Supersymmetry at the LHC

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Outline

1. Supersymmetry
2. Higgs-Boson Production via Vector-Boson Fusion
3. Reconstructing the Supersymmetric Lagrangian with SFitter
Remember the Standard Model

- Poincaré group of space-time transformations
- Gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$
- Two massless gauge bosons: gluon and photon
- $SU(2)_L \otimes U(1)_Y$ broken to $U(1)_{em}$ by the Higgs field with vacuum expectation value $v = 246$ GeV
- Massive electroweak gauge bosons: $W$ and $Z$
- One physical Higgs boson (not yet discovered, unknown mass)
- Two types of matter: colour-charged: quarks, colour-neutral: leptons and neutrinos
- Three generations each
- Yukawa coupling of matter to the Higgs proportional to the mass
Current Status

- Standard Model experimentally very well confirmed
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- Few experimental deviations
  - Dark matter
    - $\sim 23\%$ Dark Matter content in the universe
    - Possible candidate in the SM: Neutrinos
      - neutrino mass limits prevent accounting for total content
  - $M_W$
    - $\sim 1\sigma$ deviation
  - $g - 2$ of the Muon
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      - $\rightarrow$ GUT scale (?))
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      - \( \rightarrow \) GUT scale (?)
- Look for possible ultra-violet completions \( \rightarrow \) Supersymmetry
Supersymmetry

Symmetry between bosons and fermions:

\[ Q |\text{boson}\rangle = |\text{fermion}\rangle; \quad Q |\text{fermion}\rangle = |\text{boson}\rangle \]

\[ \{Q, \bar{Q}\} = 2\sigma^\mu P_\mu \]

\[ \{Q, Q\} = \{\bar{Q}, \bar{Q}\} = 0 \]

\[ [P_\mu, Q] = [P_\mu, \bar{Q}] = 0 \]

\(Q\): Supersymmetry Operator

\(P_\mu\): Generator of Lorentz translations

\(\sigma^\mu\): Pauli matrices

⇒ Non-trivial combination of space-time and gauge symmetries

- Equal number of fermionic and bosonic degrees of freedom
- Supersymmetric partner(s) to each particle
- Partners have same mass and same couplings

⇒ Higher-order corrections to the Higgs boson mass cancel
  (Relative minus sign between boson and fermion loops due to spin-statistics theorem)

⇔ Where are the 511 keV charged scalars?
Solution: Soft-supersymmetry breaking

- Explicit breaking of supersymmetry
- Adds additional mass terms to the supersymmetric partners
- Keeps automatic cancellation of quadratic divergencies

Additional ingredients:
(not absolutely necessary, but normally taken to explain the smallness of some effects)

- \( R \)-Parity conservation:
  - Discrete symmetry: \( P^R = +1 \) for SM particles, \( P^R = -1 \) for supersymmetric partners
  - Induces baryon and lepton number conservation
  - \( \Rightarrow \) Proton decay forbidden (otherwise \( \tau \approx O(\text{hours}) \))
  - \( \Rightarrow \) SUSY particles can only be produced in pairs
  - \( \Rightarrow \) Lightest supersymmetric particle (LSP) stable

- \( CP \)-conservation:
  - \( CP \)-violation strongly constrained by experimental data (EDMs)
  - Assume \( CKM \) matrix is the only source of \( CP \)-violation
  - Set all SUSY contributions to zero.
Simplest phenomenologically viable model: Minimal Supersymmetric Standard Model (MSSM)

- Supersymmetric partner to each Standard Model particle
- Two Higgs doublets $\Rightarrow$ 5 Higgs bosons $(h^0, H^0, A^0, H^\pm)$
  \[ m_{h^0} \lesssim 140 \text{ GeV} \]
- Particles with same quantum numbers mix
  - Zino, Photino, 2 neutral Higgsino $\rightarrow$ 4 Neutralino
  - Wino, charged Higgsino $\rightarrow$ 2 Chargino
Unification at the GUT scale ($\sim 10^{16}$ GeV):

- Apparent unification of gauge couplings (general feature of MSSM)
- Common scalar mass: $m_0$
- Common sfermion mass: $m_{1/2}$
- Common trilinear coupling: $A_0$

plus

- ratio of the Higgs vacuum expectation values at the electro-weak scale: $\tan \beta = \frac{v_2}{v_1}$
- one sign: $\sgn \mu$

Evolve three parameters defined at the GUT scale via renormalisation group equations down to electro-weak scale:

⇒ Weak-scale MSSM parameters
⇒ Masses and couplings
Current Fits

If TeV-scale supersymmetry is out there, what can we tell today?
→ Fits of current data to supersymmetry (only mSUGRA)

[Roszkowski, Ruiz de Austra, Trotta 2006/7]

Observables:

- Dark Matter $\Omega_{DM} h^2$
- $g - 2 \mu$
- $M_W$
- $\sin^2 \theta_W$
- $\text{BR}(b \rightarrow s \gamma)$
- $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$
- ... 

$\Rightarrow$ Predictions for SUSY mass spectrum
Current Fits

Predictions for SUSY mass spectrum

[plots by Allanach et al.]

