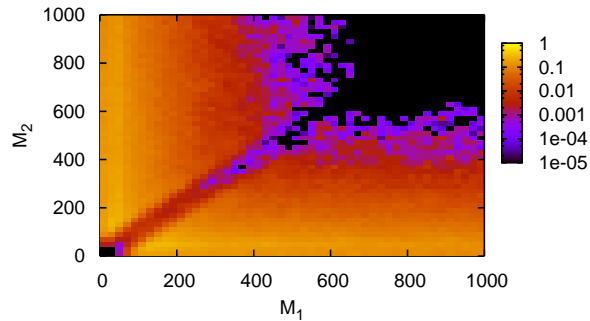


Search Strategy (1)

Full scan of 19D parameter space challenging

Four-step procedure yields better and faster results:

- Weighted-Markov-Chain run with flat pdf over full parameter space
- 5 best points additionally minimised
- (full scan, no bias on starting point)



Correlations not aligned with parameters \Rightarrow washed out in plots

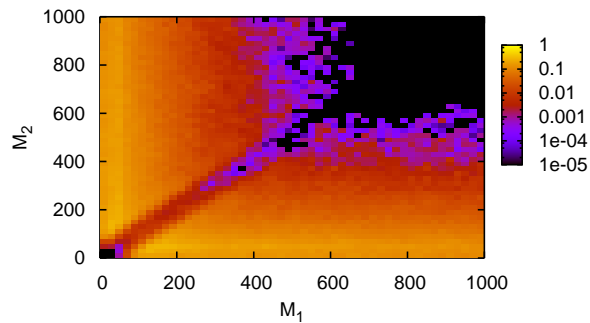
Search Strategy (2)

Full scan of 19D parameter space challenging

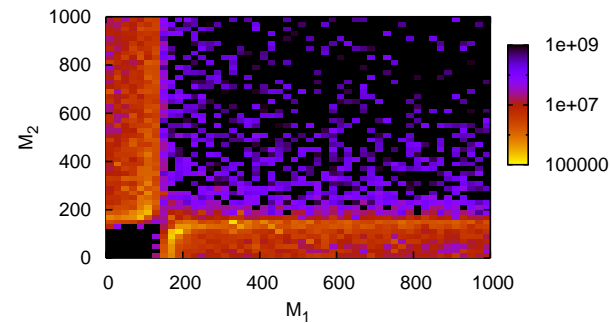
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- Weighted-Markov Chain with flat pdf on Gaugino-Higgsino subspace:
 $M_1, M_2, M_3, \mu, \tan \beta, m_t$
Additional Minuit run with 15 best solutions

Step 1

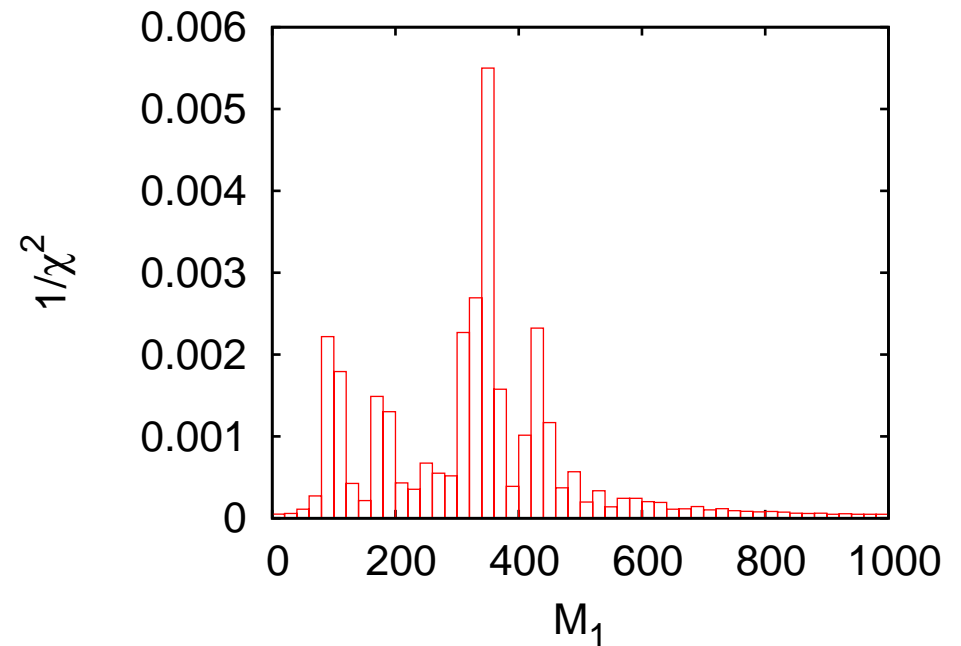
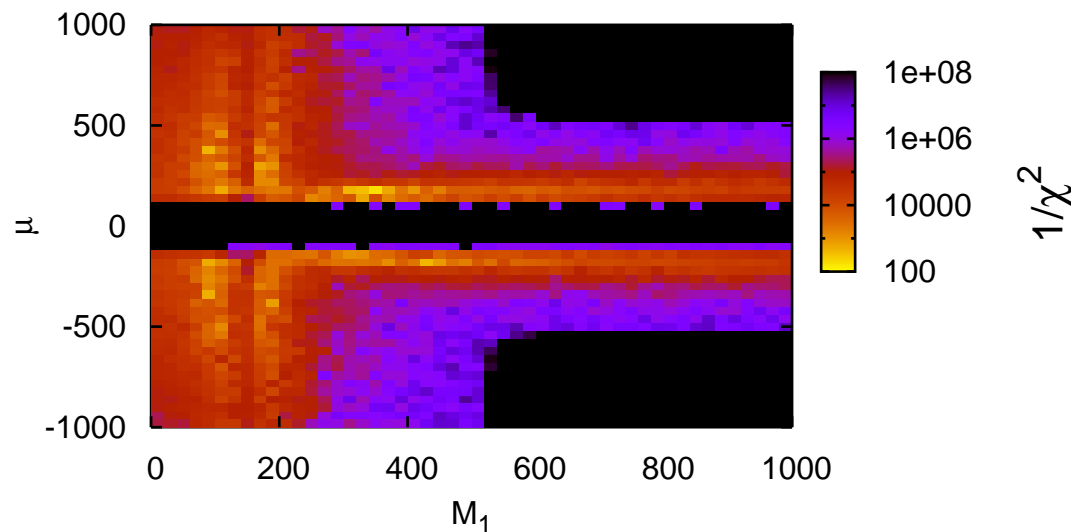


