Reconstructing the Supersymmetric Lagrangian

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arXiv:0709.3985 [hep-ph]

https://trac.lal.in2p3.fr/SFitter

Outline

- Supersymmetry
- Current Status
- Determining SUSY Parameters
- Kinematic Edges as Experimental Input
- Possible Collider Results
- Dark Matter

Supersymmetry

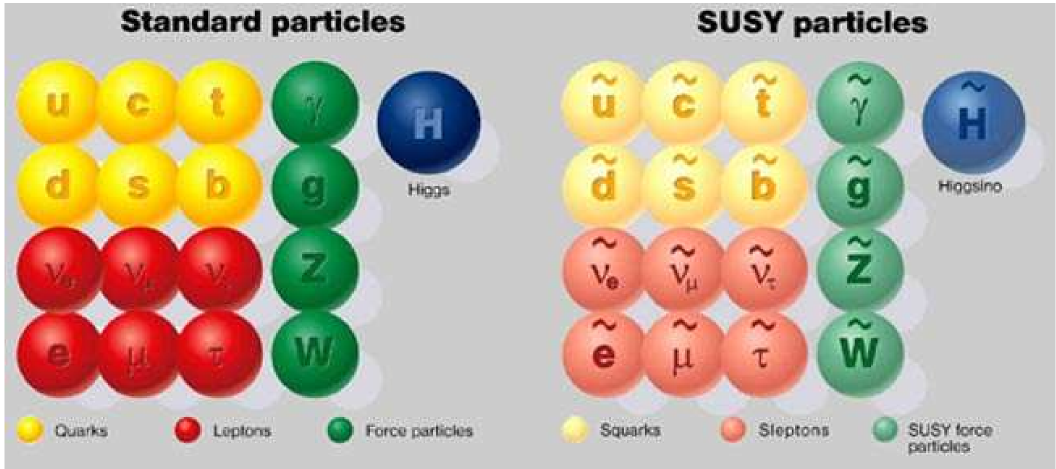
Symmetry between bosons and fermions:

$$Q\,|\text{boson}\rangle = |\text{fermion}\rangle\,; \qquad Q\,|\text{fermion}\rangle = |\text{boson}\rangle$$

Q: Supersymmetry Operator

Simplest model:
Minimal Supersymmetric Standard Model (MSSM)

- Supersymmetric partner to each Standard Model particle
- Two Higgs doublets $\Rightarrow$ 5 Higgs bosons (h^0,H^0,A^0,$H^\pm$)
- Particles with same quantum numbers mix
  (e.g. Zino, Photino, 2 Higgsino $\rightarrow$ 4 Neutralino)



mSUGRA

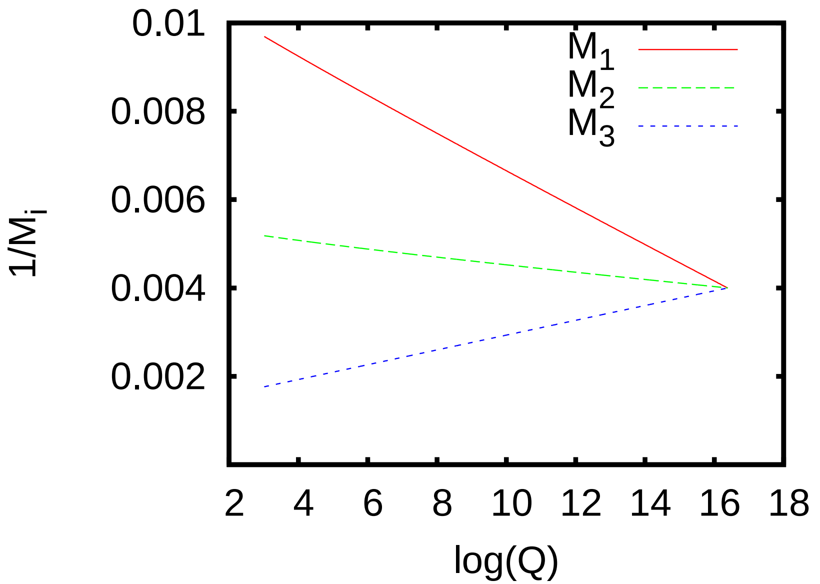
Unification at the GUT scale ($\sim 10^{16}$ GeV):

- Apparent unification of gauge couplings (general feature of MSSM)
- Common scalar mass: m_0
- Common sfermion mass: $m_{1/2}$
- Common trilinear coupling: A_0

plus

- ratio of the Higgs vacuum expectation values at the electro-weak scale: $\tan\beta = \frac{v_2}{v_1}$
- one sign: $\mathrm{sgn}\,\mu$

Evolve three parameters defined at the GUT scale via renormalisation group equations down to electro-weak scale:
$\Rightarrow$ Weak-scale MSSM parameters
$\Rightarrow$ Masses and couplings



Current Status

- Standard Model experimentally very well confirmed

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  - Dark matter

    $\sim 23\%$ Dark Matter content in the universe

    Possible candidate in the SM: Neutrinos

    $\leftrightarrow$ neutrino mass limits prevent accounting for total content
  - M_W

    $\sim 1\sigma$ deviation
  - LEP

    2.3σ excess in Higgs-like events near 98 GeV
  - $g-2$ of the Muon

    3.4σ deviation

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    $\rightarrow$ GUT scale (?))
- Look for possible ultra-violet completions $\rightarrow$ Supersymmetry

Determining SUSY parameters

nowadays:

Parameters in the Lagrangian

m_0, μ, $\tan(\beta)$, $M_{\{1,2,3\}}$, ...

Feynman diagrams, RG evolution, ...

Observables:

- Masses
- Kinematic endpoints
- Cross sections
- Branching ratios
- ...

after SUSY discovery:

Observables

m_{h^0}, $\Delta m_{\tilde{g}\chi_1^0}$, three-particle edge($\chi_4^0$,$\tilde{e}_L$,$\chi_1^0$), BR, ...

?

Lagrangian parameters

M_1	$\square \pm \square$ GeV
M_2	$\square \pm \square$ GeV
M_3	$\square \pm \square$ GeV
μ	$\square \pm \square$ GeV
$\tan\beta$	$\square \pm \square$
...	...

$\Rightarrow$ Tools to reconstruct SUSY parameters

Current Fits

$\longrightarrow$ Fits of current data to supersymmetry (only mSUGRA)

[Allanach, Cranmer, Lester, Weber 2005-8]

[Roszkowski, Ruiz de Austra, Trotta 2006-8]

[Buchmüller, Cavanaugh, De Roeck, Heinemeyer, Isidori, Olive, Paradisi, Ronga, Weber, Weiglein 2007/8]

Observables:

- Dark Matter $\Omega_{\rm DM} h^2$
- $g - 2_\mu$
- M_W
- $\sin^2\theta_W$
- BR$(b \to s\gamma)$
- BR$(B_s \to \mu^+\mu^-)$
- $\ldots$

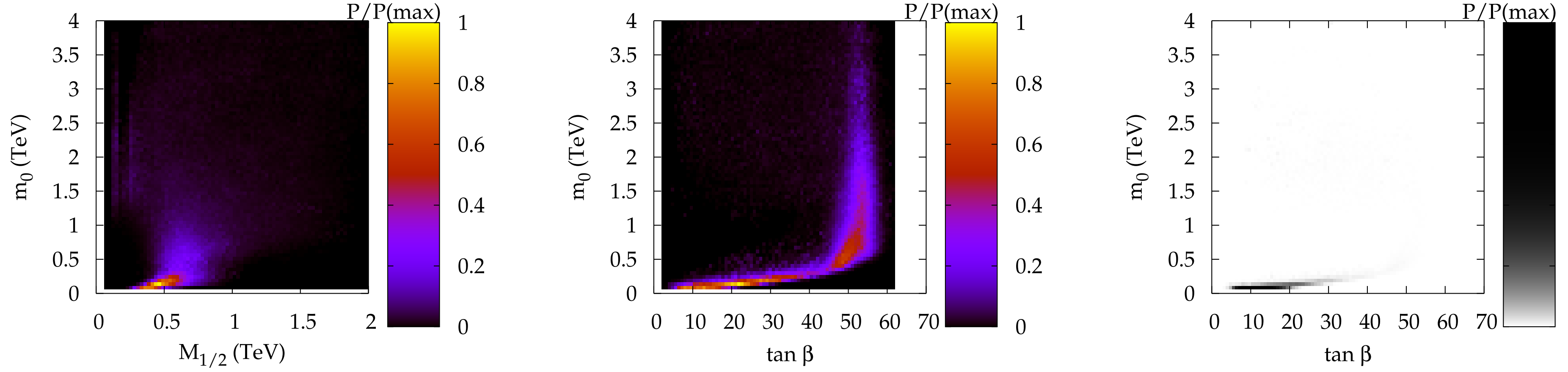
$\Rightarrow$ Predictions for SUSY mass spectrum

Current Fits (2)

Predictions for SUSY mass spectrum [plots by Allanach et al.]

Bayesian:

(flat $\tan\beta$ prior) (REWSB+same order prior)



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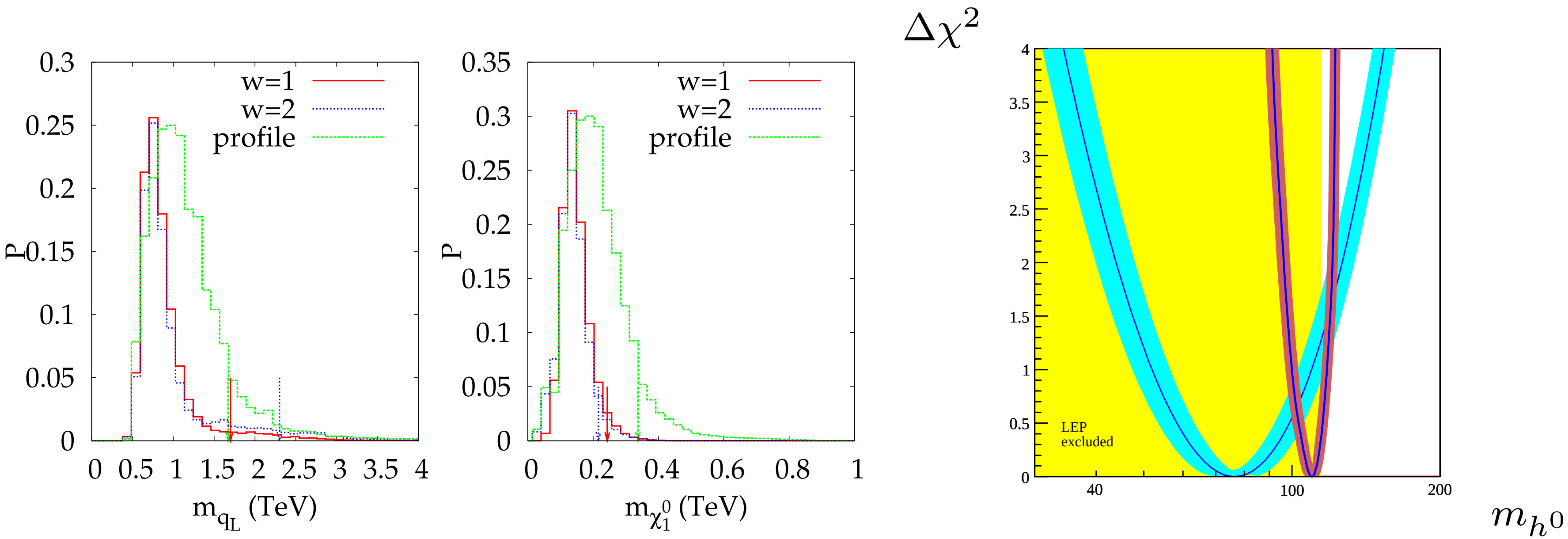

[right-hand plot by Buchmüller et al.]