- Low-energy TeV-scale SUSY fits data very well
- Mass ranges for SUSY particles
- Mass of the lightest Higgs boson compatible with LEP limit

[right-hand plot by Buchmüller et al.]
Higgs-Boson Production via Vector-Boson Fusion

(in collaboration with Wolfgang Hollik, Tilman Plehn, Heidi Rzehak)
Main (MSSM-)Higgs Boson production modes:

- **Gluon-Gluon Fusion**

- **Vector-Boson Fusion**

- **Associated Production with a Gauge Boson**

- **Associated Production with Top-Quark–Antiquark Pair**
A Higgs event

[picture by Alan Walker/CERN]
CMS event $H \rightarrow ZZ \rightarrow e^+e^- q\bar{q}$
Higgs Production via Vector Boson Fusion

- Second-largest production cross section of Higgs bosons at the LHC (after gluon-gluon fusion)
- Distinct kinematic properties:
  - 2 jets in forward regions of the detector
  - Reduced jet activity in central region
  - Central Higgs boson
- Most promising channel for early discovery of the Higgs with $h^0 \rightarrow \tau^+ \tau^-$ decay
- SM-NLO corrections completely known:
  - $O(\alpha_s) \sim 5 - 10\%$, $O(\alpha) \sim 5\%$
  - [Djouadi, Spira, Zerwas; Han, Valencia, Willenbrock]
  - [Figy, Oleari, Zeppenfeld; Berger, Campbell]
  - [Ciccolini, Denner, Dittmaier]
- Vector-boson-fusion–gluon-guon interference negligible
  - [Andersen, Binoth, Heinrich, Smillie; Bredenstein, Hagiwara, Jäger]
Corrected Tree-level Result

Conversion from the known SM result to the MSSM:

- Replace SM-Higgs boson with (SM-like) MSSM $h^0$-Boson
- Vector-Boson–Vector-Boson–Higgs coupling modified by
  \[ \Gamma_{WW h^0, ZZ h^0}^{MSSM} = \Gamma_{WW, ZZ}^{SM} \cdot \sin(\beta - \alpha) \]
- Therefore change of total cross section as
  \[ \sigma_{MSSM} = \sigma_{SM} \cdot \sin(\beta - \alpha)^2 \]

- \( \sin(\beta - \alpha)^2 \) close to 1 for large parts of the parameter space
- \( \Rightarrow \) Couplings of the $h^0$ SM-like
- Loop corrections induced by SUSY particles?
- Additional contributions to cross sections

[Please, Rainwater, Zeppenfeld 1999]
Additional Loop Corrections

- SM is subsector of the full MSSM
- \( \Rightarrow \) SM loop corrections form part of the full MSSM set
- R-parity conservation allows separation of SM and SUSY part at one-loop level

\[
\begin{align*}
\text{(loop consists either of SM} & \quad \text{or SUSY} \\
\text{ particles)}
\end{align*}
\]

- Not completely true in the Higgs sector:
  - MSSM is a (type-II) Two-Higgs doublet model (THDM)
  - Some THDM parameters fixed by SUSY relations (e.g. \( m_{h,0} \) not a free parameter any longer)
  - Renormalisation in the Higgs sector requires both SM and SUSY part so that divergencies cancel (depending on renormalization scheme)
  - Split between SM and additional SUSY contribution more difficult
SUSY = MSSM - SM

- SM part (QCD and EW) already calculated
- Simple transfer to MSSM by \( \sigma^{SM(MSSM)} = \sigma^{SM} \cdot \sin(\beta - \alpha)^2 \)
- In the end want one-loop corrections for complete MSSM
- \( \Rightarrow \) Subtract SM part from MSSM to obtain additional SUSY contribution

\[ \begin{align*}
  \text{MSSM} & \quad \text{--} \quad \text{--} \quad \text{MSSM} \\
  \text{SM} & \quad \text{--} \quad \text{--} \quad \text{SM} \quad \cdot \sin(\beta - \alpha)^2 
\end{align*} \]

using \( m_{H}^{SM} = m_{h_0}^{MSSM} \)

- Subtraction performed on amplitude level
- Cross-check that for non-Higgs couplings this corresponds to just omitting the SM particles
Strong \( O(\alpha_s) \) corrections:

\[
\begin{align*}
q & \quad \tilde{g} & \quad q' \\
\tilde{q} & \quad \tilde{q}' & \quad W, Z
\end{align*}
\]

\[
\begin{align*}
q'' & \quad q''' & \quad h^0
\end{align*}
\]

Additional possibility for pentagon diagrams:

\[
\begin{align*}
q & \quad \tilde{g} & \quad \tilde{q} & \quad h^0 \\
\tilde{q} & \quad \tilde{q}' & \quad \tilde{g}
\end{align*}
\]

\[\propto \alpha_s^2 \alpha^2\]

Using \( \alpha_s^2 \sim \alpha \) same order as tree-level

However, not same kinematic structure as tree-level diagram

\( \Rightarrow \) Greatly reduced by kinematic cuts
Numerical Results

Size of the supersymmetric corrections:

(SPS: Set of reference points which probe “typical” parts of the supersymmetric parameter space)

<table>
<thead>
<tr>
<th></th>
<th>$\Delta \sigma / \sigma$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$WWh$</td>
</tr>
<tr>
<td>SPS1a</td>
<td>-0.329</td>
</tr>
<tr>
<td>SPS1b</td>
<td>-0.162</td>
</tr>
<tr>
<td>SPS2</td>
<td>-0.147</td>
</tr>
<tr>
<td>SPS3</td>
<td>-0.146</td>
</tr>
<tr>
<td>SPS4</td>
<td>-0.258</td>
</tr>
<tr>
<td>SPS5</td>
<td>-0.606</td>
</tr>
<tr>
<td>SPS6</td>
<td>-0.226</td>
</tr>
<tr>
<td>SPS7</td>
<td>-0.206</td>
</tr>
<tr>
<td>SPS8</td>
<td>-0.157</td>
</tr>
<tr>
<td>SPS9</td>
<td>-0.094</td>
</tr>
</tbody>
</table>

⇒ Typical corrections at or below 1%
⇒ Can reach up to 4% for parameter points still allowed
⇒ Need to be included for a precision analysis of the Higgs sector
Reconstructing the Supersymmetric Lagrangian with Sfitter

(in collaboration with Rémi Lafaye, Tilman Plehn, Dirk Zerwas)
Determining SUSY parameters

nowadays:

**Parameters in the Lagrangian**

\[ m_0, \mu, \tan(\beta), M_{1,2,3}, \ldots \]

Feynman diagrams, RG evolution, ...

Observables:

- Masses
- Kinematic endpoints
- Cross sections
- Branching ratios
- …

after SUSY discovery:

**Observables**

\[ m_{h^0}, \Delta m_{\tilde{g}\chi_1^0}, \text{three-particle edge}(\chi_4^0, \tilde{e}_L, \chi_1^0), \text{BR}, \ldots \]

**Lagrangian parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_1 )</td>
<td>± GeV</td>
</tr>
<tr>
<td>( M_2 )</td>
<td>± GeV</td>
</tr>
<tr>
<td>( M_3 )</td>
<td>± GeV</td>
</tr>
<tr>
<td>( \mu )</td>
<td>± GeV</td>
</tr>
<tr>
<td>( \tan \beta )</td>
<td>±</td>
</tr>
<tr>
<td>…</td>
<td>…</td>
</tr>
</tbody>
</table>

⇒ Tools to reconstruct SUSY parameters
What SFitter does

- Set of measurements
  - LHC measurements:
    - kinematic edges, thresholds, masses, mass differences
    - cross sections, branching ratios
  - ILC measurements
  - Indirect Constraints
    - electro-weak: \( M_W, \sin^2 \theta_W; \) \( (g - 2)_\mu \)
    - flavour: \( \text{BR}(b \rightarrow s\gamma), \text{BR}(B_s \rightarrow \mu^+ \mu^-); \) dark matter: \( \Omega h^2 \)
  - or even ATLAS and CMS measurements separately

- Compare to theoretical predictions
  - Spectrum calculators: SoftSUSY, SuSPECT, ISASUSY
    [Allanach; Djouadi, Kneur, Moulata; Baer, Paige, Protopopescu, Tata]
  - LHC cross sections: Prospino2
    [Plehn et al.]
  - LC cross sections: MsmLib
    [Ganis]
  - Branching Ratios: SUSYHit (HDecay + SDecay)
    [Djouadi, Muhlleitner, Spira]
  - micrOMEGAs
    [Bélanger, Boudjema, Pukhov, Semenov]
  - g-2
    [Alexander, Stöckinger]

- Using as glue: SLHAio
  [Kreiss]
MSSM parameter space is high-dimensional:
- SM: 3+ parameters ($m_t, \alpha_s, \alpha, \ldots$)
- mSUGRA: 5 parameters ($m_0, m_{1/2}, A_0, \tan(\beta), \text{sgn}(\mu)$)
- General MSSM: 105 parameters

On loop-level observables depend on every parameter
Simple inversion of the relations not possible
⇒ Parameter scans

Error estimates on parameters in the minimum
Parameter Scans

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Find best points (best $\chi^2$) using different fitting techniques:

- Gradient search (Minuit) $\left(\begin{array}{c} + \text{Reasonably fast} \\ - \text{Limited convergence, only best fit} \end{array}\right)$
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- fixed Grid scan ($\begin{array}{c} + \text{scans complete parameter space} \\ - \text{many points needed ($O(e^N)$)} \end{array}$)
  (Simulated Annealing $\rightarrow$ Fittino)

- Weighted Markov Chains

[Bechtle, Desch, Wienemann]
Markov Chain (MC):

- Sequence of points, chosen by an algorithm (Metropolis-Hastings), only depending on its direct predecessor
- Picks a set of "average" points according to a potential $V$ (e.g. inverse log-likelihood, $1/\chi^2$)
- Point density resembles the value of $V$ (i.e. more points in region with high $V$)
- Scans high dimensional parameter spaces efficiently