Step 2



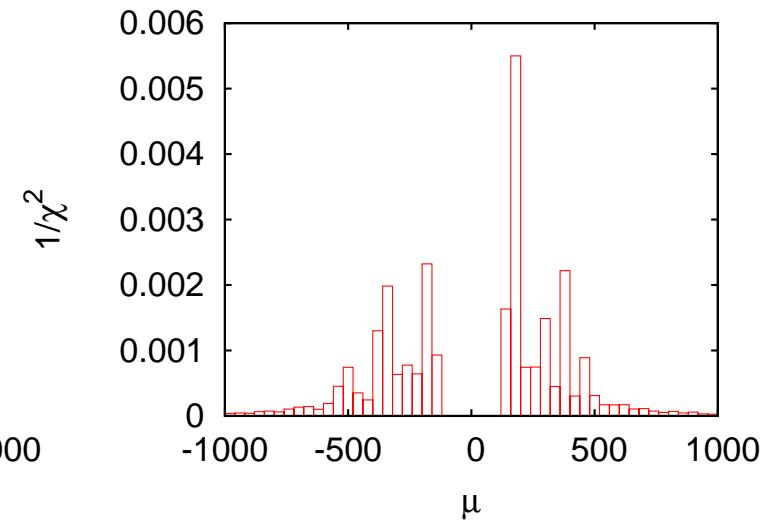
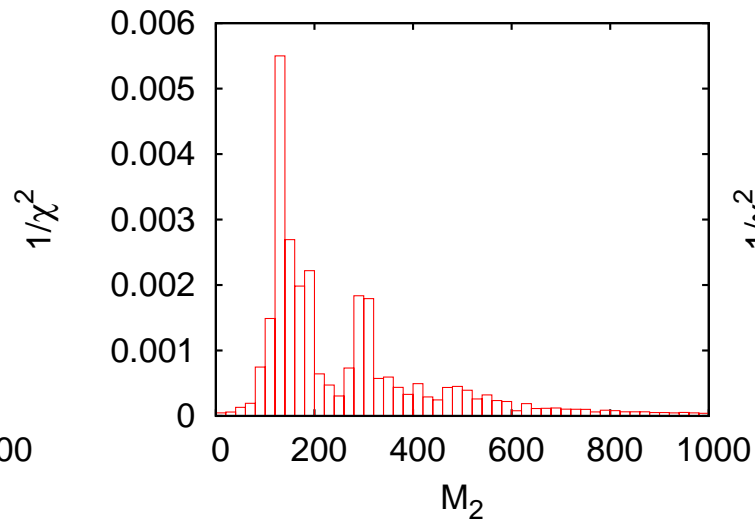
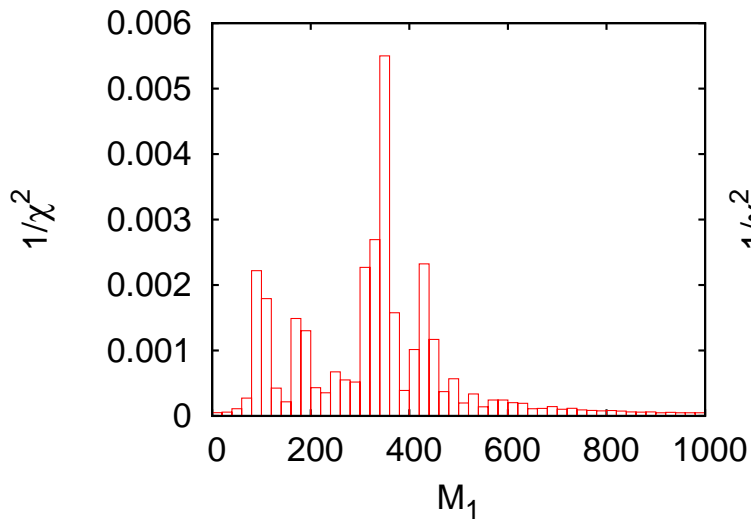
Search Strategy (2) - results