- Low-energy TeV-scale SUSY fits data very well
- $\Rightarrow$ Mass ranges for SUSY particles
- Mass of the lightest Higgs boson compatible with LEP limit

Discovery of SUSY at the LHC
$\Rightarrow$ Additional observables from collider data
$\Rightarrow$ SFitter [Lafaye, Plehn, MR, Zerwas]
(or Fittino) [Bechtle, Desch, Wienemann]
(or GFitter) [Flächer, Goebel, Haller, Höcker, Mönig, Stelzer]

What SFitter does

- Set of measurements
  - LHC measurements:
    kinematic edges, thresholds, masses, mass differences
    cross sections, branching ratios
  - ILC measurements
  - Indirect Constraints
    electro-weak: M_W, $\sin^2\theta_W$; $(g-2)_\mu$
    flavour: BR$(b \to s\gamma)$, BR$(B_s \to \mu^+\mu^-)$; dark matter: Ωh^2
  - or even ATLAS and CMS measurements separately
- Compare to theoretical predictions
  - Spectrum calculators: SoftSUSY, SuSPECT, ISASUSY
    [Allanach; Djouadi, Kneur, Moultaka; Baer, Paige, Protopopescu, Tata]
  - LHC cross sections: Prospino2 [Plehn et al.]
  - LC cross sections: MsmLib [Ganis]
  - Branching Ratios: SUSYHit (HDecay + SDecay) [Djouadi, Mühlleitner, Spira]
  - micrOMEGAs [Bélanger, Boudjema, Pukhov, Semenov]
  - g-2 [Alexander, Stöckinger]
- Using as glue: SLHAio [Kreiss]

Parameter Scans

- MSSM parameter space is high-dimensional:
  - SM: 3+ parameters (m_t, α_s, α, . . .)
  - mSUGRA: 5 parameters (m_0,$m_{1/2}, A_0, \tan(\beta), \mathrm{sgn}(\mu)$)
  - General MSSM: 105 parameters
- On loop-level observables depend on every parameter
  Simple inversion of the relations not possible
  $\Rightarrow$ Parameter scans
- Error estimates on parameters in the minimum

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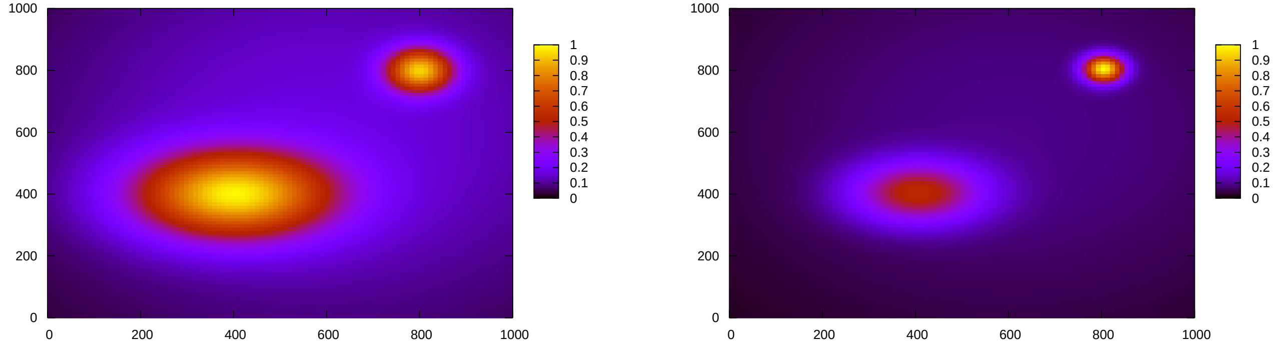
Find best points (best χ^2) using different fitting techniques:

- Gradient search (Minuit) (+ Reasonably fast; – Limited convergence, only best fit)



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- Weighted Markov Chains

Markov Chains

Markov Chain (MC):

- Sequence of points, chosen by an algorithm (Metropolis-Hastings), only depending on its direct predecessor
- Picks a set of ”typical” points according to a potential V (e.g. inverse log-likelihood, $1/\chi^2$)
- Point density resembles the value of V (i.e. more points in region with high V)
- Scans high dimensional parameter spaces efficiently [Baltz, Gondolo 2004]
- mSUGRA MC scans with current exp. limits

[Allanach, Cranmer, Lester, Weber 2005-8; Roszkowski, Ruiz de Austra, Trotta 2006-8]

Weighted Markov Chains

Weighted Markov Chains:

Improved evaluation algorithm for binning: [Plehn, MR]

- Weight points with value of V:
- Take care of
  - Overcounting because point density is already weighted ($\frac{\text{number of points}}{\sum_{\text{points}} 1/V(\text{point})}$)
    [based on Ferrenberg, Swendsen 1988]
  - Correct account for regions with zero probability
    (maintain additional chain which stores points rejected because $V(\text{point}) = 0$)

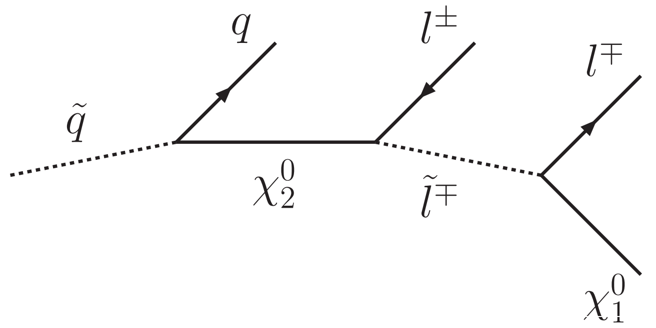
$+$ Fast scans of high-dimensional spaces $\mathcal{O}(N)$
$+$ Does not rely on shape of χ^2 (no derivatives used)
$+$ Can find secondary distinct solutions
$-$ Exact minimum difficult to find $\Rightarrow$ Additional gradient fit
$-$ Bad choice of proposal function for next point leads to bad coverage of the space

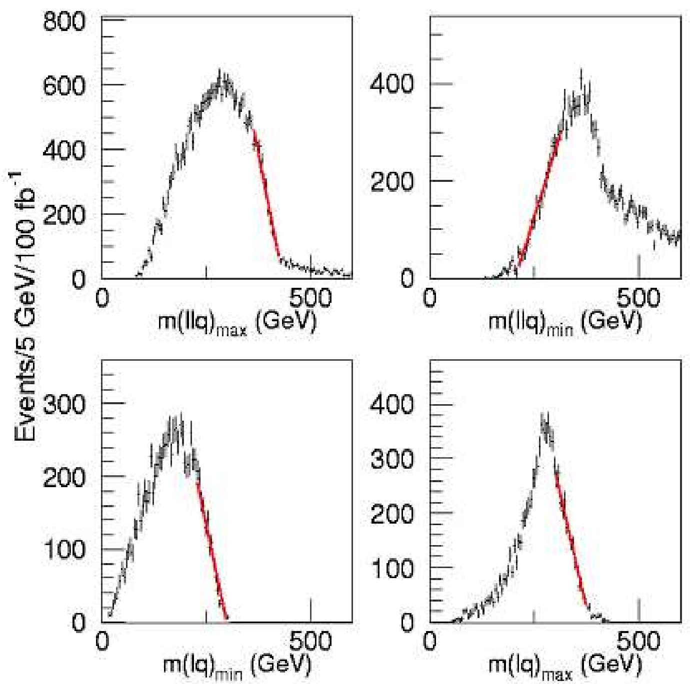
Experimental Input (Edges)

mSUGRA SPS1a as a benchmark point:

$m_0 = 100$ GeV , $m_{1/2} = 250$ GeV , $A_0 = -100$ GeV , $\tan\beta = 10$, $\text{sgn}\,\mu = +1$, $m_t = 171.4$ GeV

LHC “experimental” data from cascade decays (best precision obtainable)





Measurement	Value (GeV)	Errors (GeV) (stat)	(syst)
$(m_{llq}^{\max})$:Edge($\tilde{q}_L$,χ_2^0,χ_1^0)	446.44	1.4	4.3
$(m_{llq}^{\min})$:Thres($\tilde{q}_L$,χ_2^0,$\tilde{\mu}_R$,χ_1^0)	211.95	1.6	2.0
(m_{lq}^{low}) :Edge($\tilde{c}_L$,χ_2^0,$\tilde{\mu}_R$)	316.51	0.9	3.0
(m_{lq}^{high}):Edge($\tilde{c}_L$,χ_2^0,$\tilde{\mu}_R$,χ_1^0)	392.8	1.0	3.8
...	...	...	

Theoretical Errors:

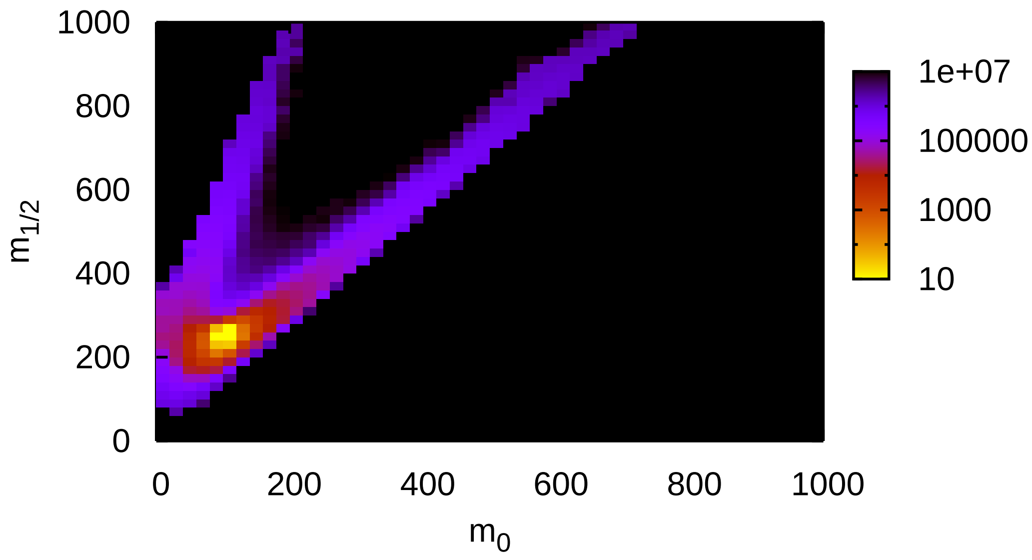
- mSUGRA: 3% for gluino and squark masses,
  1% for all other sparticle masses
- MSSM: 1% for gluino and squark masses,
  0.5% for all other sparticle masses
- m_{h^0}: 2 GeV (unknown higher order terms)

mSUGRA as a Toy Model

mSUGRA with LHC measurements (SPS1a kinematic edges):
pick one set of ”measurements”, randomly smeared from the true values

Free parameters:
m_0, $m_{1/2}$, $\tan(\beta)$, A_0, $\mathrm{sgn}(\mu)$, m_t

SFitter output 1:
Fully-dimensional exclusive likelihood map
(colour:
minimum χ^2 over all unseen parameters)



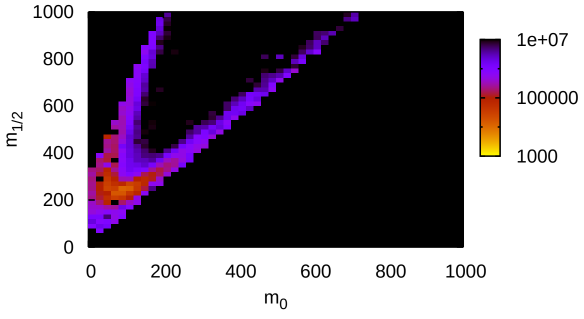
SFitter output 2:
Ranked list of minima:

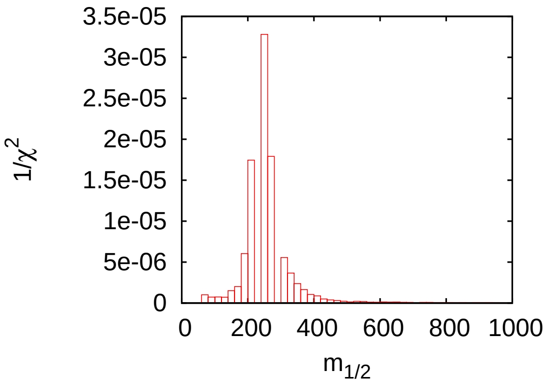
	χ^2	m_0	$m_{1/2}$	$\tan(\beta)$	A_0	μ	m_t
SPS1a		100.0	250.0	10.0	-100.0	$+$	171.4
1)	0.09	102.0	254.0	11.5	-95.2	$+$	172.4
2)	1.50	104.8	242.1	12.9	-174.4	$-$	172.3
3)	73.2	108.1	266.4	14.6	742.4	$+$	173.7
4)	139.5	112.1	261.0	18.0	632.6	$-$	173.0

Bayesian or Frequentist?