[mSUGRA MC scans with current exp. limits] [Baltz, Gondolo 2004]

[Allanach, Cranmer, Lester, Weber 2005-7; Roszkowski, Ruiz de Austra, Trotta 2006/7]
Weighted Markov Chains:

Improved evaluation algorithm for binning:  

- Weight points with value of $V$:
  - Take care of
    - Overcounting because point density is already weighted ($\frac{\text{number of points}}{\sum \text{points} \frac{1}{V(\text{point})}}$)  
      [based on Ferrenberg, Swendsen 1988]
    - Correct account for regions with zero probability
      (maintain additional chain which stores points rejected because $V(\text{point}) = 0$)

  - Fast scans of high-dimensional spaces $O(N)$
  - Does not rely on shape of $\chi^2$ (no derivatives used)
  - Can find secondary distinct solutions
    - Exact minimum difficult to find $\Rightarrow$ Additional gradient fit
    - Bad choice of proposal function for next point leads to bad coverage of the space
mSUGRA SPS1a as a benchmark point:
\[ m_0 = 100 \text{ GeV} , \ m_{1/2} = 250 \text{ GeV} , \ A_0 = -100 \text{ GeV} , \ \tan \beta = 10, \ sgn \mu = +1, \ m_t = 171.4 \text{ GeV} \]

LHC “experimental” data from cascade decays (best precision obtainable)

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Value (GeV)</th>
<th>Errors (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>([m_{llq}^{\text{max}}]:\text{Edge}(\tilde{q}_L, \chi_2^0, \chi_1^0))</td>
<td>446.44</td>
<td>1.4 4.3</td>
</tr>
<tr>
<td>([m_{llq}^{\text{min}}]:\text{Thres}(\tilde{q}_L, \chi_2^0, \tilde{\mu}_R, \chi_1^0))</td>
<td>211.95</td>
<td>1.6 2.0</td>
</tr>
<tr>
<td>([m_{lq}^{\text{low}}]:\text{Edge}(\tilde{c}_L, \chi_2^0, \tilde{\mu}_R))</td>
<td>316.51</td>
<td>0.9 3.0</td>
</tr>
<tr>
<td>([m_{lq}^{\text{high}}]:\text{Edge}(\tilde{c}_L, \chi_2^0, \tilde{\mu}_R, \chi_1^0))</td>
<td>392.80</td>
<td>1.0 3.8</td>
</tr>
</tbody>
</table>

Theoretical Errors:

- **mSUGRA**: 3% for gluino and squark masses, 1% for all other sparticle masses
- **MSSM**: 1% for gluino and squark masses, 0.5% for all other sparticle masses
- **\(m_{h^0}\)**: 2 GeV (unknown higher order terms)
mSUGRA as a Toy Model

mSUGRA with LHC measurements (SPS1a kinematic edges):
pick one set of ”measurements”, randomly smeared from the true values

Free parameters:
\( m_0, m_{1/2}, \tan(\beta), A_0, \text{sgn}(\mu), m_t \)

SFitter output 1:
Fully-dimensional exclusive likelihood map
(colour: minimum \( \chi^2 \) over all unseen parameters)

SFitter output 2:
Ranked list of minima:

<table>
<thead>
<tr>
<th></th>
<th>( \chi^2 )</th>
<th>( m_0 )</th>
<th>( m_{1/2} )</th>
<th>( \tan(\beta) )</th>
<th>( A_0 )</th>
<th>( \mu )</th>
<th>( m_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPS1a</td>
<td></td>
<td>100.0</td>
<td>250.0</td>
<td>10.0</td>
<td>-100.0</td>
<td>+</td>
<td>171.4</td>
</tr>
<tr>
<td>1)</td>
<td>0.09</td>
<td>102.0</td>
<td>254.0</td>
<td>11.5</td>
<td>-95.2</td>
<td>+</td>
<td>172.4</td>
</tr>
<tr>
<td>2)</td>
<td>1.50</td>
<td>104.8</td>
<td>242.1</td>
<td>12.9</td>
<td>-174.4</td>
<td>–</td>
<td>172.3</td>
</tr>
<tr>
<td>3)</td>
<td>73.2</td>
<td>108.1</td>
<td>266.4</td>
<td>14.6</td>
<td>742.4</td>
<td>+</td>
<td>173.7</td>
</tr>
<tr>
<td>4)</td>
<td>139.5</td>
<td>112.1</td>
<td>261.0</td>
<td>18.0</td>
<td>632.6</td>
<td>–</td>
<td>173.0</td>
</tr>
</tbody>
</table>
Error determination

