Outline

(I) $\mathcal{M}$otivating $\mathcal{S}$upersymmetry

Standard Model (SM) describes electromagnetic, weak and strong interactions

Incomplete picture of the Universe

a) observation of cold dark matter CDM

b) non-unified theoretical description of forces/gravity and matter

Supersymmetry (SUSY): Potentially decisive step towards solution in a most "natural" way.

Motivating Supersymmetry

(i) Relates "bosons $\leftrightarrow$ fermions":

$$\left.\begin{array}{l} Q|F>=|B> \\ Q|B>=|F> \end{array}\right\} \text{1 multiplet}$$

(ii) Maximal symmetry of the $\mathcal{S}$-matrix:

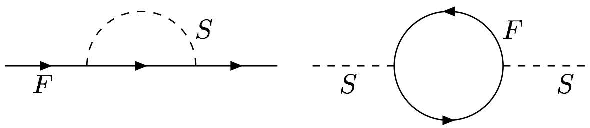
Coleman-Mandula theorem: Bosonic operators cannot extend the Poincaré algebra.
Fermionic operators: $Q \sim$ spin $\frac{1}{2} \Rightarrow$ graded Lie-algebra

(iii) Hierarchy problem:

Standard Model: ELW scale $v \sim 10^2$ GeV – GUT scale $M_{GUT} \sim 10^{16}$ GeV
Fermion masses stable against radiative corrections.

Boson masses unstable:

$$\mathcal{L}_1 = \bar{\psi}(i\partial\!\!\!/ - m_F)\psi + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}m_S^2 S^2 - \frac{\lambda_F}{2}\bar{\psi}\psi S$$



$$\begin{aligned} \delta m_F &= -\frac{3\lambda_F^2 m_F}{64\pi^2}\log\frac{\Lambda^2}{m_F^2} + \ldots \\ \delta m_S^2 &= -\frac{\lambda_F^2}{8\pi^2}\left[\Lambda^2 - m_F^2\log\frac{\Lambda^2}{m_F^2}\right] + \ldots \end{aligned}$$

F : mild log. divergence $\sim m_F \log\Lambda \to 0$ for $m_F \to 0$ ($m_F \to 0 \to \gamma_5$-symmetry)

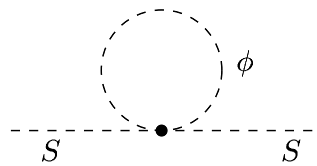
B : quadratic divergence $\sim \Lambda^2$ $\to$ canceled only by fine-tuning of the bare mass term.

Bosonic masses cannot be kept small in a natural way within the presence of high enegy scales

SUPERSYMMETRY:

Bosonic masses can be kept small in a natural way if bosons are related to fermions.

$$\mathcal{L}_2 = |\partial_\mu\phi_1|^2 + |\partial_\mu\phi_2|^2 + \frac{\lambda_S}{2}S^2(|\phi_1|^2 + |\phi_2|^2) - m_\phi^2(|\phi_1|^2 + |\phi_2|^2) \quad [\psi \leftrightarrow \phi_1, \phi_2]$$



$$\delta m_S'^2 = +\frac{\lambda_S^2}{8\pi^2}\left[\Lambda^2 - m_\phi^2\log\frac{\Lambda^2}{m_\phi^2}\right] + \ldots \quad (\pm \text{ Pauli principle})$$

SUSY: $\left.\begin{array}{l}\text{degree of freedoms : 2 fermionic} \leftrightarrow \text{ 2 bosonic} \\ \lambda_F = \lambda_S\end{array}\right\}$ $\delta m_S^2 \sim \frac{\lambda^2}{8\pi^2}(m_F^2 - m_\phi^2)\log\Lambda^2$

Low-energy Supersymmetry:

1.) Doubling of the particle spectrum

2.) Equal coupling constants in the fermionic $\sim$ bosonic couplings

3.) $m_{SM} \sim \mathcal{O}(100 \text{ GeV}) \Rightarrow m_\phi \equiv \tilde{m} \lesssim \mathcal{O}(1 \text{ TeV})$

Cancelation of quadratic divergences by extension of the symmetry; solution of the hierarchy problem by low-energetic SUSY masses $\lesssim \mathcal{O}(1 \text{ TeV})$.