- Only three neutralinos ($\chi_1^0, \chi_2^0, \chi_4^0$) with masses (97.2 GeV, 180.5 GeV, 375.6 GeV) and no charginos observable at the LHC in SPS1a
- \Rightarrow Mapping $(M_1, M_2, \mu) \rightarrow (\chi_1^0, \chi_2^0, \chi_4^0)$ not unique
- $\text{sgn } \mu$ basically undetermined by collider data
- \Rightarrow 8-fold solution



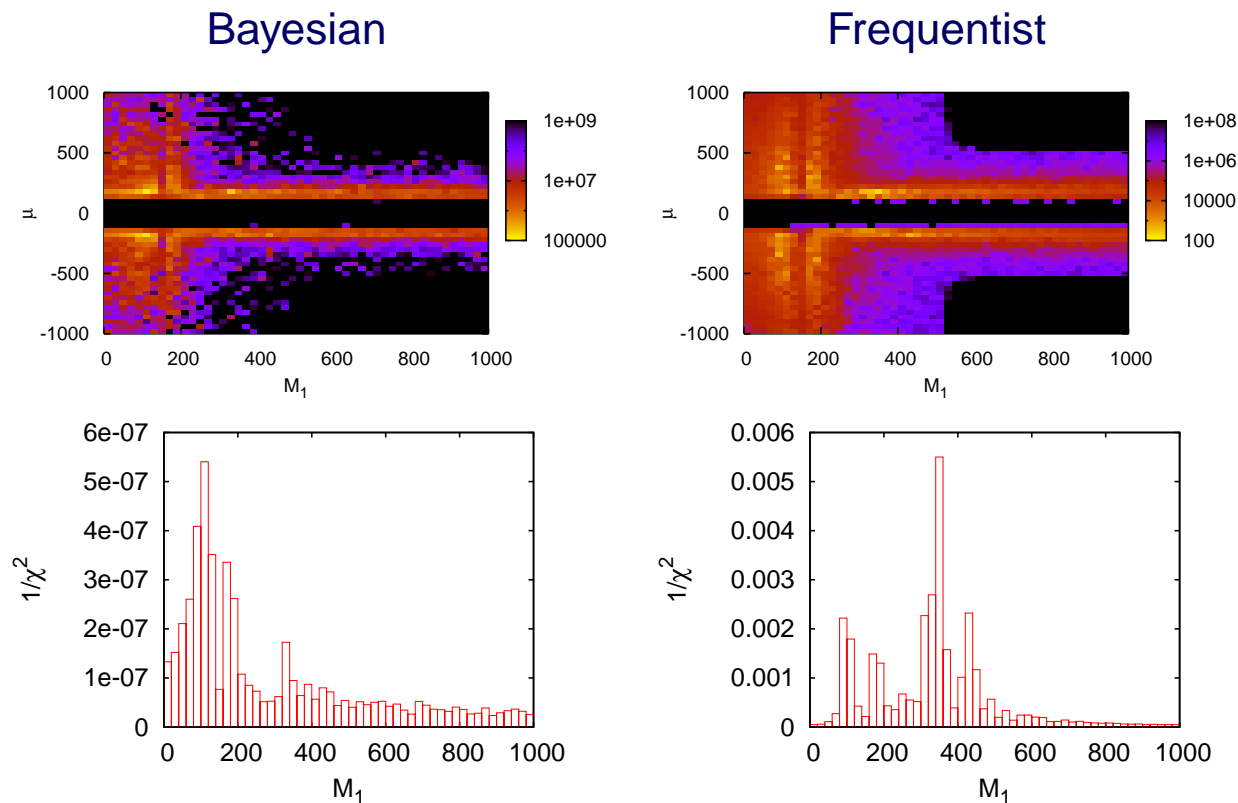
Search Strategy (2) - results

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Search Strategy (2) - results