Treatment of errors:
- All experimental errors are Gaussian
  \[ \sigma_{\text{exp}}^2 = \sigma_{\text{stat}}^2 + \sigma_{\text{syst(j)}}^2 + \sigma_{\text{syst(l)}}^2 \]
- Systematic errors from jet (\(\sigma_{\text{syst(j)}}\)) and lepton energy scale (\(\sigma_{\text{syst(l)}}\)) assumed 99% correlated each
- Theory error added as box-shaped (RFit scheme [Hoecker, Lacker, Laplace, Lediberder])
  \[ \Rightarrow -2 \log L \equiv \chi^2 = \sum_{\text{measurements}} \left\{ \begin{array}{cl}
  0 & \text{for } \left| x_{\text{data}} - x_{\text{pred}} \right| < \sigma_{\text{theo}} \\
  \left( \frac{\left| x_{\text{data}} - x_{\text{pred}} \right| - \sigma_{\text{theo}}}{\sigma_{\text{exp}}} \right)^2 & \text{for } \left| x_{\text{data}} - x_{\text{pred}} \right| \geq \sigma_{\text{theo}}
  \end{array} \right. \]

\[ \Rightarrow \text{Parameter errors:} \]

<table>
<thead>
<tr>
<th>SPS1a</th>
<th>( \Delta_{\text{theo}} - \Delta_{\text{exp}} )</th>
<th>( \Delta_{\text{theo}} - \Delta_{\text{exp}} )</th>
<th>( \Delta_{\text{theo}} - \Delta_{\text{exp}} )</th>
<th>( \Delta_{\text{theo}} - \Delta_{\text{exp}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta_{\text{zero}} )</td>
<td>( \Delta_{\text{zero}} )</td>
<td>( \Delta_{\text{gauss}} )</td>
<td>( \Delta_{\text{flat}} )</td>
</tr>
<tr>
<td></td>
<td>LHC masses</td>
<td>LHC edges</td>
<td>LHC masses</td>
<td>LHC edges</td>
</tr>
<tr>
<td>( m_0 )</td>
<td>100</td>
<td>4.11</td>
<td>0.50</td>
<td>2.97</td>
</tr>
<tr>
<td>( m_{1/2} )</td>
<td>250</td>
<td>1.81</td>
<td>0.73</td>
<td>2.99</td>
</tr>
<tr>
<td>( \tan \beta )</td>
<td>10</td>
<td>1.69</td>
<td>0.65</td>
<td>3.36</td>
</tr>
<tr>
<td>( A_0 )</td>
<td>-100</td>
<td>36.2</td>
<td>21.2</td>
<td>51.5</td>
</tr>
<tr>
<td>( m_t )</td>
<td>171.4</td>
<td>0.94</td>
<td>0.26</td>
<td>0.89</td>
</tr>
</tbody>
</table>

\[ \Rightarrow \text{Use kinematic edges for parameter determination instead of masses} \]
No need to assume specific SUSY-breaking scenario
\[ \Rightarrow \text{SUSY-breaking mechanism should be induced from data} \]

Use of Markov Chains makes scanning the **19-dimensional** parameter space feasible

Lack of sensitivity on one parameter does not slow down the scan
(no need to fix parameters)

Same SFitter output as before: Minima list and Likelihood map
Weak-scale MSSM

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  ⇒ SUSY-breaking mechanism should be induced from data

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  (no need to fix parameters)

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Full scan of 19D parameter space challenging
Four-step procedure yields better and faster results:

- Weighted-Markov-Chain run with flat pdf over full parameter space
  5 best points additionally minimised
  (full scan, no bias on starting point)
Weak-scale MSSM

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  \[ \Rightarrow \text{SUSY-breaking mechanism should be induced from data} \]
- Use of Markov Chains makes scanning the 19-dimensional parameter space feasible
- Lack of sensitivity on one parameter does not slow down the scan
  (no need to fix parameters)
- Same SFitter output as before: Minima list and Likelihood map

Full scan of 19D parameter space challenging
Four-step procedure yields better and faster results:

- Weighted-Markov-Chain run with flat pdf over full parameter space
  5 best points additionally minimised
  (full scan, no bias on starting point)
- Weighted-Markov Chain with flat pdf on Gaugino-Higgsino subspace:
  \[ M_1, M_2, M_3, \mu, \tan \beta, m_t \]
  Additional Minuit run with 15 best solutions
Search Strategy (2) - results

- Only three neutralinos \((\chi^0_1, \chi^0_2, \chi^0_4)\) with masses \((97.2 \text{ GeV}, 180.5 \text{ GeV}, 375.6 \text{ GeV})\) and no charginos observable at the LHC in SPS1a

- \(\Rightarrow\) Mapping \((M_1, M_2, \mu) \rightarrow (\chi^0_1, \chi^0_2, \chi^0_4)\) not unique

- \(\text{sgn} \, \mu\) basically undetermined by collider data

- \(\Rightarrow\) 8-fold solution
Search Strategy (2) - results

- Only three neutralinos ($\chi^0_1, \chi^0_2, \chi^0_4$) with masses (97.2 GeV, 180.5 GeV, 375.6 GeV) and no charginos observable at the LHC in SPS1a

- $\Rightarrow$ Mapping ($M_1, M_2, \mu$) $\rightarrow$ ($\chi^0_1, \chi^0_2, \chi^0_4$) not unique