SFitter provides full-dimensional log-likelihood map
$\longrightarrow$ "project" onto plotable 1- or 2-dimensional spaces

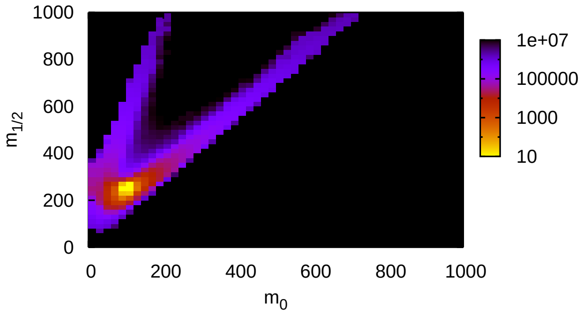
Bayesian:

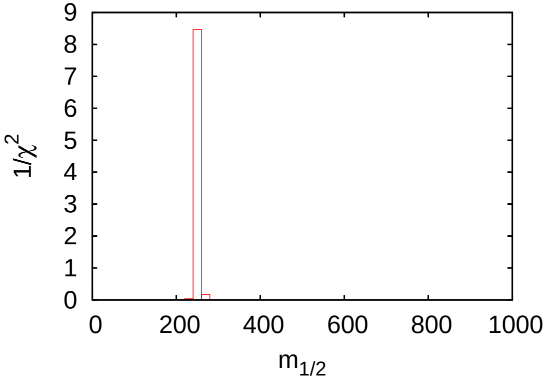




Marginalisation of χ^2 in all other directions

Frequentist:





Profile likelihood: Value of bin is value of smallest χ^2 occuring in this bin

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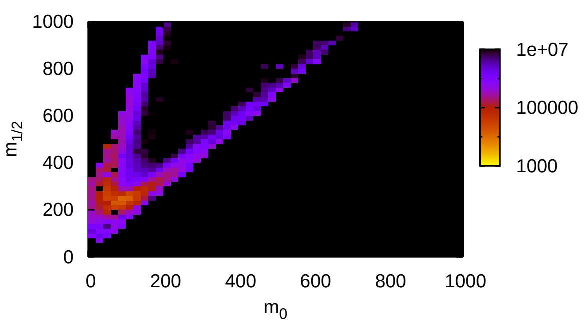

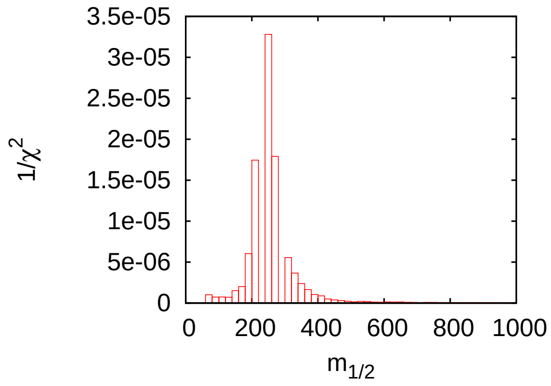

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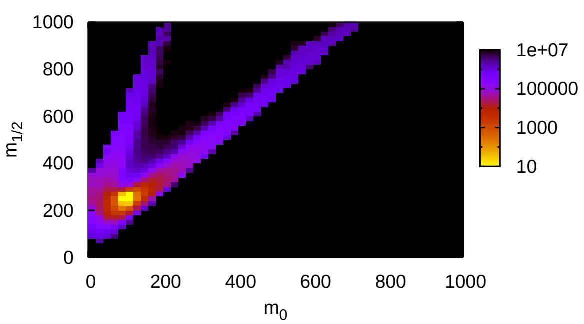

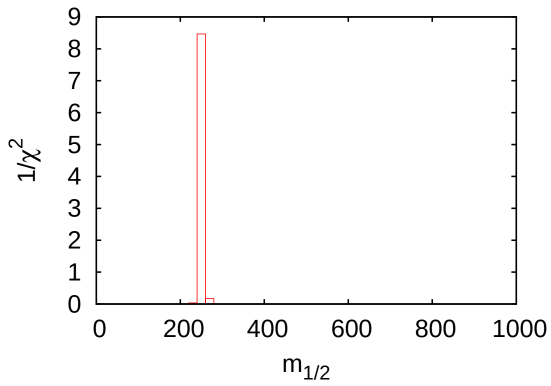

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Different methods answer different questions.
$\Rightarrow$ Bayesian and Frequentist!
Everybody can choose his/her favourite analysis ...

Purely high-scale model

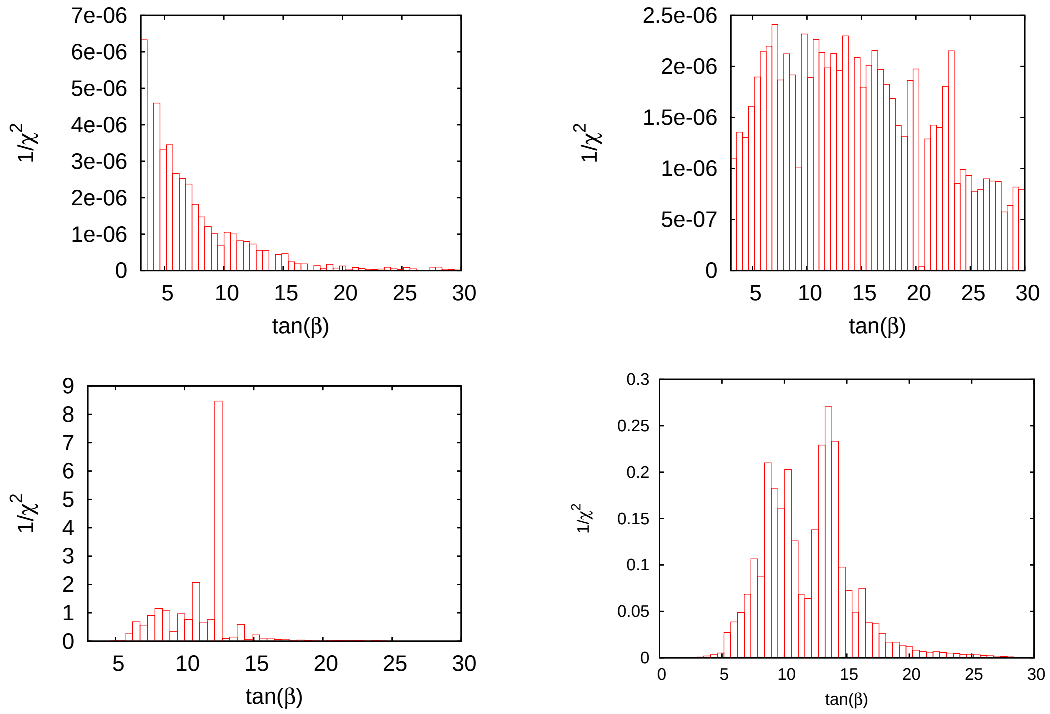
m_0, $m_{1/2}$, A_0 defined at the GUT-scale $\Leftrightarrow$ $\tan(\beta)$ defined at the weak scale

$\Rightarrow$ Replace $\tan(\beta)$ with high-scale quantity B

$\Rightarrow$ Flat prior in B yields prior $\propto \frac{1}{\tan(\beta)^2}$

SPS1a with LHC kinematic edges ($\tan(\beta)$ vs. $1/\chi^2$):

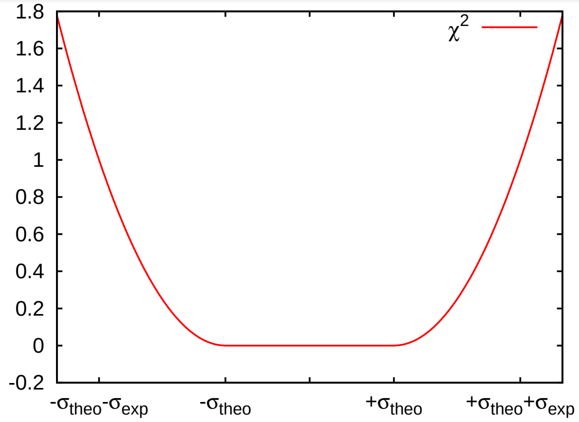
flat B prior | flat $\tan(\beta)$ prior



Bayesian:
Large influence of choice of prior
Choosing flat B prior strongly favours low values of $\tan(\beta)$.

Frequentist:
Two plots should be identical (no prior in χ^2 calculation)
Indirect influence via Markov Chain proposal function

Error determination



Treatment of errors:

- All experimental errors are Gaussian

$$\sigma^2_{\text{exp}} = \sigma^2_{\text{stat}} + \sigma^2_{\text{syst}(j)} + \sigma^2_{\text{syst}(l)}$$

- Systematic errors from jet ($\sigma_{\text{syst}(j)}$) and lepton energy scale ($\sigma_{\text{syst}(l)}$) assumed 99% correlated each
- Theory error added as box-shaped (RFit scheme [Hoecker, Lacker, Laplace, Lediberder])

$$\Rightarrow -2\log L \equiv \chi^2 = \sum_{\text{measurements}} \begin{cases} 0 & \text{for } |x_{\text{data}} - x_{\text{pred}}| < \sigma_{\text{theo}} \\ \left(\frac{|x_{\text{data}} - x_{\text{pred}}| - \sigma_{\text{theo}}}{\sigma_{\text{exp}}}\right)^2 & \text{for } |x_{\text{data}} - x_{\text{pred}}| \geq \sigma_{\text{theo}} \end{cases}$$

$\Rightarrow$ Parameter errors:

	SPS1a	$\Delta^{\text{theo}-\text{exp}}_{\text{zero}}$	$\Delta^{\text{theo}-\text{exp}}_{\text{zero}}$	$\Delta^{\text{theo}-\text{exp}}_{\text{gauss}}$	$\Delta^{\text{theo}-\text{exp}}_{\text{flat}}$
		LHC masses	LHC edges		
m_0	100	4.11	0.50	2.97	2.17
$m_{1/2}$	250	1.81	0.73	2.99	2.64
$\tan\beta$	10	1.69	0.65	3.36	2.45
A_0	-100	36.2	21.2	51.5	49.6
m_t	171.4	0.94	0.26	0.89	0.97

$\Rightarrow$ Use kinematic edges for parameter determination instead of masses

Weak-scale MSSM

- No need to assume specific SUSY-breaking scenario
  $\Rightarrow$ SUSY-breaking mechanism should be induced from data
- Use of Markov Chains makes scanning the 19-dimensional parameter space feasible
- Lack of sensitivity on one parameter does not slow down the scan
  (no need to fix parameters)
- Same SFitter output as before: Minima list and Likelihood map

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Full scan of 19D parameter space challenging
Four-step procedure yields better and faster results:

- Weighted-Markov-Chain run with flat pdf over full parameter space
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  (full scan, no bias on starting point)