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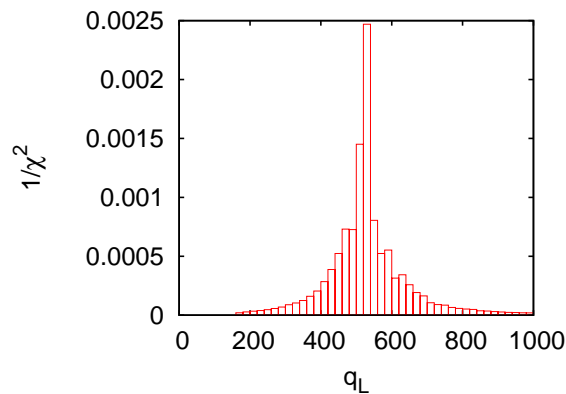
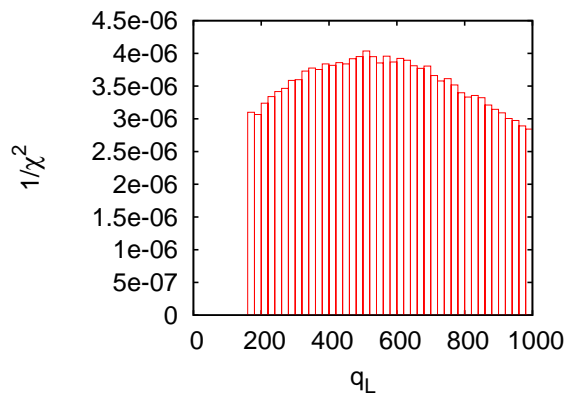
	$\mu < 0$				$\mu > 0$			
					SPS1a			
M_1	96.6	175.1	103.5	365.8	98.3	176.4	105.9	365.3
M_2	181.2	98.4	350.0	130.9	187.5	103.9	348.4	137.8
μ	-354.1	-357.6	-177.7	-159.9	347.8	352.6	178.0	161.5
$\tan \beta$	14.6	14.5	29.1	32.1	15.0	14.8	29.2	32.1
M_3	583.2	583.3	583.3	583.5	583.1	583.1	583.3	583.4
m_t	171.4	171.4	171.4	171.4	171.4	171.4	171.4	171.4

Search Strategy (3)

Full scan of 19D parameter space challenging

Four-step procedure yields better and faster results:

- Weighted-Markov-Chain run with flat pdf over full parameter space
5 best points additionally minimised
(full scan, no bias on starting point)
- Weighted-Markov Chain with flat pdf on Gaugino-Higgsino subspace:
 $M_1, M_2, M_3, \mu, \tan \beta, m_t$
Additional Minuit run with 15 best solutions
- Weighted-Markov Chain with Breit-Wigner-shaped pdf on remaining parameters for all solutions of previous step
Minimisation for best 5 points



Search Strategy (4)

Full scan of 19D parameter space challenging

Four-step procedure yields better and faster results:

- Weighted-Markov-Chain run with flat pdf over full parameter space
5 best points additionally minimised
(full scan, no bias on starting point)
- Weighted-Markov Chain with flat pdf on Gaugino-Higgsino subspace:
 $M_1, M_2, M_3, \mu, \tan \beta, m_t$
Additional Minuit run with 15 best solutions
- Weighted-Markov Chain with Breit-Wigner-shaped pdf on remaining parameters for all solutions of previous step
Minimisation for best 5 points
- Minuit run for best points of last step keeping all parameters variable

Best points

	$\mu < 0$				$\mu > 0$			
					SPS1a			
M_1	96.6	175.1	103.5	365.8	98.3	176.4	105.9	365.3
M_2	181.2	98.4	350.0	130.9	187.5	103.9	348.4	137.8
μ	-354.1	-357.6	-177.7	-159.9	347.8	352.6	178.0	161.5
$\tan \beta$	14.6	14.5	29.1	32.1	15.0	14.8	29.2	32.1
M_3	583.2	583.3	583.3	583.5	583.1	583.1	583.3	583.4
$M_{\tilde{\tau}_L}$	114.9	2704.3	128.3	4794.2	128.0	229.9	3269.3	118.6
$M_{\tilde{\tau}_R}$	348.8	129.9	1292.7	130.1	2266.5	138.5	129.9	255.1
$M_{\tilde{\mu}_L}$	192.7	192.7	192.7	192.9	192.6	192.6	192.7	192.8
$M_{\tilde{\mu}_R}$	131.1	131.1	131.1	131.3	131.0	131.0	131.1	131.2
$M_{\tilde{e}_L}$	186.3	186.4	186.4	186.5	186.2	186.2	186.4	186.4
$M_{\tilde{e}_R}$	131.5	131.5	131.6	131.7	131.4	131.4	131.5	131.6
$M_{\tilde{q}^3_L}$	497.1	497.2	494.1	494.0	495.6	495.6	495.8	495.0
$M_{\tilde{t}_R}$	1073.9	920.3	547.9	950.8	547.9	460.5	978.2	520.0
$M_{\tilde{b}_R}$	497.3	497.3	500.4	500.9	498.5	498.5	498.7	499.6
$M_{\tilde{q}_L}$	525.1	525.2	525.3	525.5	525.0	525.0	525.2	525.3
$M_{\tilde{q}_R}$	511.3	511.3	511.4	511.5	511.2	511.2	511.4	511.5
$A_t (-)$	-252.3	-348.4	-477.1	-259.0	-470.0	-484.3	-243.4	-465.7
$A_t (+)$	384.9	481.8	641.5	432.5	739.2	774.7	440.5	656.9
m_A	350.3	725.8	263.1	1020.0	171.6	156.5	897.6	256.1
m_t	171.4	171.4	171.4	171.4	171.4	171.4	171.4	171.4

