Reconstructing Supersymmetry

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arXiv:0709.3985 [hep-ph]

Determining SUSY parameters

nowadays:

Parameters in the Lagrangian

m_0, μ, $\tan(\beta)$, $M_{\{1,2,3\}}$, ...

Feynman diagrams, RG evolution, ...

Observables:

- Masses
- Kinematic endpoints
- Cross sections
- Branching ratios
- ...

after SUSY discovery:

Observables

m_{h^0}, $\Delta m_{\tilde{g}\chi_1^0}$, three-particle edge($\chi_4^0$,$\tilde{e}_L$,$\chi_1^0$), BR, ...

?

Lagrangian parameters

M_1	$\square \pm \square$ GeV
M_2	$\square \pm \square$ GeV
M_3	$\square \pm \square$ GeV
μ	$\square \pm \square$ GeV
$\tan\beta$	$\square \pm \square$
...	...

$\Rightarrow$ Tools to reconstruct SUSY parameters

What SFitter does

- Take a set of measurements
  (LHC kinematic edges, thresholds, masses, mass differences, cross sections, BRs;
  Indirect constraints electro-weak: M_W, $\sin^2\theta_W$, $(g-2)_\mu$;
  flavour: BR$(b \to s\gamma)$, BR$(B_s \to \mu^+\mu^-)$; dark matter: Ωh^2
- Compare to theoretical predictions
  Spectrum calculators: SoftSUSY, SuSPECT, ISASUSY
  Cross sections and BRs: Prospino2, MsmLib, SUSYHit (HDecay + SDecay)
  Indirect Constraints: micrOMEGAs, $(g-2)_\mu$
- Find best points (best χ^2) using different fitting techniques:
  - Gradient search (Minuit) (+ Reasonably fast; − Limited convergence, only best fit)
  - fixed Grid scan (+ scans complete parameter space; − many points needed ($\mathcal{O}(e^N)$))
  - (Simulated Annealing → Fittino) [Bechtle, Desch, Wienemann]
  - Weighted Markov Chains
- Two types of output: exclusive likelihood map $p(d|m)$, list of best points with errors
- Remove directions to plot maps:
  - Bayesian: marginalisation: $p(m|d) = p(d|m)p(m)$ with prior (theory bias) $p(m)$
  - Frequentist: profile likelihood: best-fitting parameter point $\max_m p(d|m)$

Weighted Markov Chains

Markov Chains:

- Picks a set of “typical” points according to a potential V (e.g. inverse log-likelihood, $1/\chi^2$)
- Scans higs-dimensional parameter spaces efficiently

[Baltz, Gondolo; Allanach, Cranmer et al.; Roszkowski et al.]

Weighted Markov Chains:

Improved evaluation algorithm for binning: [Plehn, MR]

- Weight points with value of V:
- Take care of
  - Overcounting because point density is already weighted ($\frac{\text{number of points}}{\sum_{\text{points}} 1/V(\text{point})}$)
    [based on Ferrenberg, Swendsen 1988]
  - Correct account for regions with zero probability
    (maintain additional chain which stores points rejected because $V(\text{point}) = 0$)

$+$ Fast scans of high-dimensional spaces $\mathcal{O}(N)$
$+$ Does not rely on shape of χ^2 (no derivatives used)
$+$ Can find secondary distinct solutions
$-$ Exact minimum difficult to find $\Rightarrow$ Additional gradient fit
$-$ Bad choice of proposal function for next point leads to bad coverage of the space

Weak-scale MSSM

- No need to assume specific SUSY-breaking scenario
  $\Rightarrow$ SUSY-breaking mechanism should be induced from data
- Use of Markov Chains makes scanning the 19-dimensional parameter space feasible
- Lack of sensitivity on one parameter does not slow down the scan
  (no need to fix parameters)
- Experimental Input: LHC kinematic edges
- Parameter Point: SPS1a
  $m_0 = 100$ GeV , $m_{1/2} = 250$ GeV , $A_0 = -100$ GeV , $\tan\beta = 10$, $\mathrm{sgn}\,\mu = +1$

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Full scan of 19D parameter space challenging
Four-step procedure yields better and faster results:

- Weighted-Markov-Chain run with flat pdf over full parameter space
  5 best points additionally minimised
  (full scan, no bias on starting point)

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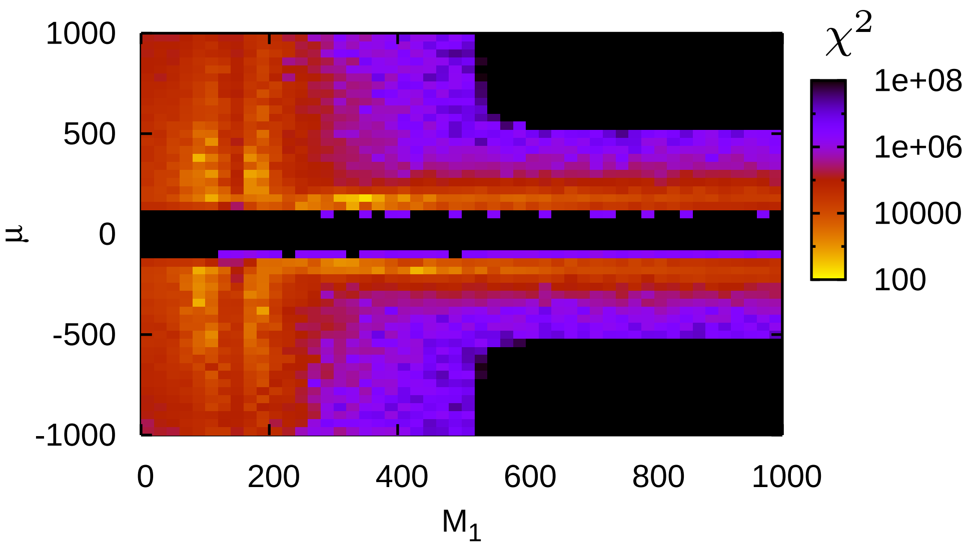
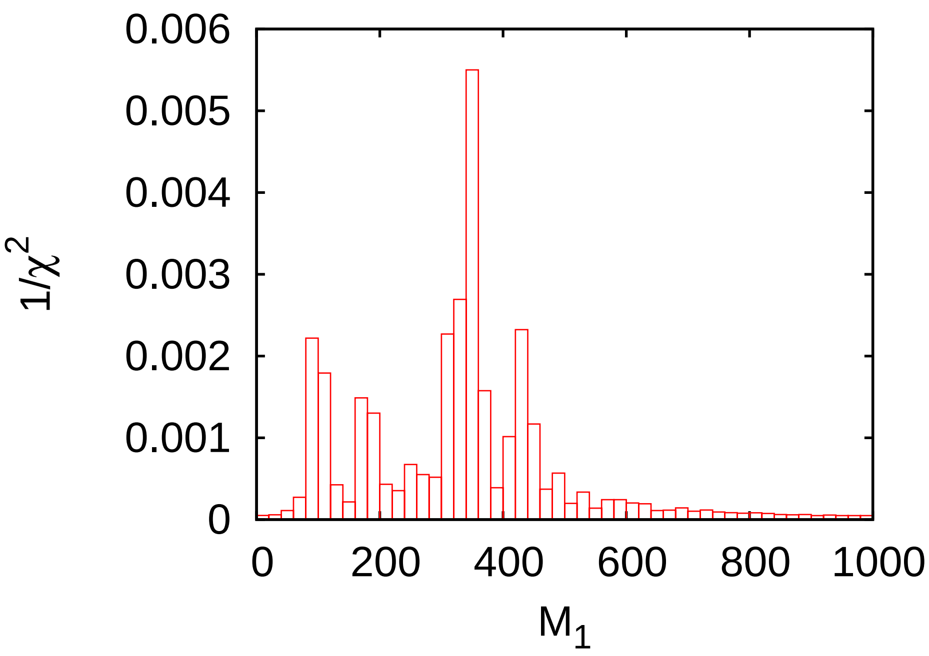
- Weighted-Markov Chain with flat pdf on Gaugino-Higgsino subspace:
  M_1, M_2, M_3, μ, $\tan\beta$, m_t
  Additional Minuit run with 15 best solutions