(iv) Higgs mechanism generated via radiative corrections (for $m_t \sim 100...200$ GeV)

(v) Unification of ELM + weak + strong couplings

$$\frac{1}{\alpha_i(Q^2)} = \frac{1}{\alpha_i} - \frac{b_i}{2\pi} \log Q^2 \text{ for } i = U(1), SU(2), SU(3)$$

SM: No single crossing point: order of magnitude deficit.

SUSY: Unification of couplings $\delta\alpha/\alpha \approx 1.5\%$. Depends solely on quantum numbers, independent of mass spectrum beyond ~ 1 TeV.

(vi) Cold Dark Matter (CDM) If SUSY particles assigned conserved multiplicative quantum number,

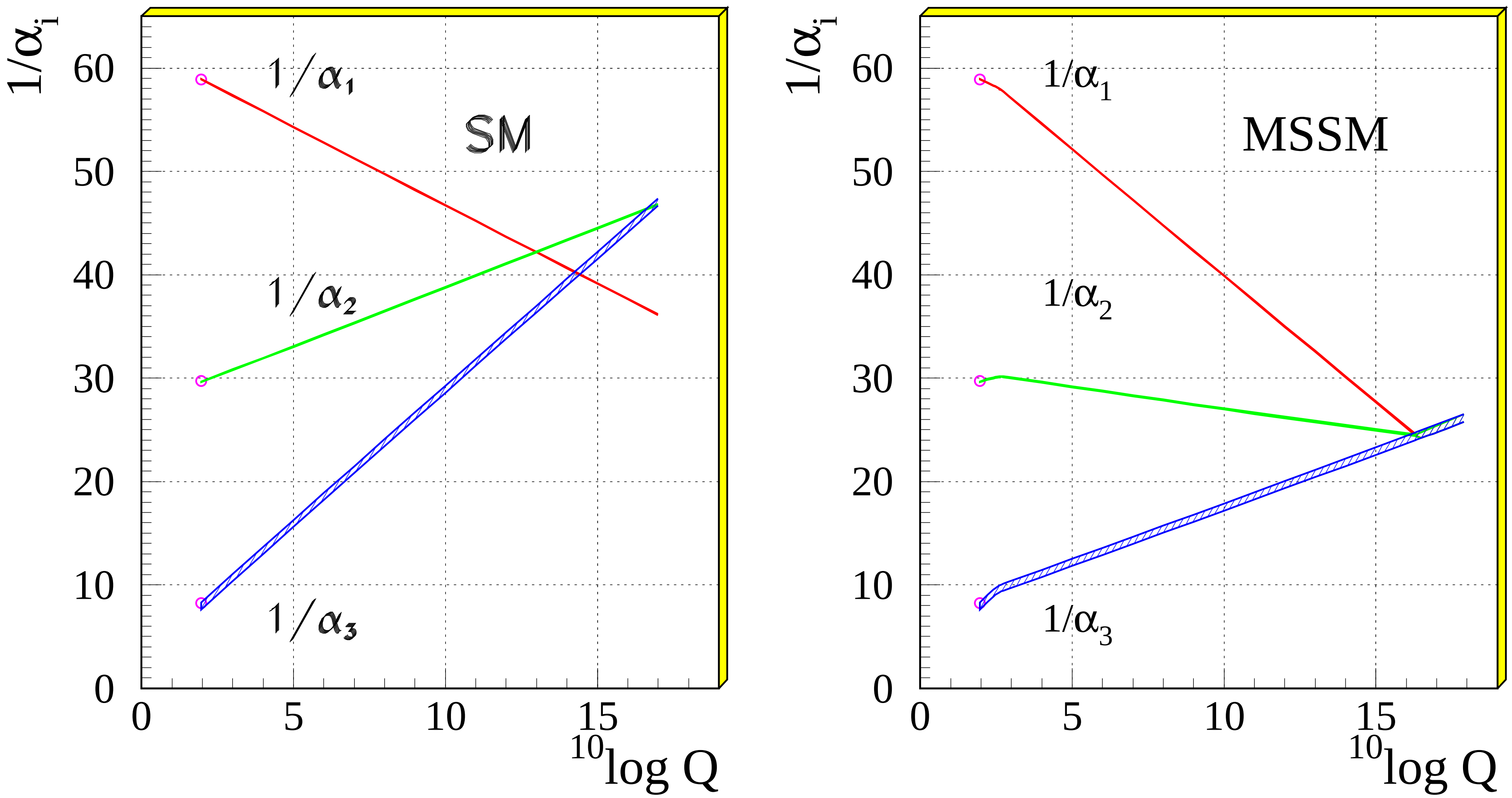
R-parity $= +1$ SM, $= -1$ SUSY, then

SUSY particles prod. pairwise in SM collisions   lightest SUSY particle stable: CDM candidate

Dark Matter (DM) ~ 25 % of the universe

(vii) Local SUSY: enforces gravity

Unification of the Coupling Constants in the SM and the minimal MSSM



(II) $\mathcal{S}$upersymmetry $\mathcal{I}$ntroduction

a) SUSY algebra

Uniqueness of SUSY extension: Coleman-Mandula theorem: Invariance under shifts, rotations, Lorentz-transformations is maximal set for symmetry transformations involving space-time coordinates: beyond $P_\mu, M_{\mu\nu}$ no more conserved bosonic observables.

HOWEVER: fermionic transformations are not forbidden $\Rightarrow$