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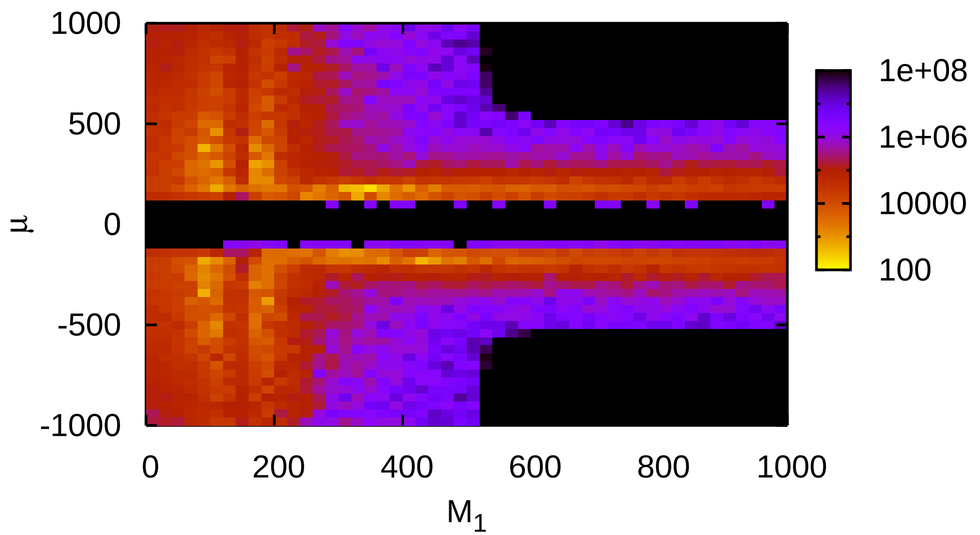
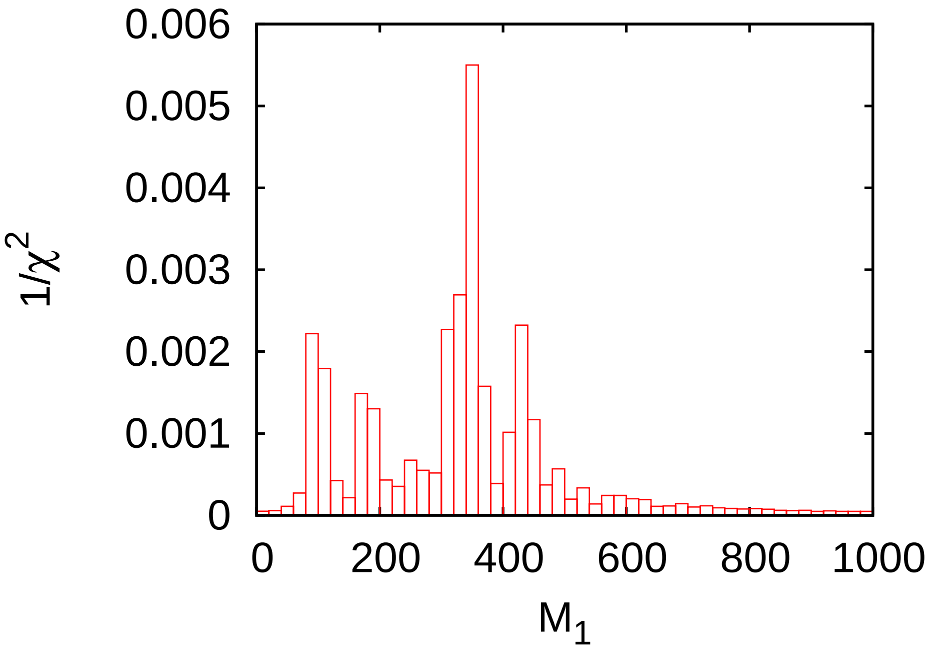
- Weighted-Markov Chain with flat pdf on Gaugino-Higgsino subspace:
  M_1, M_2, M_3, μ, $\tan\beta$, m_t
  Additional Minuit run with 15 best solutions

Search Strategy (2) - results

- Only three neutralinos (χ_1^0, χ_2^0, χ_4^0) with masses (97.2 GeV , 180.5 GeV , 375.6 GeV) and no charginos observable at the LHC in SPS1a
- $\Rightarrow$ Mapping (M_1, M_2, μ) $\rightarrow$ (χ_1^0, χ_2^0, χ_4^0) not unique
- $\mathrm{sgn}\,\mu$ basically undetermined by collider data
- $\Rightarrow$ 8-fold solution





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	$\mu < 0$				$\mu > 0$			
					SPS1a			
M_1	96.6	175.1	103.5	365.8	98.3	176.4	105.9	365.3
M_2	181.2	98.4	350.0	130.9	187.5	103.9	348.4	137.8
μ	-354.1	-357.6	-177.7	-159.9	347.8	352.6	178.0	161.5
$\tan\beta$	14.6	14.5	29.1	32.1	15.0	14.8	29.2	32.1
M_3	583.2	583.3	583.3	583.5	583.1	583.1	583.3	583.4
m_t	171.4	171.4	171.4	171.4	171.4	171.4	171.4	171.4

Search Strategy (3)

Full scan of 19D parameter space challenging
Four-step procedure yields better and faster results:

- Weighted-Markov-Chain run with flat pdf over full parameter space
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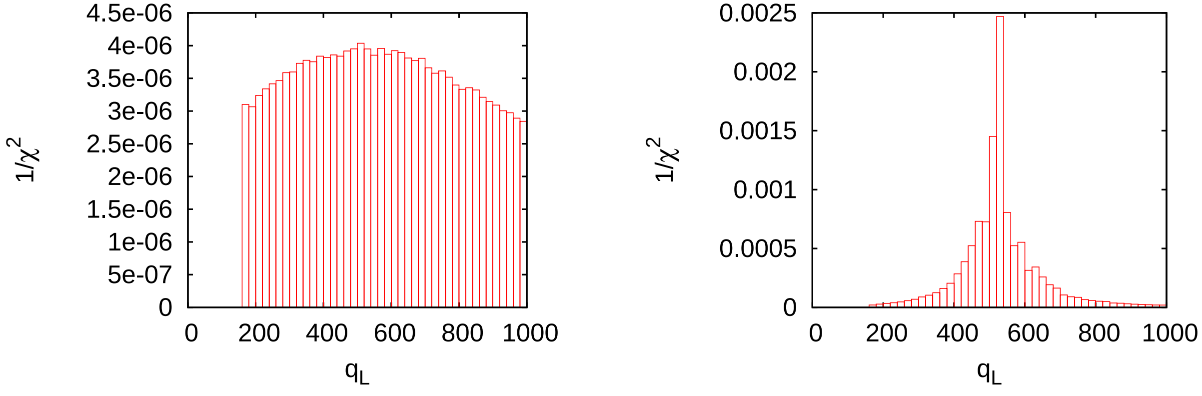
  (full scan, no bias on starting point)
- Weighted-Markov Chain with flat pdf on Gaugino-Higgsino subspace:
  $M_1, M_2, M_3, \mu, \tan\beta, m_t$
  Additional Minuit run with 15 best solutions
- Weighted-Markov Chain with Breit-Wigner-shaped pdf on remaining parameters for all solutions of previous step
  Minimisation for best 5 points



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  M_1, M_2, M_3, μ, $\tan\beta$, m_t
  Additional Minuit run with 15 best solutions
- Weighted-Markov Chain with Breit-Wigner-shaped pdf on remaining parameters for all solutions of previous step
  Minimisation for best 5 points
- Minuit run for best points of last step keeping all parameters variable

Best points

	$\mu < 0$				$\mu > 0$			
					SPS1a			
M_1	96.6	175.1	103.5	365.8	98.3	176.4	105.9	365.3
M_2	181.2	98.4	350.0	130.9	187.5	103.9	348.4	137.8
μ	-354.1	-357.6	-177.7	-159.9	347.8	352.6	178.0	161.5
$\tan\beta$	14.6	14.5	29.1	32.1	15.0	14.8	29.2	32.1
M_3	583.2	583.3	583.3	583.5	583.1	583.1	583.3	583.4
$M_{\tilde{\tau}_L}$	114.9	2704.3	128.3	4794.2	128.0	229.9	3269.3	118.6
$M_{\tilde{\tau}_R}$	348.8	129.9	1292.7	130.1	2266.5	138.5	129.9	255.1
$M_{\tilde{\mu}_L}$	192.7	192.7	192.7	192.9	192.6	192.6	192.7	192.8
$M_{\tilde{\mu}_R}$	131.1	131.1	131.1	131.3	131.0	131.0	131.1	131.2
$M_{\tilde{e}_L}$	186.3	186.4	186.4	186.5	186.2	186.2	186.4	186.4
$M_{\tilde{e}_R}$	131.5	131.5	131.6	131.7	131.4	131.4	131.5	131.6
$M_{\tilde{q}3_L}$	497.1	497.2	494.1	494.0	495.6	495.6	495.8	495.0
$M_{\tilde{t}_R}$	1073.9	920.3	547.9	950.8	547.9	460.5	978.2	520.0
$M_{\tilde{b}_R}$	497.3	497.3	500.4	500.9	498.5	498.5	498.7	499.6
$M_{\tilde{q}_L}$	525.1	525.2	525.3	525.5	525.0	525.0	525.2	525.3
$M_{\tilde{q}_R}$	511.3	511.3	511.4	511.5	511.2	511.2	511.4	511.5
$A_t\ (-)$	-252.3	-348.4	-477.1	-259.0	-470.0	-484.3	-243.4	-465.7
$A_t\ (+)$	384.9	481.8	641.5	432.5	739.2	774.7	440.5	656.9
m_A	350.3	725.8	263.1	1020.0	171.6	156.5	897.6	256.1
m_t	171.4	171.4	171.4	171.4	171.4	171.4	171.4	171.4

Degenerate Solutions

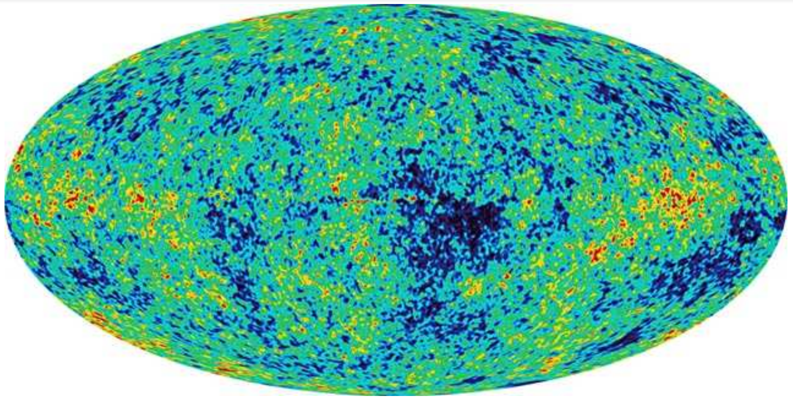
- In total 19 parameters constrained by 22 measurements
- Measurements constructed from only 15 underlying masses
- $\Rightarrow$ Complete determination of parameter set not possible
- Five parameters not well constrained
  - m_A
    $\leftarrow$ no heavy Higgses measurable
  - $M_{\tilde{t}_R}$
  - A_t
    $\leftarrow$ stop sector parameters do not enter edge measurements
  - $M_{\tilde{\tau}_L}$ or $M_{\tilde{\tau}_R}$
    $\leftarrow$ only the lighter stau measured
  - $\tan\beta$
    $\leftarrow$ change can always be accomodated by rotating M_1, M_2, $M_{\tilde{q}_3}, \ldots$
- Single common link: m_{h^0}
- $\Rightarrow$ 4-dimensional hyperplane in parameter space undetermined
- Can still assign errors to some of the badly determined parameters

Error analysis

- Technical procedure as in mSUGRA case:
  - 10000 smeared data sets
  - Minimum determined for each data set individually
  - Error determined from fit with Gaussian
- Most constrained parameters determinable with $\sim 5\%$ accuracy
- Inclusion of theory errors leads to an increase of factor 2 on the parameter errors
- ILC data complementary to LHC
- Combination of the two experiments allows for precise determination of all parameters

	LHC	ILC	LHC+ILC	SPS1a
M_1	102.1± 7.8	103.0± 1.1	103.1±0.84	103.1
$M_{\tilde{e}_R}$	135.0± 8.3	135.8± 0.81	135.9±0.77	135.8
m_A	406.3±$\mathcal{O}(10^3)$	393.8± 1.6	393.9± 1.6	394.9
$M_{\tilde{t}_R}$	415.8±$\mathcal{O}(10^2)$	440.0±$\mathcal{O}(4 \cdot 10^2)$	410.7±48.4	408.3

Dark Matter



[NASA/WMAP Science Team]

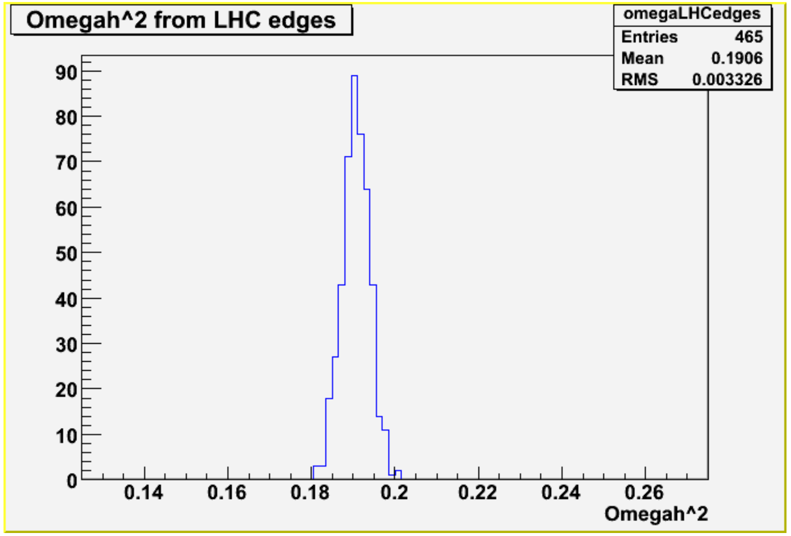
Content of the universe:

- 73% Dark energy
- 4% Ordinary matter
- 23% Dark matter

MSSM: χ_1^0 as LSP ideal candidate for cold dark matter (CDM): massive, weakly interacting

- SFitter: Determine Lagrangian parameters $\Rightarrow$ Spectrum and couplings
- e.g. micrOMEGAs: Calculate relic density $\Omega_{\text{CDM}} h^2 = n_{\text{LSP}} m_{\text{LSP}}$ [Bélanger et al.]
- $\Rightarrow$ Prediction of $\Omega_{\text{CDM}} h^2$
  LHC : $\Omega_{\text{CDM}} h^2 = 0.1906 \pm 0.0033$
  LHC+ILC: $\Omega_{\text{CDM}} h^2 = 0.1910 \pm 0.0003$
  (improvement by one order of magnitude)
- Compare with experiment
  (Measurement of the fluctuations of the
  cosmic microwave background):

WMAP: $\Omega_{\text{CDM}} h^2 = 0.1277 \pm 0.008$ [astro-ph/0603449]
Planck: $\Omega_{\text{CDM}} h^2 = \quad ? \pm 0.0016$



Summary & Outlook

The SFitter program:

- High-dimensional parameter scans important to determine Lagrangian parameters from observables
- Improved Weighted Markov Chain algorithm can do this efficiently
- Two types of SFitter output: Likelihood map and list of best points

SUSY analysis:

- mSUGRA:
  - Can reconstruct SPS1a from (simulated) LHC data
  - Bayesian output significantly dependent on priors
- weak-scale MSSM:
  - Reconstruction works as well
  - Degenerate solutions in gaugino-higgsino-sector and
  - General underdetermination of parameter space, in particular $\tan\beta$
- Test of Dark-Matter hypothesis seems possible
- SFitter (despite its name) not tied to SUSY
  $\rightarrow$ extend to other models/problems

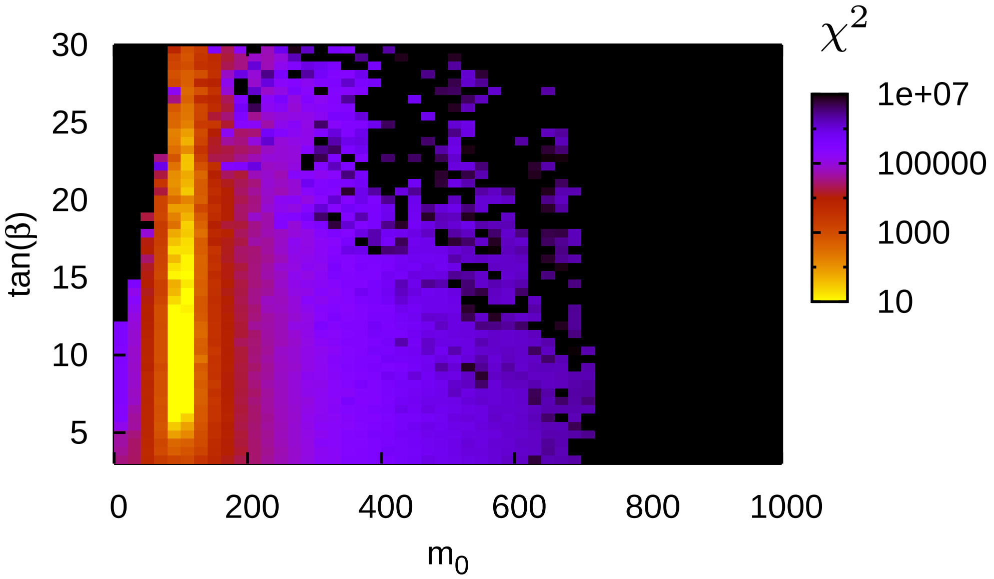
Backup Slides

Improving on $\tan\beta$

- difficult even in mSUGRA
  Analysis with LHC kinematic edges as before, but now assume mSUGRA scenario:

	SPS1a	$\Delta^{\text{theo}-\text{exp}}_{\text{zero}}$	$\Delta^{\text{theo}-\text{exp}}_{\text{zero}}$	$\Delta^{\text{theo}-\text{exp}}_{\text{gauss}}$	$\Delta^{\text{theo}-\text{exp}}_{\text{flat}}$
		LHC masses	LHC edges		
$\tan\beta$	10	1.69	0.65	3.36	2.45

profile likelihood:



Improving on $\tan\beta$

- difficult even in mSUGRA
- LHC rates: $\tan\beta$ from heavy Higgses (for large $\tan\beta$) [Kinnunen, Lehti, Moortgat, Nikitenko, Spira]
  $\propto (\tan\beta)^2$
  $\longrightarrow$ in general: challenging
  $\longrightarrow$ for SPS1a: no heavy Higgses observed

Improving on $\tan\beta$

- difficult even in mSUGRA
- LHC rates: $\tan\beta$ from heavy Higgses (for large $\tan\beta$) [Kinnunen, Lehti, Moortgat, Nikitenko, Spira]
- $B_s \to \mu^+\mu^-$ [Jäger, Spannowsky, SFitter]
  - $\propto (\tan\beta)^6$
  - LHCb will be able to probe the SM value
  - Errors largely theory-dominated (main source: f_{B_s} from lattice simulations)
  - In a simple proof-of-concept analysis: $\tan\beta = 30 \pm 6.5$

Improving on $\tan\beta$

- difficult even in mSUGRA
- LHC rates: $\tan\beta$ from heavy Higgses (for large $\tan\beta$) [Kinnunen, Lehti, Moortgat, Nikitenko, Spira]
- $B_s \to \mu^+\mu^-$ [Jäger, Spannowsky, SFitter]
- anomaluos magnetic moment of the muon [review: Stöckinger]

$\tan\beta$ from other sectors

Electro-weak sector: Anomalous Magnetic Moment of the Muon $(g-2)_\mu$ [Alexander, Kreiss, SFitter]

- Currently 3.4σ deviation from Standard Model
- Leading order $\simeq 130 \cdot 10^{-11}\ \tan\beta\ \text{sgn}(\mu)\ \left(\frac{100\text{ GeV}}{M_{\text{SUSY}}}\right)^2$
- $\Rightarrow$ Favours one sign of $\mu \to +$
- $\Rightarrow$ Linearly sensitive on $\tan\beta$

	LHC	LHC $\otimes (g-2)\mu$	SPS1a
$\tan\beta$	**10.0± 4.5**	**10.3± 2.0**	**10.0**
M_1	102.1± 7.8	102.7± 5.9	103.1
M_2	193.3± 7.8	193.2± 5.8	192.9
M_3	577.2±14.5	578.2± 12.1	577.9
μ	350.5±14.5	352.5± 10.8	353.7
$M_{\tilde{\mu}_R}$	135.0± 8.3	135.6± 6.3	135.8
$M_{\tilde{q}_R}$	507.3±17.5	507.6± 15.8	508.1

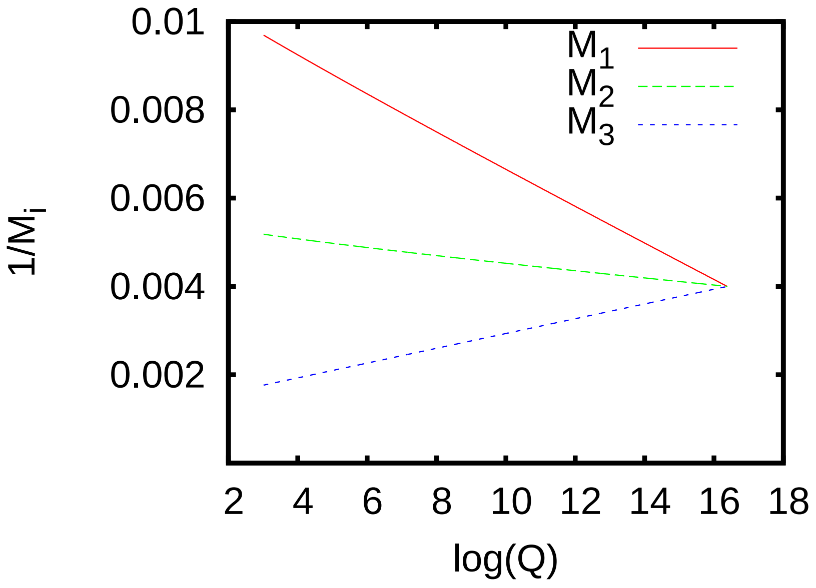
$\Rightarrow$ Need to combine information on $\tan\beta$ from all sectors

Testing Unification

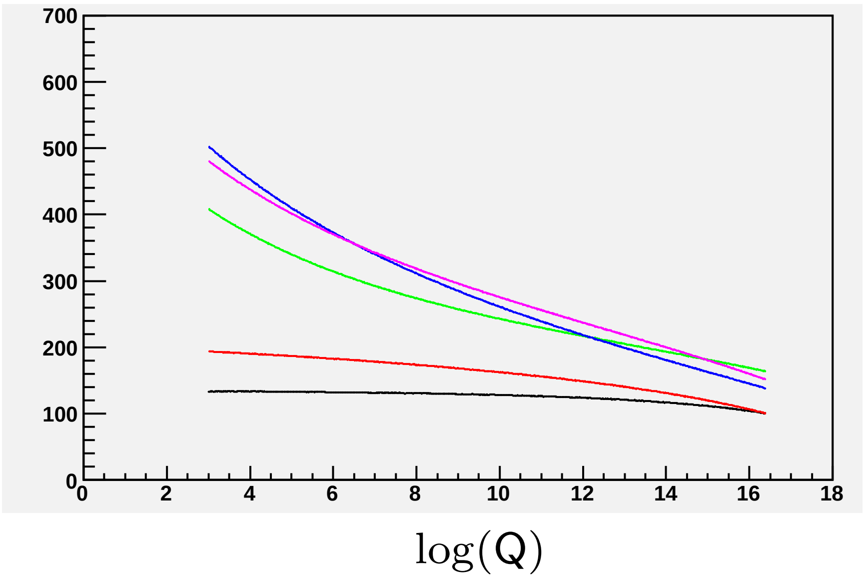
Apparent unification of gauge coupling parameters in the MSSM

Question arises: Do other parameters unify as well?
⇒ Should be tested by bottom-up running from weak scale to Planck scale
⇒ Can give hints about supersymmetry breaking
(e.g. test scalar-mass sum rules with a sliding scale) [Schmaltz et al.]

Bottom-up running of gaugino masses and 3rd-generation sfermion masses:



$M_{\tilde{\tau}_R}, M_{\tilde{\tau}_L}, M_{\tilde{t}_R}, M_{\tilde{b}_R}, M_{\tilde{q3}_L}$; $\Delta M_3 = -10$ GeV