Degenerate Solutions

- In total 19 parameters constrained by 22 measurements
- Measurements constructed from only 15 underlying masses
- \Rightarrow Complete determination of parameter set not possible
- Five parameters not well constrained
 - m_A
 - \leftarrow no heavy Higgses measurable
 - $M_{\tilde{t}_R}$
 - A_t
 - \leftarrow stop sector parameters do not enter edge measurements
 - $M_{\tilde{\tau}_L}$ or $M_{\tilde{\tau}_R}$
 - \leftarrow only the lighter stau measured
 - $\tan \beta$
 - \leftarrow change can always be accommodated by rotating $M_1, M_2, M_{\tilde{q}_3}, \dots$
- Single common link: m_{h^0}
- \Rightarrow 4-dimensional hyperplane in parameter space undetermined
- Can still assign errors to some of the badly determined parameters

Error analysis

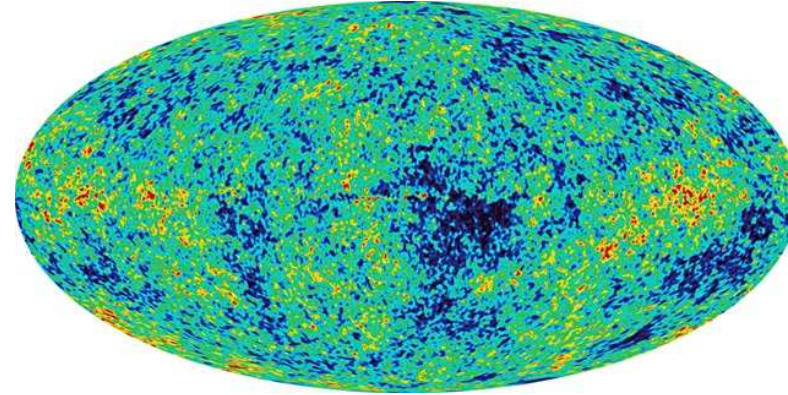
- Technical procedure as in mSUGRA case:
 - 10000 smeared data sets
 - Minimum determined for each data set individually
 - Error determined from fit with Gaussian
- Most constrained parameters determinable with $\sim 5\%$ accuracy
- Inclusion of theory errors leads to an increase of factor 2 on the parameter errors
- ILC data complementary to LHC
- Combination of the two experiments allows for precise determination of all parameters

	LHC		ILC		LHC+ILC	SPS1a
M_1	102.1 ± 7.8		103.0 ± 1.1		103.1 ± 0.84	103.1
$M_{\tilde{e}_R}$	135.0 ± 8.3		135.8 ± 0.81		135.9 ± 0.77	135.8
m_A	$406.3 \pm \mathcal{O}(10^3)$		393.8 ± 1.6		393.9 ± 1.6	394.9
$M_{\tilde{t}_R}$	$415.8 \pm \mathcal{O}(10^2)$		$440.0 \pm \mathcal{O}(4 \cdot 10^2)$		410.7 ± 48.4	408.3

Dark Matter

Content of the universe:

- 73% Dark energy
- 4% Ordinary matter
- 23% Dark matter



[NASA/WMAP
Science Team]

MSSM: χ_1^0 as LSP ideal candidate for cold dark matter (CDM): massive, weakly interacting

- SFitter: Determine Lagrangian parameters \Rightarrow Spectrum and couplings
- e.g. micrOMEGAs: Calculate relic density $\Omega_{\text{CDM}} h^2 = n_{\text{LSP}} m_{\text{LSP}}$