(Q_α = left-handed Weyl spinor: $\gamma_5 Q = -Q$)

Poincaré algebra:

$$\begin{aligned}
[P_\mu, P_\nu] &= 0 \\
[P_\mu, M_{\rho\sigma}] &= i(g_{\mu\rho}P_\sigma - g_{\mu\sigma}P_\rho) \\
[M_{\mu\nu}, M_{\rho\sigma}] &= i(-g_{\mu\rho}M_{\nu\sigma} + g_{\mu\sigma}M_{\nu\rho} + g_{\nu\rho}M_{\mu\sigma} - g_{\nu\sigma}M_{\mu\rho})
\end{aligned}$$

$\oplus$ SUSY algebra: of fermionic Q's

$$\begin{aligned}
\{Q_\alpha, Q_\beta\} &= \{\bar{Q}_\alpha, \bar{Q}_\beta\} = 0 \\
\{Q_\alpha, \bar{Q}_\beta\} &= 2\sigma^\mu_{\alpha\beta}P_\mu \\
[Q_\alpha, P_\mu] &= [\bar{Q}_\alpha, P_\mu] = 0 \\
[Q_\alpha, M_{\mu\nu}] &= \tfrac{1}{2}(\sigma_{\mu\nu})_{\alpha\beta}Q_\beta
\end{aligned}$$

SUSY multiplets combine fermions with bosons spectrum: $M_{12} = J_3$, $\sigma_{12} = \sigma_3$

$J_3\bar{Q}_{1,2} = \bar{Q}_{1,2}\left(J_3 \pm \frac{1}{2}\right) \Rightarrow \bar{Q}_{1(2)}$ generates (destroys), $Q_{1(2)}$ destroys (generates) spin $\frac{1}{2}$

multiplet structure: particle multiplet $|\lambda>, |\lambda + \frac{1}{2}>$
antiparticle multiplet $|-\lambda>, |-\lambda - \frac{1}{2}>$

Generalization: $Q_{\alpha=1,2}^{i=1,2,\dots,N}$: $\{Q_\alpha^i, Q_\beta^j\} = \{\bar{Q}_\alpha^i, \bar{Q}_\beta^j\} = 0, \quad \{Q_\alpha^i, \bar{Q}_\beta^j\} = 2\delta_{ij}\sigma_{\alpha\beta}^\mu P_\mu$