- $\text{sgn } \mu$ basically undetermined by collider data

- $\Rightarrow$ 8-fold solution

<table>
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<tr>
<th></th>
<th>$\mu &lt; 0$</th>
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<td>96.6 175.1 103.5 365.8</td>
<td>98.3 176.4 105.9 365.3</td>
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<td>181.2 98.4 350.0 130.9</td>
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<tr>
<td>$m_t$</td>
<td>171.4 171.4 171.4 171.4</td>
<td>171.4 171.4 171.4 171.4</td>
</tr>
</tbody>
</table>
Search Strategy (3+4)

Full scan of 19D parameter space challenging
Four-step procedure yields better and faster results:

- Weighted-Markov-Chain run with flat pdf over full parameter space
  5 best points additionally minimised
  (full scan, no bias on starting point)

- Weighted-Markov Chain with flat pdf on Gaugino-Higgsino subspace:
  \( M_1, M_2, M_3, \mu, \tan \beta, m_t \)
  Additional Minuit run with 15 best solutions

- Weighted-Markov Chain with Breit-Wigner-shaped pdf on remaining parameters for all
  solutions of previous step
  Minimisation for best 5 points

- Minuit run for best points of last step keeping all parameters variable
### Best points

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<tr>
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<td>98.3 176.4 105.9 365.3</td>
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<tr>
<td>$M_2$</td>
<td>181.2 98.4 350.0 130.9</td>
<td>187.5 103.9 348.4 137.8</td>
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<tr>
<td>$\mu$</td>
<td>-354.1 -357.6 -177.7 -159.9</td>
<td>347.8 352.6 178.0 161.5</td>
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<tr>
<td>$\tan \beta$</td>
<td>14.6 14.5 29.1 32.1</td>
<td>15.0 14.8 29.2 32.1</td>
</tr>
<tr>
<td>$M_3$</td>
<td>583.2 583.3 583.3 583.5</td>
<td>583.1 583.1 583.3 583.4</td>
</tr>
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<td>128.0 229.9 3269.3 118.6</td>
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<td>2266.5 138.5 129.9 255.1</td>
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<td>192.7 192.7 192.7 192.9</td>
<td>192.6 192.6 192.7 192.8</td>
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<td>131.4 131.4 131.5 131.6</td>
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<tr>
<td>$M_{\tilde{\epsilon}_L}$</td>
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<td>1073.9 920.3 547.9 950.8</td>
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<td>$A t (-)$</td>
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<td>498.5 498.5 498.7 499.6</td>
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<tr>
<td>$A t (+)$</td>
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<td>-470.0 -484.3 -243.4 -465.7</td>
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<td></td>
<td>384.9 481.8 641.5 432.5</td>
<td>739.2 774.7 440.5 656.9</td>
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<td></td>
<td>350.3 725.8 263.1 1020.0</td>
<td>171.6 156.5 897.6 256.1</td>
</tr>
<tr>
<td></td>
<td>171.4 171.4 171.4 171.4</td>
<td>171.4 171.4 171.4 171.4</td>
</tr>
</tbody>
</table>
Degenerate Solutions

- In total 19 parameters constrained by 22 measurements
- Measurements constructed from only 15 underlying masses
- Complete determination of parameter set not possible
- Five parameters not well constrained
  - \( m_A \)
    - no heavy Higgses measurable
  - \( M_{\tilde{t}_R} \)
  - \( A_t \)
    - stop sector parameters do not enter edge measurements
  - \( M_{\tilde{\tau}_L} \) or \( M_{\tilde{\tau}_R} \)
    - only the lighter stau measured
  - \( \tan \beta \)
    - change can always be accommodated by rotating \( M_1, M_2, M_{\tilde{q}_3}, \ldots \)
- Single common link: \( m_{h^0} \)
- 4-dimensional hyperplane in parameter space undetermined
- Can still assign errors to some of the badly determined parameters
Error analysis

Technical procedure as in mSUGRA case:
- 10000 smeared data sets
- Minimum determined for each data set individually
- Error determined from fit with Gaussian

Most constrained parameters determinable with $\sim 5\%$ accuracy

Inclusion of theory errors leads to an increase of factor 2 on the parameter errors

ILC data complementary to LHC

Combination of the two experiments allows for precise determination of all parameters

<table>
<thead>
<tr>
<th></th>
<th>LHC</th>
<th>ILC</th>
<th>LHC+ILC</th>
<th>SPS1a</th>
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<tbody>
<tr>
<td>$M_1$</td>
<td>102.1 ± 7.8</td>
<td>103.0 ± 1.1</td>
<td>103.1 ± 0.84</td>
<td>103.1</td>
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<tr>
<td>$M_{\tilde{e}_R}$</td>
<td>135.0 ± 8.3</td>
<td>135.8 ± 0.81</td>
<td>135.9 ± 0.77</td>
<td>135.8</td>
</tr>
<tr>
<td>$m_A$</td>
<td>406.3 ± $O(10^3)$</td>
<td>393.8 ± 1.6</td>
<td>393.9 ± 1.6</td>
<td>394.9</td>
</tr>
<tr>
<td>$M_{\tilde{t}_R}$</td>
<td>415.8 ± $O(10^2)$</td>
<td>440.0 ± $O(4 \cdot 10^2)$</td>
<td>410.7 ± 48.4</td>
<td>408.3</td>
</tr>
</tbody>
</table>
Higgs-boson production via Vector Boson Fusion important discovery mode for the Higgs boson and to study electroweak symmetry breaking