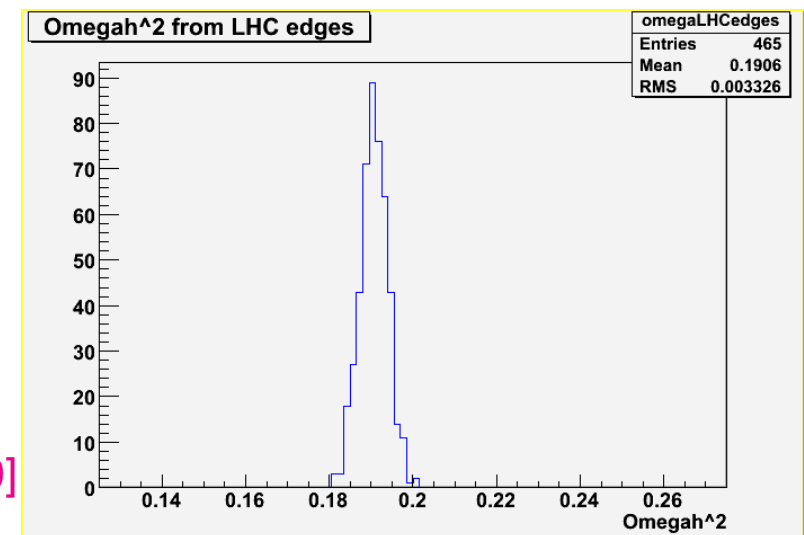
[Bélanger et al.]

- \Rightarrow Prediction of $\Omega_{\text{CDM}} h^2$
LHC : $\Omega_{\text{CDM}} h^2 = 0.1906 \pm 0.0033$
LHC+ILC: $\Omega_{\text{CDM}} h^2 = 0.1910 \pm 0.0003$
(improvement by one order of magnitude)

- Compare with experiment
(Measurement of the fluctuations of the cosmic microwave background):

WMAP: $\Omega_{\text{CDM}} h^2 = 0.1277 \pm 0.008$ [[astro-ph/0603449](#)]