Maximum: $|-\lambda_{max} > \dots | -\lambda_{max} + \frac{N}{2} >= |\lambda_{max} >$ consistent field theory: $\lambda_{max} = 2 \Rightarrow \boxed{N \leq 8}$

Same number of fermions and bosons in one multiplet

$\{Q, \bar{Q}\} = 2\sigma_\mu P^\mu$ LHS:

$$\begin{aligned} & \mathrm{Tr} < (-1)^{N_F} Q\bar{Q} > && + \mathrm{Tr} < (-1)^{N_F} \bar{Q}Q > \\ = & \sim && + \mathrm{Tr} < Q(-1)^{N_F} \bar{Q} > \\ = & \sim && - \mathrm{Tr} < (-1)^{N_F} Q\bar{Q} >= 0 \end{aligned}$$

RHS: $2\sigma^\mu \mathrm{Tr}[(-1)^{N_F} P_\mu] = 0 \qquad \Rightarrow \Delta = \mathrm{Tr}(-1)^{N_F} = 0 \Rightarrow$

$$\boxed{\text{number of bosons} = \text{number of fermions}}$$

Masses of fermions and bosons are equal

$[P_\mu, Q] = [P_\mu, \bar{Q}] = 0$

$$\begin{aligned} H|B> &= m_B|B> \\ QH|B> &= HQ|B> = H|F> \quad \Rightarrow \\ m_B|F> &= m_F|F> \end{aligned}$$

$$\boxed{m_B = m_F}$$

Experimentally: no SUSY partners to SM particles with identical mass observed $\Rightarrow$

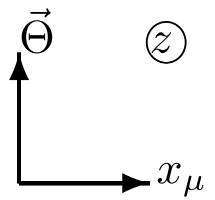
If SUSY is realized in nature, it must be broken.

b) Superfields

Superfield: combination of fields belonging to the same multiplet into one single field.

pairings:	fermion f	$\sim$	sfermion $\tilde{f}$	[spin $\frac{1}{2}$ - spin 0]
	gauge field W	$\sim$	gaugino $\tilde{W}$	[spin 1 - spin $\frac{1}{2}$]
	Higgs H	$\sim$	Higgsino $\tilde{H}$	[spin 0 - spin $\frac{1}{2}$]

combined to superfields defined in superspace.

Superspace: 

$z = [x_\mu; \theta, \bar{\theta}]$ $\quad x_\mu$ = space-time, $\theta_{1,2}, \bar{\theta}_{1,2}$ fermionic coordinates

Grassmann numbers: $\{\theta_i, \theta_j\} = 0 \Rightarrow \theta_i^2 = 0 \qquad f(\theta_i) = a + b\theta_i \qquad \int d\theta_i = 0$ and $\int d\theta_i \theta_i = 1$

$$\frac{\partial}{\partial\theta_i}\theta_i = 1 \qquad \frac{\partial}{\partial\theta_i} f = b \qquad \int d\theta_i f = b$$

Metric: $\theta_\alpha, \bar{\theta}_\alpha$ $(\alpha = 1, 2)$ $(\epsilon^{12} = -\epsilon^{21} = 1)$

$$\begin{aligned} \theta^2 &= \theta_a \theta^a = \epsilon^{ab}\theta_a\theta_b = 2\theta_1\theta_2 \\ &= -\theta^1\theta_1 - \theta^2\theta_2 = \theta_1\theta^1 + \theta_2\theta^2 \end{aligned}$$

Superfields generated by shift from origin: $\Phi(0) \rightarrow \Phi(x) = G\Phi(0)G^{-1}$, where G is the translation operator

SUSY shift: $\quad \Phi(x; \theta, \bar{\theta}) = G\Phi(0; 0, 0)G^{-1}$

$$G = \exp[i(-xP + \theta Q + \bar{\theta}\bar{Q})]$$

$$G(0;\theta',\bar\theta')\,G(x;\theta,\bar\theta)=G(x'';\theta'',\bar\theta'') \qquad \begin{aligned}\theta'' &= \theta+\theta' \\ \bar\theta'' &= \bar\theta+\bar\theta' \\ x'' &= x-i[\theta'\sigma\bar\theta-\theta\sigma\bar\theta'] \quad \sigma=(\mathbf{1},\tilde\sigma)\end{aligned}$$