Supersymmetric QCD corrections strongly suppressed

Supersymmetric electro-weak corrections modify cross section on the percent level

Parameter scans important to determine Lagrangian parameters from observables

Low-energy supersymmetry fits precision data very well

Problem of high-dimensional parameter spaces

Markov Chains can do this effectively

Improved algorithm developed

Two types of output: Likelihood map and list of best points

SFitter (despite its name) not tied to SUSY

→ extend to other models/problems
Backup Slides
Metropolis-Hastings Algorithm

Start point

Probability density function (PDF)

Suggested point

\[
\frac{V(\text{suggested point})}{V(\text{start point})} \overset{?}{>} \text{random number } \in [0, 1)
\]

Yes

Replace

No

Add point to chain
### Experimental Input (edges)

<table>
<thead>
<tr>
<th>(Obs)</th>
<th>= (meas) ± (exp) ± (theo)</th>
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</thead>
<tbody>
<tr>
<td>$m_{h^0}$</td>
<td>$109.53 \pm 0.25 \pm 2.0$</td>
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<tr>
<td>$m_t$</td>
<td>$171.4 \pm 1.0 \pm 0.0$</td>
</tr>
<tr>
<td>$\Delta m_{\tilde{\mu}_L,\chi^0_1}$</td>
<td>$106.26 \pm 1.6 \pm 0.1$</td>
</tr>
<tr>
<td>$\Delta m_{\tilde{g},\chi^0_1}$</td>
<td>$509.96 \pm 2.3 \pm 6.0$</td>
</tr>
<tr>
<td>$\Delta m_{\tilde{c}_R,\chi^0_1}$</td>
<td>$450.52 \pm 10.0 \pm 4.2$</td>
</tr>
<tr>
<td>$\Delta m_{\tilde{g},\tilde{b}_1}$</td>
<td>$98.971 \pm 1.5 \pm 1.0$</td>
</tr>
<tr>
<td>$\Delta m_{\tilde{g},\tilde{b}_2}$</td>
<td>$64.016 \pm 2.5 \pm 0.7$</td>
</tr>
<tr>
<td>Edge($\chi^0_2,\tilde{\mu}_R,\chi^0_1$)</td>
<td>$79.757 \pm 0.03 \pm 0.08 \quad (m_{ll}^{\text{max}})$</td>
</tr>
<tr>
<td>Edge($\tilde{c}_L,\chi^0_2,\chi^0_1$)</td>
<td>$446.44 \pm 1.4 \pm 4.3 \quad (m_{llq}^{\text{max}})$</td>
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<tr>
<td>Edge($\tilde{c}_L,\chi^0_2,\tilde{\mu}_R$)</td>
<td>$316.51 \pm 0.9 \pm 3.0 \quad (m_{lq}^{\text{low}})$</td>
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<tr>
<td>Edge($\tilde{c}_L,\chi^0_2,\tilde{\mu}_R,\chi^0_1$)</td>
<td>$392.8 \pm 1.0 \pm 3.8 \quad (m_{lq}^{\text{high}})$</td>
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<tr>
<td>Edge($\chi^0_4,\tilde{\mu}_R,\chi^0_1$)</td>
<td>$257.41 \pm 2.3 \pm 0.3 \quad (m_{ll}^{\text{max}}(\chi^0_4))$</td>
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<tr>
<td>Edge($\chi^0_4,\tilde{\tau}_L,\chi^0_1$)</td>
<td>$82.993 \pm 5.0 \pm 0.8 \quad (m_{\tau\tau}^{\text{max}})$</td>
</tr>
<tr>
<td>Threshold($\tilde{c}_L,\chi^0_2,\tilde{\mu}_R,\chi^0_1$)</td>
<td>$211.95 \pm 1.6 \pm 2.0 \quad (m_{llq}^{\text{min}})$</td>
</tr>
<tr>
<td>Threshold($\tilde{b}_1,\chi^0_2,\tilde{\mu}_R,\chi^0_1$)</td>
<td>$211.95 \pm 1.6 \pm 2.0 \quad (m_{llb}^{\text{min}})$</td>
</tr>
</tbody>
</table>
mSUGRA around Minima – positive $\mu$

Bayesian

Frequentist
mSUGRA around Minima – negative $\mu$

Bayesian

Frequentist
Error determination

Minuit output not usable for flat theory errors:

- Migrad function depends on parabolic approximation
- Cannot determine $\Delta \chi^2$ for Minos to yield 68% CL intervals

⇒ Need more general approach
- Perform 10,000 toy experiments with measurements smeared around correct value
- Minimise each toy experiment
- Plot resulting distribution of parameter points and fit with Gaussian