Planck: $\Omega_{\text{CDM}} h^2 = ? \pm 0.0016$

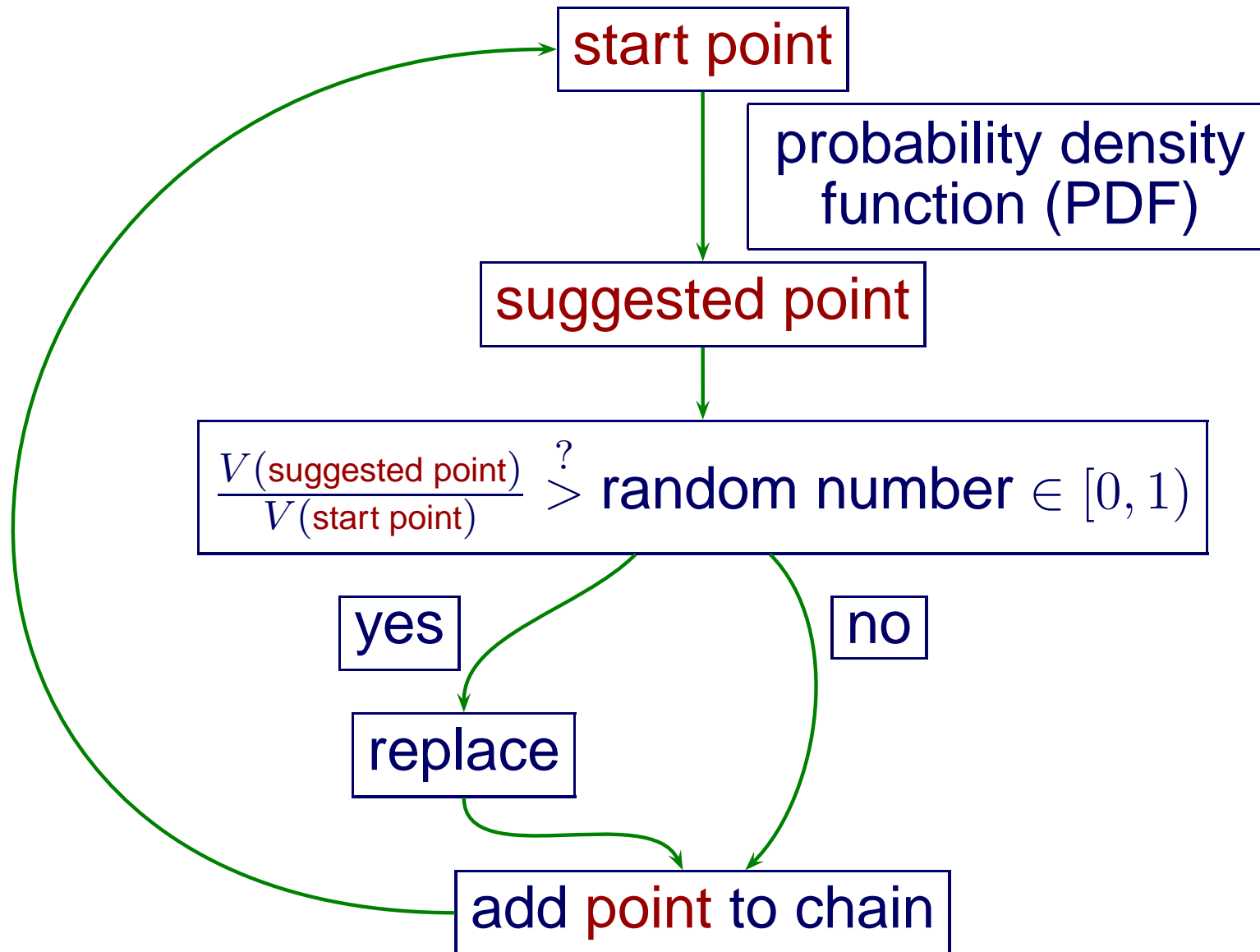


Summary & Outlook

- Current status: Low-energy supersymmetry fits precision data very well
- Parameter scans important to determine Lagrangian parameters from observables
- Problem of high-dimensional parameter spaces
- Markov Chains can do this effectively
- Improved algorithm developed
- Two types of output: Likelihood map and list of best points
- Both Bayesian and Frequentist from likelihood map
- Bayesian output significantly dependent on priors
- Tested with mSUGRA SPS1a:
can reconstruct SPS1a from (simulated) LHC data
- Repeated procedure with weak-scale MSSM:
reconstruction works as well
- SFitter (despite its name) not tied to SUSY
→ extend to other models/problems

Backup Slides

Metropolis-Hastings Algorithm



Experimental Input (edges)

(Obs)	= (meas) \pm (exp) \pm (theo)		
m_{h^0}	= 109.53 \pm 0.25 \pm 2.0		
m_t	= 171.4 \pm 1.0 \pm 0.0		
$\Delta m_{\tilde{\mu}_L, \chi_1^0}$	= 106.26 \pm 1.6 \pm 0.1		
$\Delta m_{\tilde{g}, \chi_1^0}$	= 509.96 \pm 2.3 \pm 6.0		
$\Delta m_{\tilde{c}_R, \chi_1^0}$	= 450.52 \pm 10.0 \pm 4.2		
$\Delta m_{\tilde{g}, \tilde{b}_1}$	= 98.971 \pm 1.5 \pm 1.0		
$\Delta m_{\tilde{g}, \tilde{b}_2}$	= 64.016 \pm 2.5 \pm 0.7		
Edge($\chi_2^0, \tilde{\mu}_R, \chi_1^0$)	= 79.757 \pm 0.03 \pm 0.08	(m_{ll}^{\max})	
Edge($\tilde{c}_L, \chi_2^0, \chi_1^0$)	= 446.44 \pm 1.4 \pm 4.3	(m_{llq}^{\max})	
Edge($\tilde{c}_L, \chi_2^0, \tilde{\mu}_R$)	= 316.51 \pm 0.9 \pm 3.0	(m_{lq}^{low})	
Edge($\tilde{c}_L, \chi_2^0, \tilde{\mu}_R, \chi_1^0$)	= 392.8 \pm 1.0 \pm 3.8	(m_{lq}^{high})	
Edge($\chi_4^0, \tilde{\mu}_R, \chi_1^0$)	= 257.41 \pm 2.3 \pm 0.3	$(m_{ll}^{\max}(\chi_4^0))$	
Edge($\chi_4^0, \tilde{\tau}_L, \chi_1^0$)	= 82.993 \pm 5.0 \pm 0.8	$(m_{\tau\tau}^{\max})$	
Threshold($\tilde{c}_L, \chi_2^0, \tilde{\mu}_R, \chi_1^0$)	= 211.95 \pm 1.6 \pm 2.0	(m_{llq}^{\min})	
Threshold($\tilde{b}_1, \chi_2^0, \tilde{\mu}_R, \chi_1^0$)	= 211.95 \pm 1.6 \pm 2.0	(m_{llb}^{\min})	