Differential representation:
$$\begin{aligned} P &= i\partial \\ Q &= \frac{\partial}{\partial\theta}-i\sigma\bar\theta\partial \\ \bar Q &= \frac{\partial}{\partial\bar\theta}-i\theta\sigma\partial \end{aligned}$$

Powers of superfields are superfields $\rightsquigarrow$ Superfields provide a reducible representation of the SUSY algebra. Irreducible representations are obtained by imposing additional constraints on the superfield (must be invariant under SUSY trafos).

Chiral Superfield

Defined by covariant condition

$$\boxed{\text{Chiral Superfield: } \bar D\Phi_- = 0}$$

with $\underline{\bar D = \frac{\partial}{\partial\bar\theta}+i\theta\sigma\partial}$: SUSY-invariant $[\bar D, Q] = \ldots = 0$

antichiral superfield: $D\Phi_+ = 0 \quad D=\frac{\partial}{\partial\theta}+i\sigma\bar\theta\partial$

Solution: $\Phi_- = \Phi_-(x+i\theta\sigma\bar\theta;\theta,0)\equiv\Phi_-(y;\theta)$ is general chiral superfield

$$\bar D\Phi_- = -i\theta\sigma\partial\Phi+i\theta\sigma\partial\Phi = 0$$

Taylor expansion $[y = x + i\theta\sigma\bar{\theta}] \rightsquigarrow$

$$\boxed{\begin{array}{c}\Phi_-(y) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \\ y_\mu = x_\mu + i\theta\sigma_\mu\bar{\theta}\end{array}}$$

$$\Phi : \begin{array}{lcl} \phi & = & \text{1 complex scalar field for the description of sleptons and squarks} \\ \psi & = & \text{1 left-handed Weyl spinor field for the description of leptons and quarks} \\ F & = & \text{1 non-propagating auxiliary field} \end{array}$$

Gauge Superfield Defined by: $V = V^\dagger$ hermitesch

gauge trafo: $V \to V + \chi + \chi^\dagger \to$ minimal field content:

Wess-Zumino gauge

$[\chi^{(\dagger)}$ (anti)chiral superfield: $\Phi \to e^{-g\chi}\Phi$, $\Phi^\dagger \to \Phi^\dagger e^{-g\chi^\dagger}]$

$$\boxed{V = -\theta\sigma_\mu\bar{\theta}A^\mu(x) + i\theta^2\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}^2\theta\lambda(x) + \theta^2\bar{\theta}^2 D(x)}$$

$$\hat{V} : \begin{array}{lcl} A_\mu & = & \text{real vector field} \\ \lambda & = & \text{spin } \tfrac{1}{2} \text{ vectorino field} \\ D & = & \text{non-propagating auxiliary field} \end{array}$$

V vector potential $[A_\mu]$

V_a field strength $[F_{\mu\nu} = \partial_\nu A_\mu - \partial_\mu A_\nu]$: $V_a = -\frac{1}{4}\bar{D}^2 D_a V$

$$V_a = i\lambda_a - \theta_b[2\delta_a^{\ b} D - \tfrac{i}{2}(\sigma_{\mu\nu})^b{}_a F^{\mu\nu}] - \theta^2(\sigma\partial)_{a\dot{a}}\bar{\lambda}^{\dot{a}}|_{y=x+i\theta\sigma\bar{\theta}}$$