Search Strategy (2) - results

- Only three neutralinos (χ_1^0, χ_2^0, χ_4^0) with masses (97.2 GeV , 180.5 GeV , 375.6 GeV) and no charginos observable at the LHC in SPS1a
- $\Rightarrow$ Mapping (M_1, M_2, μ) $\rightarrow$ (χ_1^0, χ_2^0, χ_4^0) not unique
- $\operatorname{sgn}\mu$ basically undetermined by collider data
- $\Rightarrow$ 8-fold solution





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	$\mu < 0$				$\mu > 0$			
					SPS1a			
M_1	96.6	175.1	103.5	365.8	98.3	176.4	105.9	365.3
M_2	181.2	98.4	350.0	130.9	187.5	103.9	348.4	137.8
μ	-354.1	-357.6	-177.7	-159.9	347.8	352.6	178.0	161.5
$\tan\beta$	14.6	14.5	29.1	32.1	15.0	14.8	29.2	32.1
M_3	583.2	583.3	583.3	583.5	583.1	583.1	583.3	583.4
m_t	171.4	171.4	171.4	171.4	171.4	171.4	171.4	171.4

Search Strategy (3+4)

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  Additional Minuit run with 15 best solutions
- Weighted-Markov Chain with Breit-Wigner-shaped pdf on remaining parameters for all solutions of previous step
  Minimisation for best 5 points
- Minuit run for best points of last step keeping all parameters variable

Under-determination of parameter space:

- Only 15 independent measurements for 19 parameters
- 5 parameters only determined via m_h^0:
  m_A, $M_{\tilde{t}_R}$, A_t, heavier of $M_{\tilde{\tau}_L}$ or $M_{\tilde{\tau}_R}$, $\tan\beta$
- Can still assign errors to some of these parameters

Error analysis

Errors using LHC kinematic edges as input data:

	no theory errors		with theory errors		SPS1a
$\tan\beta$	9.8$\pm$	2.3	10.0$\pm$	4.5	10.0
M_1	101.5$\pm$	4.6	102.1$\pm$	7.8	103.1
M_2	191.7$\pm$	4.8	193.3$\pm$	7.8	192.9
M_3	575.7$\pm$	7.7	577.2$\pm$	14.5	577.9
μ	350.9$\pm$	7.3	350.5$\pm$	14.5	353.7
$M_{\tilde{\mu}_R}$	134.0$\pm$	4.8	135.0$\pm$	8.3	135.8
$M_{\tilde{q}_R}$	506.2$\pm$	11.7	507.3$\pm$	17.5	508.1

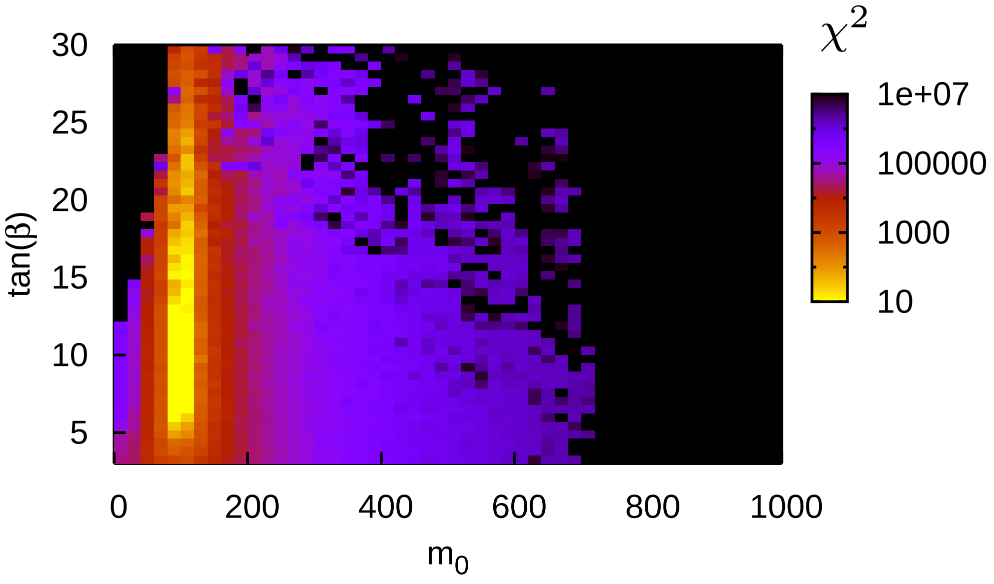
- Theory errors box-shaped (RFit scheme [Hoecker, Lacker, Laplace, Lediberder])
- Errors determined via 10,000 Toy Monte Carlos
  - Generate pseudo measurements smeared according to errors
  - Plot distribution of best-fit points
- Errors assume we know which of the discrete solutions is correct
- Need to factor in ignorance of this as well