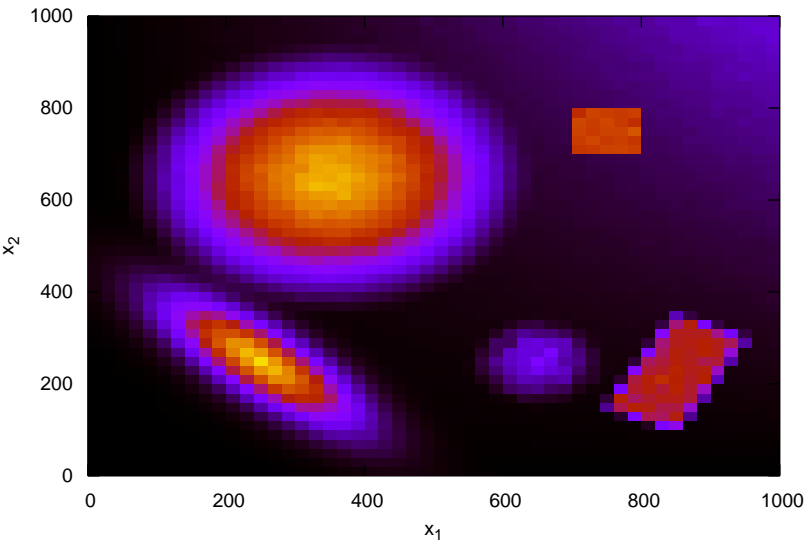
MSSM errors

	LHC		ILC		LHC+ILC		SPS1a
$\tan \beta$	10.0 ± 4.5		13.4 ± 6.8		12.3 ± 5.3		10.0
M_1	102.1 ± 7.8		103.0 ± 1.1		103.1 ± 0.84		103.1
M_2	193.3 ± 7.8		193.4 ± 3.1		193.2 ± 2.3		192.9
M_3	577.2 ± 14.5		fixed 500		579.7 ± 12.8		577.9
$M_{\tilde{\tau}_L}$	$227.8 \pm \mathcal{O}(10^3)$		183.8 ± 16.6		187.3 ± 12.9		193.6
$M_{\tilde{\tau}_R}$	$164.1 \pm \mathcal{O}(10^3)$		143.9 ± 17.9		140.1 ± 14.1		133.4
$M_{\tilde{\mu}_L}$	193.2 ± 8.8		194.4 ± 1.1		194.5 ± 1.0		194.4
$M_{\tilde{\mu}_R}$	135.0 ± 8.3		135.9 ± 1.0		136.0 ± 0.89		135.8
$M_{\tilde{e}_L}$	193.3 ± 8.8		194.4 ± 0.89		194.4 ± 0.84		194.4
$M_{\tilde{e}_R}$	135.0 ± 8.3		135.8 ± 0.81		135.9 ± 0.77		135.8
$M_{\tilde{q}^3_L}$	481.4 ± 22.0		$507.2 \pm \mathcal{O}(4 \cdot 10^2)$		486.6 ± 19.5		480.8
$M_{\tilde{t}_R}$	$415.8 \pm \mathcal{O}(10^2)$		$440.0 \pm \mathcal{O}(4 \cdot 10^2)$		410.7 ± 48.4		408.3
$M_{\tilde{b}_R}$	501.7 ± 17.9		fixed 500		504.0 ± 17.4		502.9
$M_{\tilde{q}_L}$	524.6 ± 14.5		fixed 500		526.1 ± 7.2		526.6
$M_{\tilde{q}_R}$	507.3 ± 17.5		fixed 500		508.4 ± 16.7		508.1
A_τ	fixed 0		$633.2 \pm \mathcal{O}(10^4)$		$139.6 \pm \mathcal{O}(10^4)$		-249.4
A_t	-509.1 ± 86.7		$-516.1 \pm \mathcal{O}(10^3)$		-500.1 ± 143.4		-490.9
A_b	fixed 0		fixed 0		$-686.2 \pm \mathcal{O}(10^4)$		-763.4
m_A	$406.3 \pm \mathcal{O}(10^3)$		393.8 ± 1.6		393.9 ± 1.6		394.9
μ	350.5 ± 14.5		343.7 ± 3.1		354.8 ± 2.8		353.7
m_t	171.4 ± 1.0		171.4 ± 0.12		171.4 ± 0.12		171.4

Example

Test function (5-dim):

- Small Hypersphere $r = 100$, $V_{\max} = 75$ @ (650, 250, 350, 350, 350)
- Cuboid $d = (173, 120, 200, 200, 200)$, $V_{\max} = 60$ @ (850, 225, 650, 650, 650)
- Cube $d = (100, 100, 300, 300, 300)$, $V_{\max} = 25$ @ (750, 750, 450, 450, 450)
- Gaussian $\sigma = (50, 150, 150, 150, 150)$, $V_{\max} = 16$ @ (250, 250, 550, 550, 550)
- Big Hypersphere $r = 300$, $V_{\max} = 12$ @ (350, 650, 650, 650, 650)
- Background $V = 0.1 + 4 \cdot 10^{-30} \cdot x_1^2 x_2^2 x_3^2 x_4^2 x_5^2$



1. $V=74.929$ @(655.00, 253.72, 347.83, 348.57, 349.59)
2. $V=59.972$ @(850.04, 224.99, 650.00, 649.99, 654.56)
3. $V=58.219$ @(849.97, 225.01, 587.08, 650.01, 650.02)
4. $V=25.110$ @(750.00, 749.99, 450.00, 450.01, 450.01)
5. $V=16.042$ @(245.45, 253.44, 552.51, 542.58, 544.75)
6. $V=12.116$ @(350.70, 650.40, 650.36, 650.40, 650.38)
7. ...

Plot Details

- Parameters: $x_1, \dots, x_5 \in [0, 1000]$
- Bins: 50×50
- PDF: Breit-Wigner ($\frac{1}{1+\Delta x_i^2/\sigma^2}$) with $\sigma = 100$
- Number of Markov chains: 9
- Number of points per chain: 10^7
- Number of function evaluations: 33,797,153
- Acceptance ratio: 0.19
- Final r (measure of convergence): 1.815
- CPU time (3 GHz): 150 min