c) Interactions Action: SUSY and $[U(1)$-$]$gauge invariant

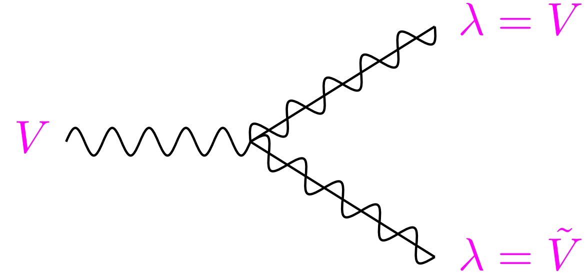
$$\begin{aligned} S &= S_V + S_m + S_W : \\ S_V &= \tfrac{1}{4}\int d^6z\, V_\alpha V^\alpha + h.c. && \text{gauge-field contribution} \\ S_m &= \int d^8z\, \Phi^\dagger e^{gV}\Phi && \text{matter- + gauge interactions} \\ S_W &= \int d^6z\, W[\Phi] + h.c. && \text{super potential: } W = \tfrac{m}{2}\Phi^2 + \tfrac{\lambda}{6}\Phi^3 \end{aligned}$$

$$[d^8z = d^4x\, d^2\theta\, d^2\bar{\theta},\ d^6z = d^4x\, d^2\theta \to d^4x]$$

- **Gauge fields:** $S_V = \frac{1}{4}\int d^4x\, d^2\theta\, V_\alpha V^\alpha + h.c. = \int d^4x\, [-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}i\bar{\lambda}\not{\partial}\lambda + \frac{1}{2}D^2]$

non-Abelian: $\frac{1}{2}i\bar{\lambda}\not{D}\lambda$

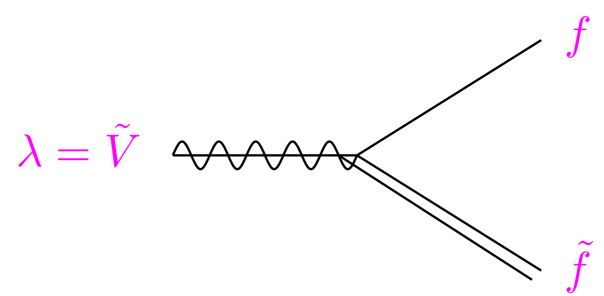
Feynman: gauge interactions and $\lambda\lambda V$



- **Matter-gauge interactions:** $S_m = \int d^4x\, d^2\theta\, d^2\bar\theta\, \Phi^\dagger e^{gV} \Phi$

$$= \int d^4x \left[|D_\mu \phi|^2 + i\bar\psi_L \not{D} \psi_L + i\sqrt{2} g [\phi^* \overline{\psi^c_L} \lambda_L - \phi \overline{\lambda_L} \psi^c_L] + g|\phi|^2 D + |F|^2 \right]$$

Feynman: gauge interactions and $\lambda f \tilde f$

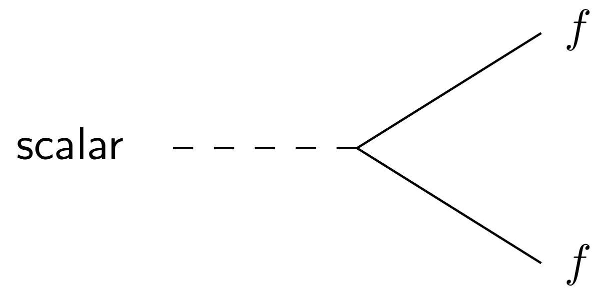


$\boxed{\text{Identity of couplings: } g(f\tilde f \tilde V) = g(ffV)}$

- **Superpotential:** $S_W = \int d^4x\, d^2\theta\, W[\Phi] + h.c.$ $\quad W = a\Phi + b\Phi^2 + c\Phi^3$

$$= \int d^4x \left[-\frac{1}{2} \frac{\partial^2 W}{\partial\phi \partial\phi} \overline{\psi^c_L} \psi_L + F \frac{\partial W}{\partial \phi} \right] + h.c.$$

Feynman: Yukawa interactions ffscalar



- **Auxiliary fields D and F:** Euler-Lagrange: $\partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \Phi} = \frac{\partial \mathcal{L}}{\partial \Phi}$

Auxiliary field D: in $S_V + S_m$: $D + g\phi^* \phi = 0 \Rightarrow$

$$\boxed{\mathcal{L}_D = -\frac{1}{2} D^2 = -\frac{1}{2} g^2 |\phi|^4}$$

Quartic scalar coupling generated by supersymmetric gauge interactions
with strength $\sim$ gauge coupling^2

