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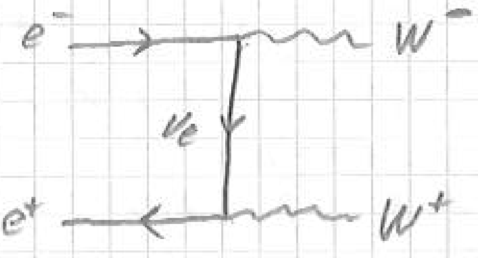
Theoretische Teilchenphysik II, Blatt 4

5 a) $e^-(p_1) + e^+(p_2) \to W^-(p_3) + W^+(p_4)$



+ 3-Eichboson-Wechselwirkungs-Diagramme

Elektron masselos $\Rightarrow$ $\sum_\lambda u(p_1,\lambda)\bar u(p_1,\lambda) = \not p_1$

$\sum_\lambda v(p_2,\lambda)\bar v(p_2,\lambda) = \not p_2$

W-Boson im Hochenergielimes:

$$\sum_\lambda \varepsilon_\mu(q,\lambda)\varepsilon_\nu^*(q,\lambda) \simeq \frac{q_\mu q_\nu}{M_W^2}$$

Also hat das Matrixelement die folgende Form:

$$\mathcal{M}_{fi} = \bar v(p_2)\, ig\gamma^\mu P_L \frac{i}{\not p_1 - \not p_3}\, ig\gamma^\nu P_L u(p_1)\, \varepsilon_\mu^*(p_3)\, \varepsilon_\nu^*(p_4)$$

$$\left(g = \frac{e}{\sqrt{2}\, s_W}\right)$$

quadriert und summiert/gemittelt:

$$\overline{\sum} |\mathcal{M}_{fi}|^2 = \underbrace{\frac{1}{4}}_{\text{Mittelung Fermion-Spins}} \cdot g^4 \cdot \qquad (P_{L/R}\gamma^\mu = \gamma^\mu P_{R/L};\ P_{L/R}^2 = P_{L/R})$$

$$\cdot \mathrm{Tr}\left[\not p_2 \gamma^\mu P_L (\not p_1 - \not p_3)\gamma^\nu P_L \not p_1 \gamma^\rho P_L (\not p_1 - \not p_3)\gamma^\sigma P_L\right]$$

$$\cdot \left(\frac{1}{(p_1-p_3)^2}\right)^2 \frac{p_{4\mu} p_{4\sigma}}{M_W^2} \frac{p_{3\nu} p_{3\rho}}{M_W^2}$$

$$= \frac{g^4}{4(p_1-p_3)^4 M_W^4} \mathrm{Tr}\left[\not p_2 \not p_4 (\not p_1 - \not p_3)\not p_3 \not p_1 \not p_3 (\not p_1 - \not p_3)\not p_4 P_L\right]$$

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$$= \frac{g^4}{4(p_1-p_3)^4 M_W^4} \mathrm{Tr}\Big[\underbrace{\cancel{p_4} \cancel{p_2} \cancel{p_4}}_{= -M_W^2 \cancel{p_2} + 2 p_2\cdot p_4 \cancel{p_4}} \overbrace{(\cancel{p_1}-\cancel{p_3})}^{=-(\cancel{p_2}-\cancel{p_4})} \underbrace{\cancel{p_3} \cancel{p_1} \cancel{p_3}}_{= -\cancel{p_1}\cancel{p_3}\cancel{p_3} + 2 p_1\cdot p_3 \cancel{p_3}} (\cancel{p_1}-\cancel{p_3})\, P_R \Big]$$

$$= -M_W^2 \cancel{p_1} + 2 p_1\cdot p_3 \cancel{p_3}$$

$$= \frac{-g^4}{4(p_1-p_3)^4 M_W^4} \mathrm{Tr}\Big[(-M_W^2 \cancel{p_2} + 2 p_2\cdot p_4 \cancel{p_4})(\cancel{p_2}-\cancel{p_4}) (-M_W^2 \cancel{p_1} + 2 p_1\cdot p_3 \cancel{p_3})(\cancel{p_1}-\cancel{p_3})\, P_R \Big]$$

$$= \frac{-g^4}{4(p_1-p_3)^4 M_W^4} \mathrm{Tr}\Big[(M_W^2 \overbrace{\cancel{p_2}\cancel{p_4}}^{=-\cancel{p_4}\cancel{p_2} + 2p_2\cdot p_4} + 2 p_2\cdot p_4 \cancel{p_4}\cancel{p_2} - 2 M_W^2\, p_2\cdot p_4) (M_W^2 \underbrace{\cancel{p_1}\cancel{p_3}}_{=-\cancel{p_3}\cancel{p_1} + 2p_1\cdot p_3} + 2 p_1\cdot p_3 \cancel{p_3}\cancel{p_1} - 2 M_W^2\, p_1\cdot p_3)\, P_R \Big]$$

$$= \frac{-g^4}{4\underbrace{(p_1-p_3)^4}_{t^2} M_W^4} \mathrm{Tr}\Big[(-t\, \cancel{p_4}\cancel{p_2})(-t\, \cancel{p_3}\cancel{p_1})\, P_R \Big]$$

$$= \frac{-g^4}{4M_W^4} \frac{1}{2}\cdot 4 \cdot \left(p_4\cdot p_2\, p_3\cdot p_1 + p_4\cdot p_1\, p_2\cdot p_3 - p_4\cdot p_3\, p_2\cdot p_1 \right)$$

$$= \frac{-g^4}{8M_W^4} \Big((M_W^2 - t)^2 + \underbrace{(M_W^2 - u)^2}_{=(s+t-M_W^2)^2} - s(s-2M_W^2) \Big)$$

$$\overset{\text{führende Ordnung in } s,t}{=} \frac{-g^4}{8M_W^4} \left(t^2 + \cancel{s^2} + 2s\cdot t + t^2 - \cancel{s^2} \right)$$

$$= \frac{-g^4}{4M_W^4}\, t\cdot(s+t)$$