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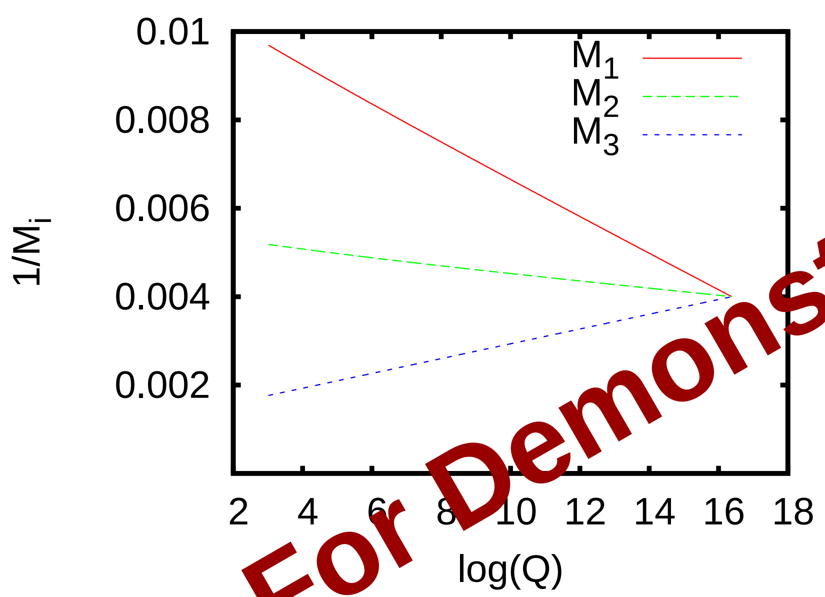
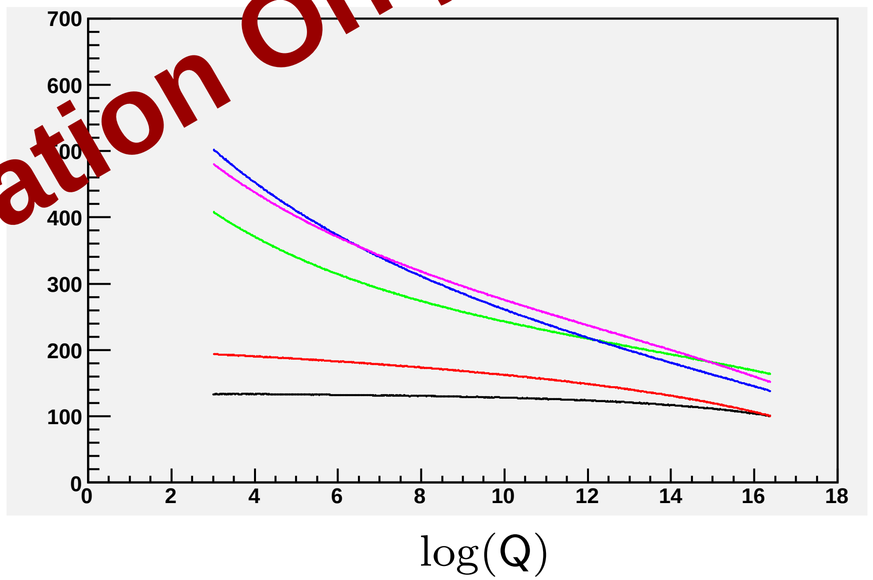
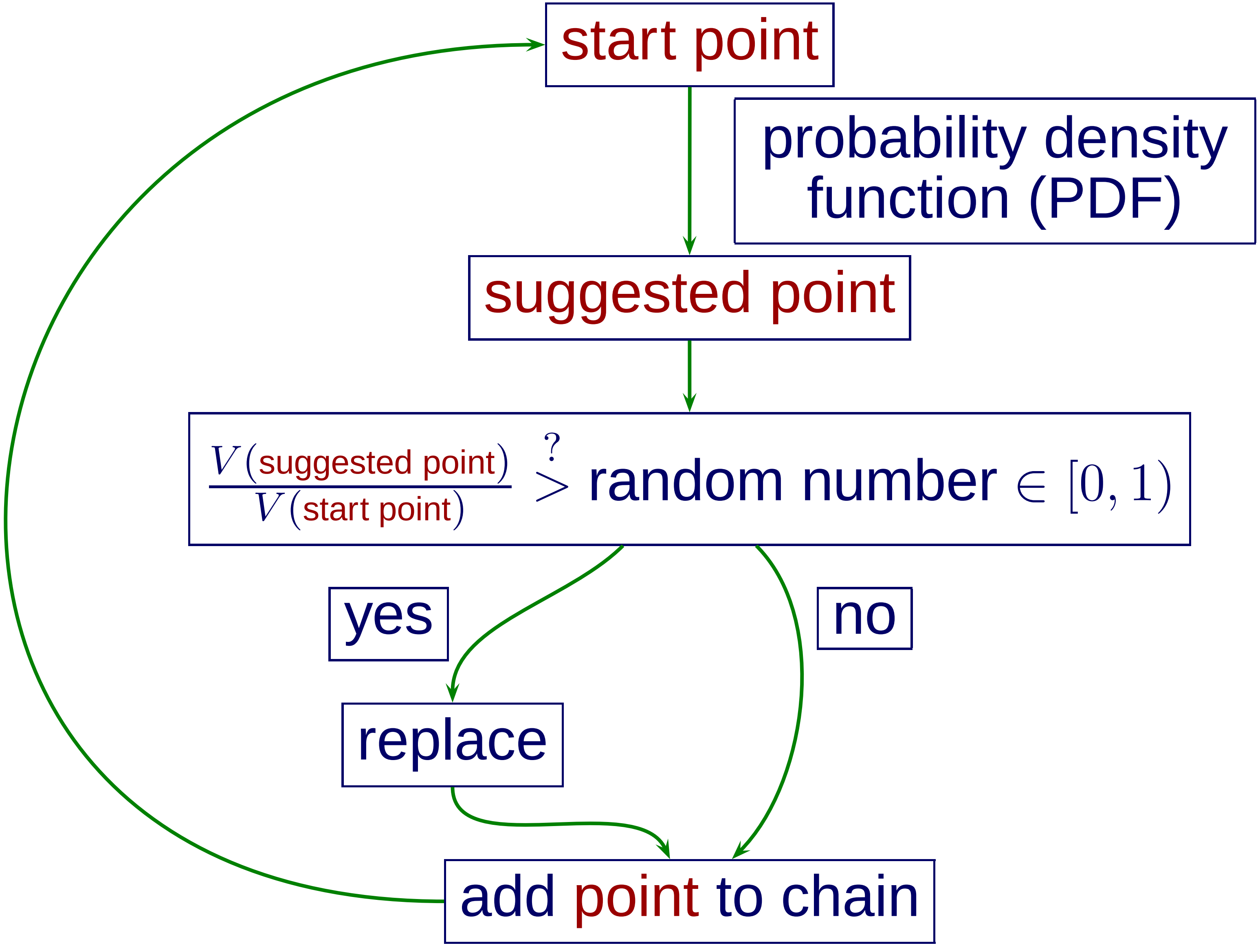


$M_{\tilde{\tau}_R}, M_{\tilde{\tau}_L}, M_{\tilde{t}_R}, M_{\tilde{b}_R}, M_{\tilde{q}3_L}$; $\Delta M_3 = -10$ GeV



Metropolis-Hastings Algorithm

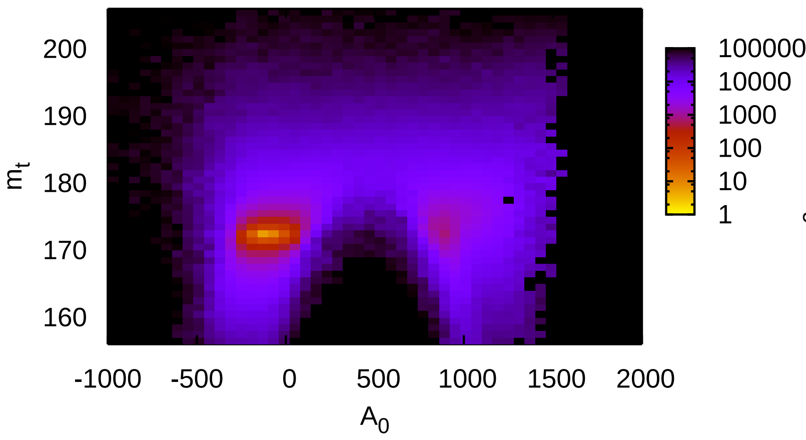


Experimental Input (edges)

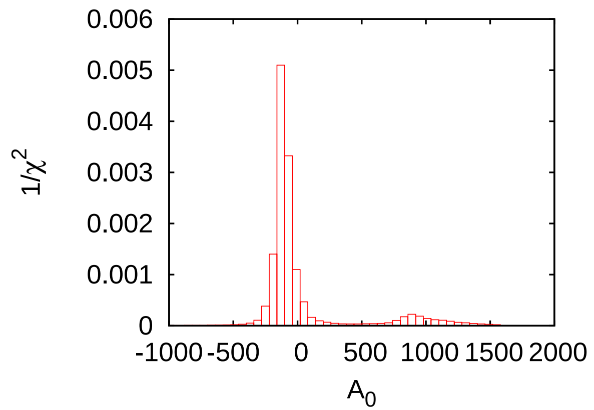
(Obs)	= (meas)	± (exp)	± (theo)	
m_{h^0}	$= 109.53 \pm$	$0.25 \pm$	2.0	
m_t	$= 171.4 \pm$	$1.0 \pm$	0.0	
$\Delta m_{\tilde{\mu}_L,\chi_1^0}$	$= 106.26 \pm$	$1.6 \pm$	0.1	
$\Delta m_{\tilde{g},\chi_1^0}$	$= 509.96 \pm$	$2.3 \pm$	6.0	
$\Delta m_{\tilde{c}_R,\chi_1^0}$	$= 450.52 \pm$	$10.0 \pm$	4.2	
$\Delta m_{\tilde{g},\tilde{b}_1}$	$= 98.971 \pm$	$1.5 \pm$	1.0	
$\Delta m_{\tilde{g},\tilde{b}_2}$	$= 64.016 \pm$	$2.5 \pm$	0.7	
Edge(χ_2^0,$\tilde{\mu}_R$,χ_1^0)	$= 79.757 \pm$	$0.03 \pm$	0.08	(m_{ll}^{max})
Edge($\tilde{c}_L$,χ_2^0,χ_1^0)	$= 446.44 \pm$	$1.4 \pm$	4.3	(m_{llq}^{max})
Edge($\tilde{c}_L$,χ_2^0,$\tilde{\mu}_R$)	$= 316.51 \pm$	$0.9 \pm$	3.0	(m_{lq}^{low})
Edge($\tilde{c}_L$,χ_2^0,$\tilde{\mu}_R$,χ_1^0)	$= 392.8 \pm$	$1.0 \pm$	3.8	(m_{lq}^{high})
Edge(χ_4^0,$\tilde{\mu}_R$,χ_1^0)	$= 257.41 \pm$	$2.3 \pm$	0.3	$(m_{ll}^{\text{max}}(\chi_4^0))$
Edge(χ_4^0,$\tilde{\tau}_L$,χ_1^0)	$= 82.993 \pm$	$5.0 \pm$	0.8	$(m_{\tau\tau}^{\text{max}})$
Threshold($\tilde{c}_L$,χ_2^0,$\tilde{\mu}_R$,χ_1^0)	$= 211.95 \pm$	$1.6 \pm$	2.0	(m_{llq}^{min})
Threshold($\tilde{b}_1$,χ_2^0,$\tilde{\mu}_R$,χ_1^0)	$= 211.95 \pm$	$1.6 \pm$	2.0	(m_{llb}^{min})