Flat theory errors

Gaussian theory errors
### MSSM errors

<table>
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<th>SPS1a</th>
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<td>$\tan \beta$</td>
<td>10.0± 4.5</td>
<td>13.4± 6.8</td>
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<tr>
<td>$M_1$</td>
<td>102.1± 7.8</td>
<td>103.0± 1.1</td>
<td>103.1± 0.84</td>
<td>103.1</td>
</tr>
<tr>
<td>$M_2$</td>
<td>193.3± 7.8</td>
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<td>$M_3$</td>
<td>577.2± 14.5</td>
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<td>579.7± 12.8</td>
<td>577.9</td>
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<tr>
<td>$M_{\tilde{\tau}_L}$</td>
<td>227.8±$\mathcal{O}(10^3)$</td>
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<td>$M_{\tilde{\tau}_R}$</td>
<td>164.1±$\mathcal{O}(10^3)$</td>
<td>143.9± 17.9</td>
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<td>$M_{\tilde{\mu}_L}$</td>
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<td>194.4± 0.89</td>
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<td>171.4± 0.12</td>
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Dark Matter

Content of the universe:

- 73% Dark energy
- 4% Ordinary matter
- 23% Dark matter

MSSM: $\chi^0_1$ as LSP ideal candidate for cold dark matter (CDM): massive, weakly interacting

- SFitter: Determine Lagrangian parameters $\Rightarrow$ Spectrum and couplings
- e.g. micrOMEGAs: Calculate relic density $\Omega_{CDM} h^2 = n_{LSP} m_{LSP}$
  $\Rightarrow$ Prediction of $\Omega_{CDM} h^2$
    - LHC: $\Omega_{CDM} h^2 = 0.1906 \pm 0.0033$
    - LHC+ILC: $\Omega_{CDM} h^2 = 0.1910 \pm 0.0003$
      (improvement by one order of magnitude)
- Compare with experiment
  (Measurement of the fluctuations of the cosmic microwave background):
    - WMAP: $\Omega_{CDM} h^2 = 0.1277 \pm 0.008$ [astro-ph/0603449]
    - Planck: $\Omega_{CDM} h^2 = ? \pm 0.0016$
Apparent unification of gauge coupling parameters in the MSSM

Question arises: Do other parameters unify as well?
⇒ Should be tested by bottom-up running from weak scale to Planck scale
⇒ Can give hints about supersymmetry breaking
   (e.g. test scalar-mass sum rules with a sliding scale)

Bottom-up running of gaugino masses and 3rd-generation sfermion masses:

\[ M_{\tilde{\tau}_R}, M_{\tilde{\tau}_L}, M_{\tilde{\ell}_R}, M_{\tilde{b}_R}, M_{\tilde{q}_3L} ; \Delta M_3 = -10 \text{ GeV} \]
Apparent unification of gauge coupling parameters in the MSSM

Question arises: Do other parameters unify as well?
⇒ Should be tested by bottom-up running from weak scale to Planck scale
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Bottom-up running of gaugino masses and 3rd-generation sfermion masses:

\[ M_{\tilde{\tau}_R}, M_{\tilde{\tau}_L}, M_{\tilde{t}_R}, M_{\tilde{b}_R}, M_{\tilde{q}_{3L}}; \Delta M_3 = -10 \text{ GeV} \]
Example

Test function (5-dim):

- Small Hypersphere $r = 100$, $V_{\text{max}} = 75$ @ $(650, 250, 350, 350, 350)$
- Cuboid $d = (173, 120, 200, 200, 200)$, $V_{\text{max}} = 60$ @ $(850, 225, 650, 650, 650)$
- Cube $d = (100, 100, 300, 300, 300)$, $V_{\text{max}} = 25$ @ $(750, 750, 450, 450, 450)$
- Gaussian $\sigma = (50, 150, 150, 150, 150)$, $V_{\text{max}} = 16$ @ $(250, 250, 550, 550, 550)$
- Big Hypersphere $r = 300$, $V_{\text{max}} = 12$ @ $(350, 650, 650, 650, 650)$
- Background $V = 0.1 + 4 \cdot 10^{-30} \cdot x_1^2 x_2^2 x_3^2 x_4^2 x_5^2$

1. $V = 74.929$ @ $(655.00, 253.72, 347.83, 348.57, 349.59)$
2. $V = 59.972$ @ $(850.04, 224.99, 650.00, 649.99, 654.56)$
3. $V = 58.219$ @ $(849.97, 225.01, 587.08, 650.01, 650.02)$
4. $V = 25.110$ @ $(750.00, 749.99, 450.00, 450.01, 450.01)$
5. $V = 16.042$ @ $(245.45, 253.44, 552.51, 542.58, 544.75)$
6. $V = 12.116$ @ $(350.70, 650.40, 650.36, 650.40, 650.38)$
7. ...
Plot Details

- **Parameters:** $x_1, \ldots, x_5 \in [0, 1000]$
- **Bins:** $50 \times 50$
- **PDF:** Breit-Wigner $(\frac{1}{1+\Delta x_i^2/\sigma^2})$ with $\sigma = 100$
- **Number of Markov chains:** 9
- **Number of points per chain:** $10^7$
- **Number of function evaluations:** 33, 797, 153
- **Acceptance ratio:** 0.19
- **Final $r$ (measure of convergence):** 1.815
- **CPU time (3 GHz):** 150 min