Improving on $\tan\beta$

- difficult even in mSUGRA
  Analysis with LHC kinematic edges as before, but now assume mSUGRA scenario:

	SPS1a	$\Delta_{zero}^{theo-exp}$	$\Delta_{zero}^{theo-exp}$	$\Delta_{gauss}^{theo-exp}$	$\Delta_{flat}^{theo-exp}$
		LHC masses	LHC edges		
$\tan\beta$	10	1.69	0.65	3.36	2.45

profile likelihood:



Improving on $\tan\beta$

- difficult even in mSUGRA
- LHC rates: $\tan\beta$ from heavy Higgses (for large $\tan\beta$) [Kinnunen, Lehti, Moortgat, Nikitenko, Spira]
  $\propto (\tan\beta)^2$
  $\rightarrow$ in general: challenging
  $\rightarrow$ for SPS1a: no heavy Higgses observed

Improving on $\tan\beta$

- difficult even in mSUGRA
- LHC rates: $\tan\beta$ from heavy Higgses (for large $\tan\beta$) [Kinnunen, Lehti, Moortgat, Nikitenko, Spira]
- $B_s \to \mu^+\mu^-$ [Jäger, Spannowsky, SFitter]
  - $\propto (\tan\beta)^6$
  - LHCb will be able to probe the SM value
  - Errors largely theory-dominated (main source: f_{B_s} from lattice simulations)
  - In a simple proof-of-concept analysis: $\tan\beta = 30 \pm 6.5$

Improving on $\tan\beta$

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- LHC rates: $\tan\beta$ from heavy Higgses (for large $\tan\beta$) [Kinnunen, Lehti, Moortgat, Nikitenko, Spira]
- $B_s \to \mu^+\mu^-$ [Jäger, Spannowsky, SFitter]
- anomaluos magnetic moment of the muon [review: Stöckinger]

$\tan\beta$ from other sectors

Electro-weak sector: Anomalous Magnetic Moment of the Muon $(g-2)_\mu$ [Alexander, Kreiss, SFitter]

- Currently 3.4σ deviation from Standard Model
- Leading order $\simeq 130 \cdot 10^{-11}\ \tan\beta\ \mathrm{sgn}(\mu)\ \left(\frac{100\ \text{GeV}}{M_{\text{SUSY}}}\right)^2$
- $\Rightarrow$ Favours one sign of $\mu \rightarrow +$
- $\Rightarrow$ Linearly sensitive on $\tan\beta$

	LHC	LHC $\otimes (g-2)\mu$	SPS1a
$\tan\beta$	**10.0± 4.5**	**10.3± 2.0**	**10.0**
M_1	102.1± 7.8	102.7± 5.9	103.1
M_2	193.3± 7.8	193.2± 5.8	192.9
M_3	577.2±14.5	578.2± 12.1	577.9
μ	350.5±14.5	352.5± 10.8	353.7
$M_{\tilde{\mu}_R}$	135.0± 8.3	135.6± 6.3	135.8
$M_{\tilde{q}_R}$	507.3±17.5	507.6± 15.8	508.1

$\Rightarrow$ Need to combine information on $\tan\beta$ from all sectors

Summary & Outlook

The SFitter program:

- High-dimensional parameter scans important to determine Lagrangian parameters from observables
- Improved Weighted Markov Chain algorithm can do this efficiently
- Two types of SFitter output: Likelihood map and list of best points

MSSM analysis:

- Degenerate solutions in gaugino-higgsino-sector and general underdetermination of parameter space, in particular $\tan\beta$
- Large $\tan\beta$ easier due to more measurements being available
- Additional $(g-2)_\mu$ measurement greatly reduces errors
- Need information from all sectors to constrain it