mSUGRA around Minima – positive μ

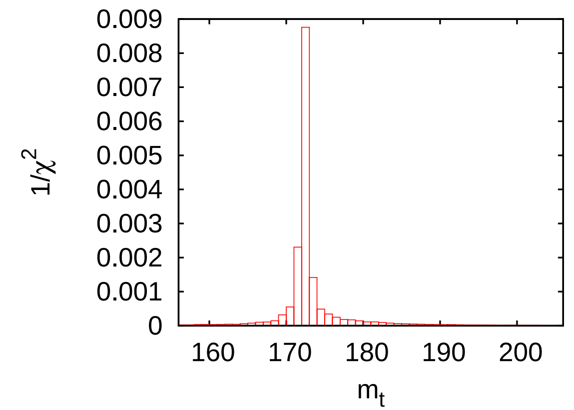
Bayesian



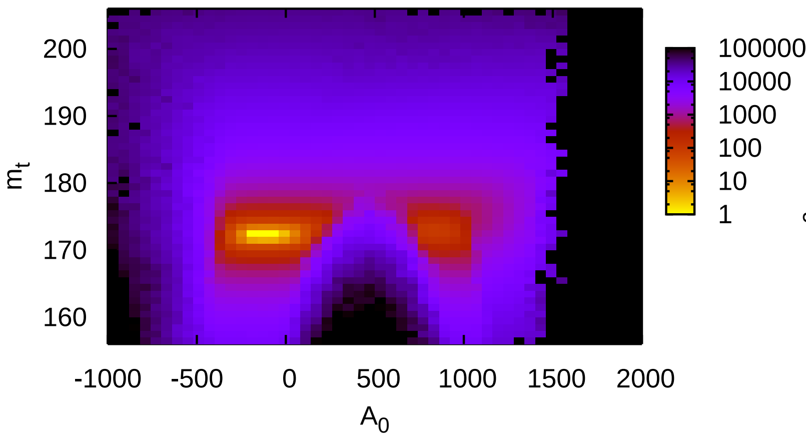
A_0



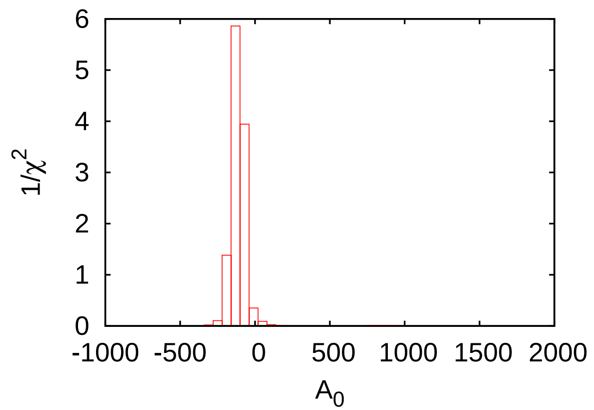
m_t



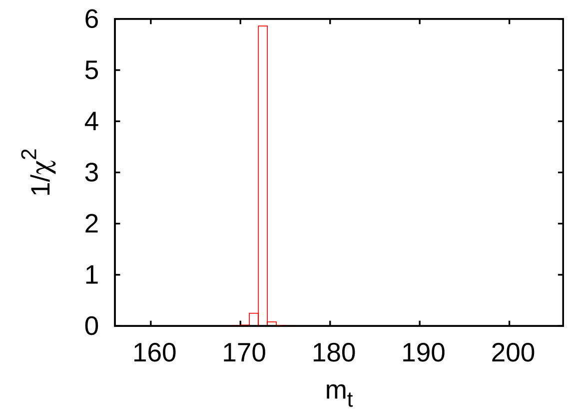
Frequentist



A_0

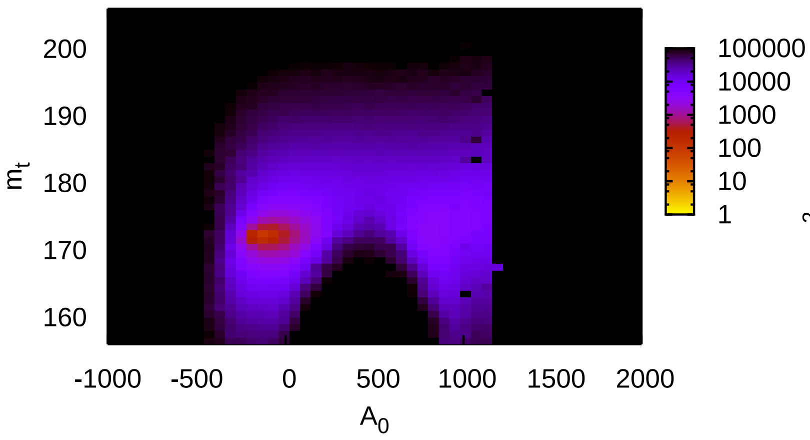


m_t

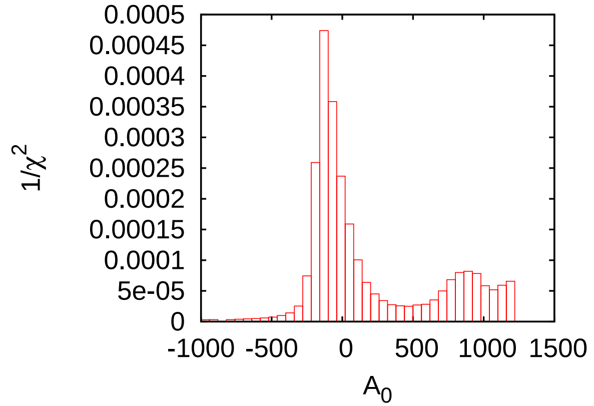


mSUGRA around Minima – negative μ

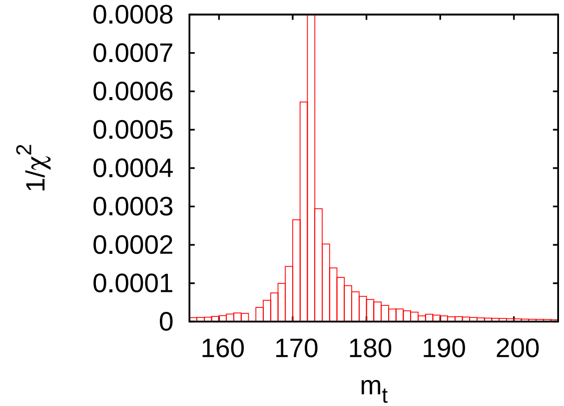
Bayesian



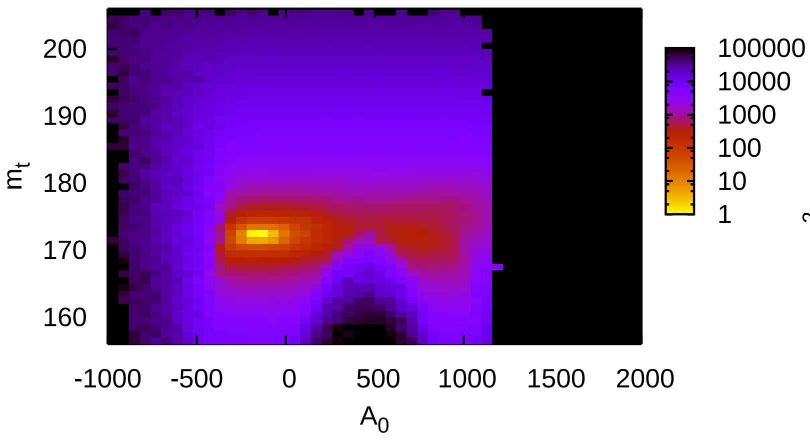
A_0



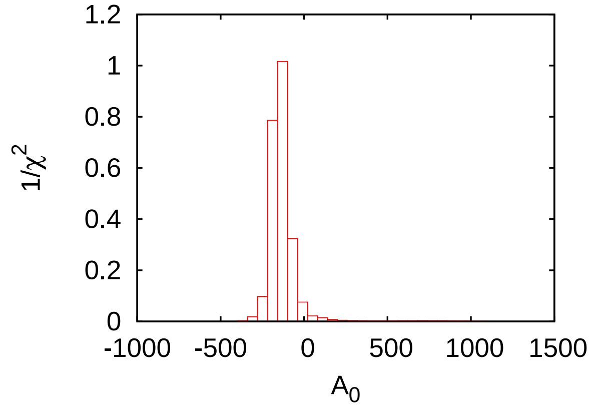
m_t



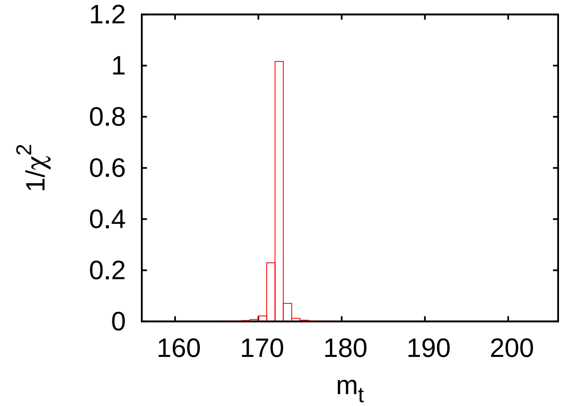
Frequentist



A_0



m_t



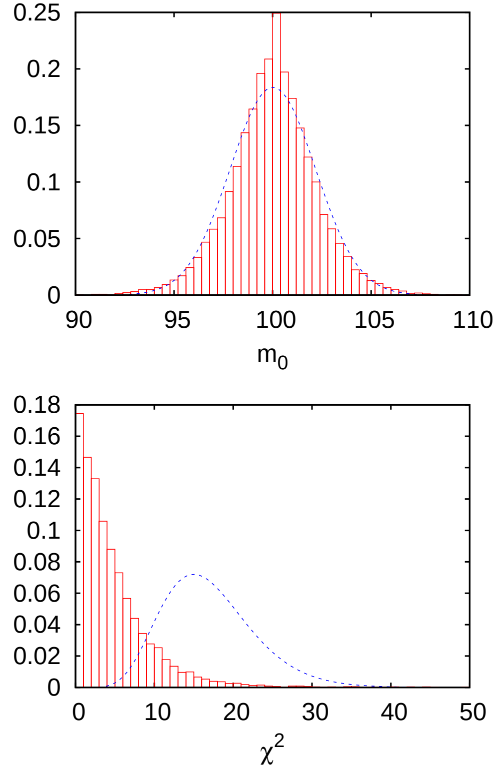
Error determination

Minuit output not usable for flat theory errors:

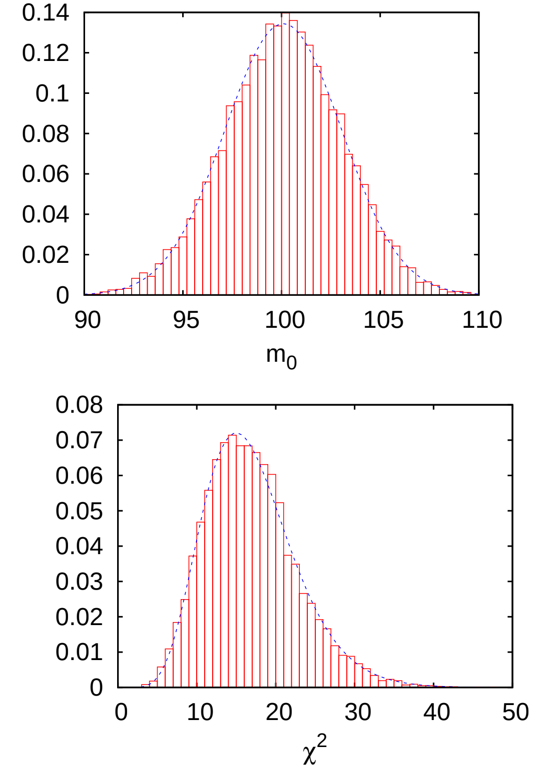
- Migrad function depends on parabolic approximation
- Cannot determine $\Delta\chi^2$ for Minos to yield 68% CL intervals

$\Rightarrow$ Need more general approach

- Perform $10,000$ toy experiments with measurements smeared around correct value
- Minimise each toy experiment
- Plot resulting distribution of parameter points and fit with Gaussian



Flat theory errors

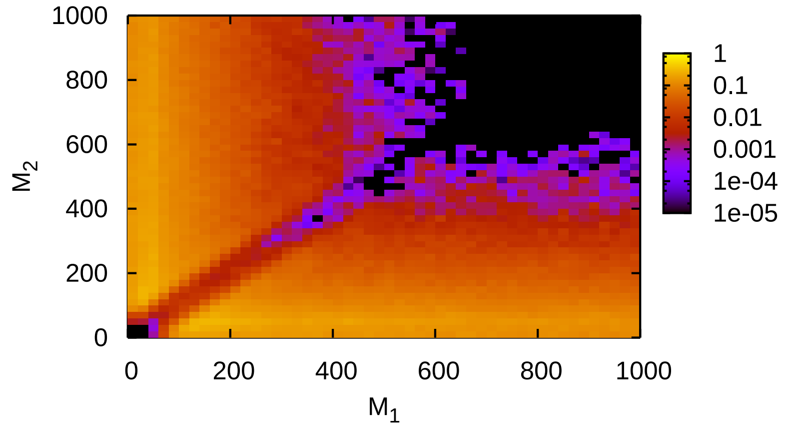


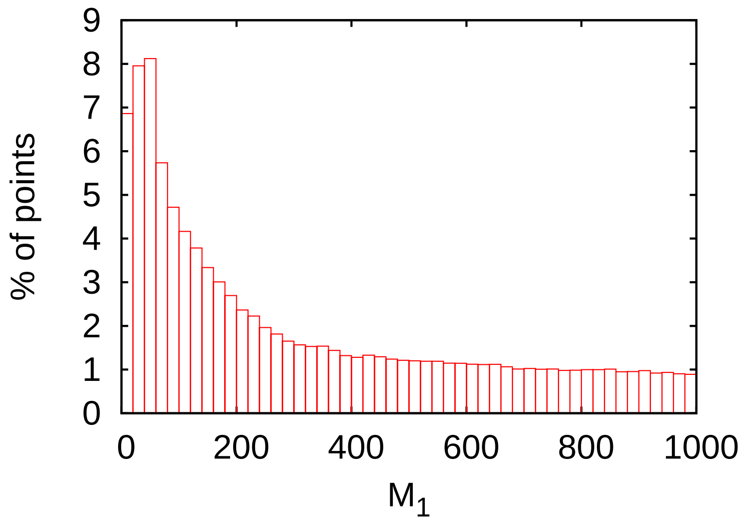
Gaussian theory errors

Weak-scale MSSM

- No need to assume specific SUSY-breaking scenario
  $\Rightarrow$ SUSY-breaking mechanism should be induced from data
- Use of Markov Chains makes scanning the 19-dimensional parameter space feasible
- Lack of sensitivity on one parameter does not slow down the scan
  (no need to fix parameters)
- Same SFitter output as before: Minima list and Likelihood map

MSSM using SPS1a spectrum and LHC kinematic edges:
(Bayesian, full parameter space)

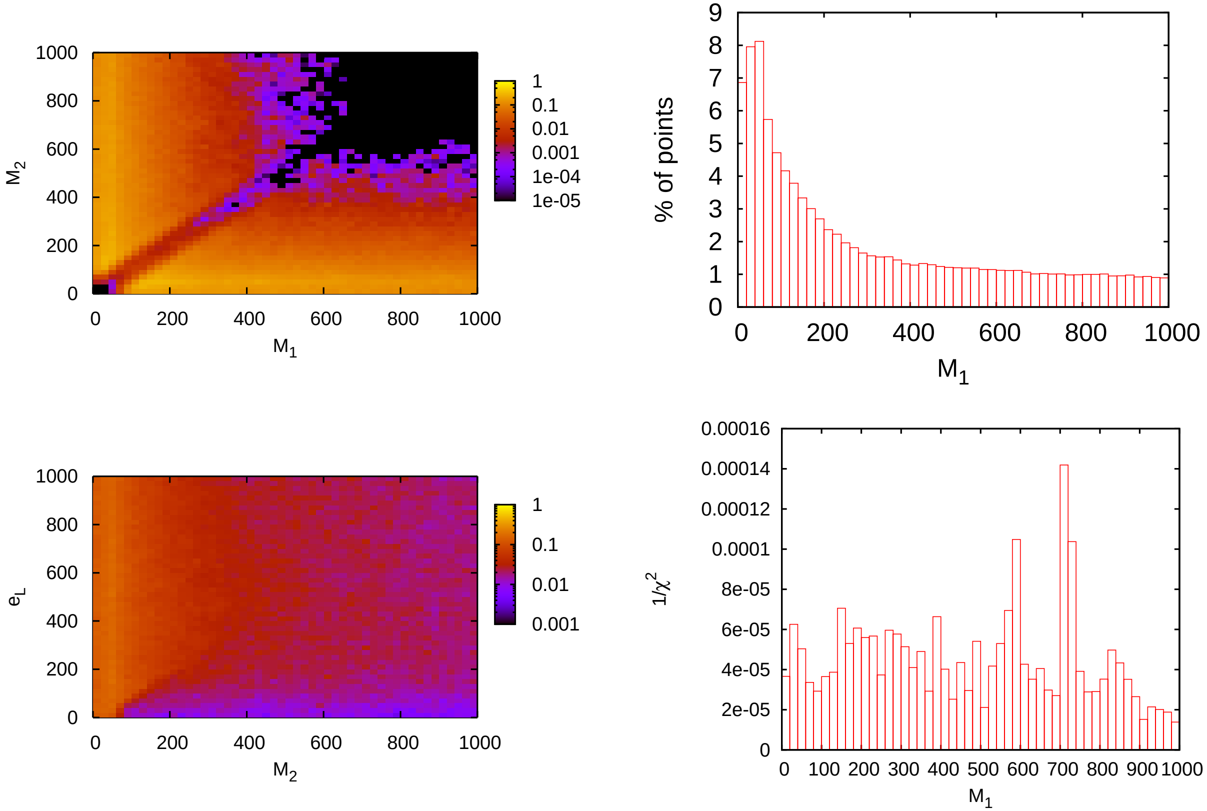




Search Strategy (1)

Full scan of 19D parameter space challenging
Four-step procedure yields better and faster results:

- Weighted-Markov-Chain run with flat pdf over full parameter space
  5 best points additionally minimised
  (full scan, no bias on starting point)

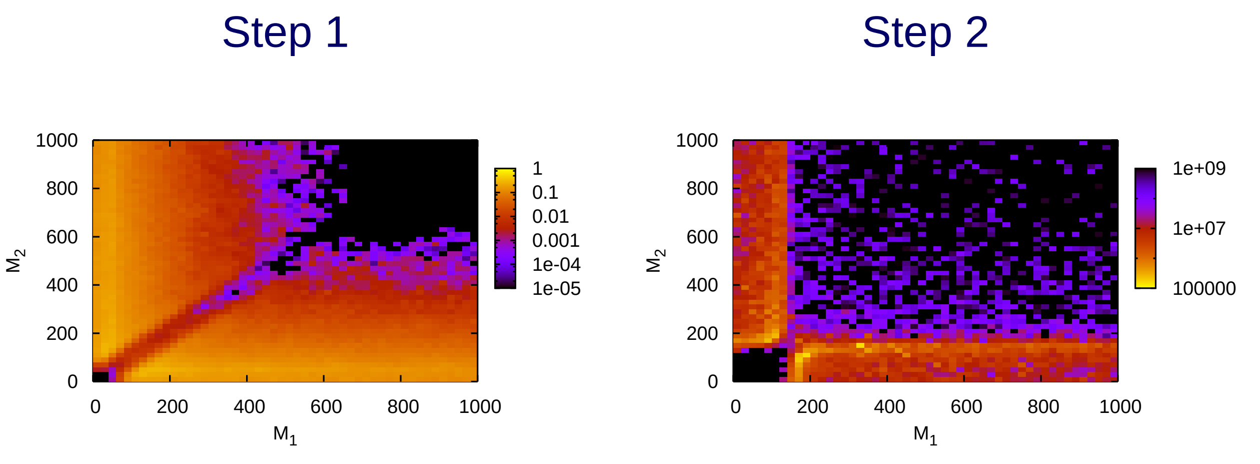


Correlations not aligned with parameters ⇒ washed out in plots

Search Strategy (2)

Full scan of 19D parameter space challenging
Four-step procedure yields better and faster results:

- Weighted-Markov-Chain run with flat pdf over full parameter space
  5 best points additionally minimised
  (full scan, no bias on starting point)
- Weighted-Markov Chain with flat pdf on Gaugino-Higgsino subspace:
  M_1, M_2, M_3, μ, $\tan\beta$, m_t
  Additional Minuit run with 15 best solutions



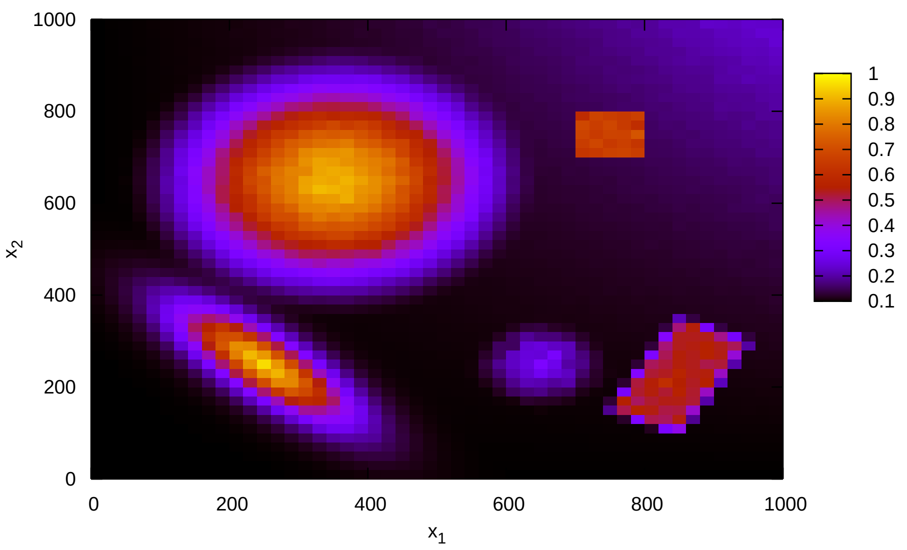
MSSM errors

	LHC	ILC	LHC+ILC	SPS1a
$\tan\beta$	10.0± 4.5	12.1± 7.0	12.6± 6.2	10.0
M_1	102.1± 7.8	103.3± 1.1	103.2± 0.95	103.1
M_2	193.3± 7.8	194.1± 3.3	193.3± 2.6	192.9
M_3	577.2± 14.5	fixed 500	581.0± 15.1	577.9
$M_{\tilde{\tau}_L}$	227.8±$\mathcal{O}(10^3)$	190.7± 9.1	190.3± 9.8	193.6
$M_{\tilde{\tau}_R}$	164.1±$\mathcal{O}(10^3)$	136.1± 10.3	136.5± 11.1	133.4
$M_{\tilde{\mu}_L}$	193.2± 8.8	194.5± 1.3	194.5± 1.2	194.4
$M_{\tilde{\mu}_R}$	135.0± 8.3	135.9± 0.87	136.0± 0.79	135.8
$M_{\tilde{e}_L}$	193.3± 8.8	194.4± 0.91	194.4± 0.84	194.4
$M_{\tilde{e}_R}$	135.0± 8.3	135.8± 0.82	135.9± 0.73	135.8
$M_{\tilde{q}3_L}$	481.4± 22.0	499.4± $\mathcal{O}(10^2)$	493.1± 23.2	480.8
$M_{\tilde{t}_R}$	415.8±$\mathcal{O}(10^2)$	434.7±$\mathcal{O}(4\cdot 10^2)$	412.7± 63.2	408.3
$M_{\tilde{b}_R}$	501.7± 17.9	fixed 500	502.4± 23.8	502.9
$M_{\tilde{q}_L}$	524.6± 14.5	fixed 500	526.1± 7.2	526.6
$M_{\tilde{q}_R}$	507.3± 17.5	fixed 500	509.0± 19.2	508.1
A_τ	fixed 0	613.4± $\mathcal{O}(10^4)$	764.7±$\mathcal{O}(10^4)$	-249.4
A_t	-509.1± 86.7	-524.1± $\mathcal{O}(10^3)$	-493.1± 262.9	-490.9
A_b	fixed 0	fixed 0	199.6±$\mathcal{O}(10^4)$	-763.4
$A_{l1,2}$	fixed 0	fixed 0	fixed 0	-251.1
$A_{u1,2}$	fixed 0	fixed 0	fixed 0	-657.2
$A_{d1,2}$	fixed 0	fixed 0	fixed 0	-821.8
m_A	406.3±$\mathcal{O}(10^3)$	393.8± 1.6	393.7± 1.6	394.9
μ	350.5± 14.5	354.8± 3.1	354.7± 3.0	353.7

Example

Test function (5-dim):

- Small Hypersphere $r = 100$, $V_{\text{max}} = 75$ @ $(650, 250, 350, 350, 350)$
- Cuboid $d = (173, 120, 200, 200, 200)$, $V_{\text{max}} = 60$ @ $(850, 225, 650, 650, 650)$
- Cube $d = (100, 100, 300, 300, 300)$, $V_{\text{max}} = 25$ @ $(750, 750, 450, 450, 450)$
- Gaussian $\sigma = (50, 150, 150, 150, 150)$, $V_{\text{max}} = 16$ @ $(250, 250, 550, 550, 550)$
- Big Hypersphere $r = 300$, $V_{\text{max}} = 12$ @ $(350, 650, 650, 650, 650)$
- Background $V = 0.1 + 4 \cdot 10^{-30} \cdot x_1^2 x_2^2 x_3^2 x_4^2 x_5^2$




```
1. V=74.929@(655.00,253.72,347.83,348.57,349.59)
2. V=59.972@(850.04,224.99,650.00,649.99,654.56)
3. V=58.219@(849.97,225.01,587.08,650.01,650.02)
4. V=25.110@(750.00,749.99,450.00,450.01,450.01)
5. V=16.042@(245.45,253.44,552.51,542.58,544.75)
6. V=12.116@(350.70,650.40,650.36,650.40,650.38)
7. ...
```

Plot Details

- Parameters: $x_1, \ldots, x_5 \in [0, 1000]$
- Bins: 50×50
- PDF: Breit-Wigner ($\frac{1}{1+\Delta x_i^2/\sigma^2}$) with $\sigma = 100$
- Number of Markov chains: 9
- Number of points per chain: 10^7
- Number of function evaluations: $33,797,153$
- Acceptance ratio: 0.19
- Final r (measure of convergence): 1.815
- CPU time (3 GHz): 150 min