$\Rightarrow$ Higgs mass small [quartic Higgs coupling $\sim g^2$]

Auxiliary field F: in $S_m + S_W$: $F + \frac{\partial W}{\partial \phi} = 0 \Rightarrow$

$$\boxed{\mathcal{L}_F = -\left| \frac{\partial W}{\partial \phi} \right|^2}$$

Summary: Potential V generated by D, F terms

$$\boxed{V = \frac{1}{2} D^2 + |F|^2}$$

$$|F|^2 = \left| \frac{\partial W}{\partial \phi} \right|^2, \quad D^2 = \sum_a |g_a \phi t^a \phi|^2$$

d) Supersymmetry Breaking General supersymmetric system:

same number of fermions and bosons in one multiplet

masses of fermions and bosons are equal

Supersymmetric ground state has vanishing energy Algebra: $\{Q_\alpha, \bar{Q}_\beta\} = 2\sigma^\mu_{\alpha\beta} P_\mu \Rightarrow$

$H = \frac{1}{4}\sum_\alpha [Q_\alpha Q^\dagger_\alpha + Q^\dagger_\alpha Q_\alpha] \rightsquigarrow < 0|H|0 >= \frac{1}{4}\sum_\alpha \left[||Q^\dagger_\alpha|0>||^2 + ||Q_\alpha|0>||^2\right] \Rightarrow$

$$\boxed{\begin{array}{c} <H> \ \gtrsim 0 \\ \text{SUSY ground state } <0|H|0>=0 \Leftrightarrow Q_\alpha|0>=0,\, Q^\dagger_\alpha|0>=0 \text{ for all } Q_\alpha, Q^\dagger_\alpha \end{array}}$$

$<H> \neq 0:$ SUSY broken: $V = \frac{1}{2}D^2 + |F|^2:$ $D \neq 0$ D breaking: Fayet-Iliopoulos

$F \neq 0$ F breaking: O'Raifeartaigh

$\rightarrow$ Ferrara-Girardello-Palumbo mass sum rule

$<H> = 0$ in SUSY: $\Lambda_{cosmo} = 0$

Scheme cannot be realized by spontaneous symmetry breaking in visible eigenworld:

Ferrara-Girardello-Palumbo sum rule: $\boxed{\sum(-1)^{2J}(2J+1)m_J^2 = 0}$ supermultiplet

Example: $m^2_{\tilde{e}_L} + m^2_{\tilde{e}_R} - 2m^2_e = 0$ not compatible with observation $\Rightarrow$

Possible solution: RHS $\neq 0$ through supergravitation sector $\sim m^2_{3/2}$ $\rightarrow$ soft, i.e. renormalizable SUSY breaking in Standard sector (through explicit mass terms)

Breaking in hidden sector communicated by gravity/gauge/messengers to eigenworld

$$\boxed{\text{hidden SUSY}} \xrightarrow[]{\text{gravity, gauge,...}} \boxed{\text{SUSY eigenworld}}$$

Soft SUSY breaking Define SUSY breaking operators such that no new quadratic divergences are generated

Realisation:

- spontaneous SUSY breaking in "hidden sector"
- mediated in eigen-world through gravitation etc.
- apparent explicit SUSY breaking in eigen-world
  - identity between Yukawa/gauge couplings preserved
  - SUSY masses $\neq$ SM masses
  - scalar couplings in superpotential changed

SUSY breaking Lagrangian

$$\boxed{\begin{array}{lcll} \mathcal{L}_{break} & = & -\frac{1}{2}M_i\bar{\lambda}_i\lambda_i & \text{for gauginos} \\ & - & m_{\tilde{f}}^2|\tilde{f}|^2 + \ldots & \text{for sfermions,Higgs} \\ & - & W_2(\phi) - W_3(\phi) & \text{superpotential} \end{array}}$$

(III) $\mathcal{M}$inimal $\mathcal{S}$upersymmetry $\mathcal{M}$odel

The SM alone cannot be formulated as SUSY theory ($N \leq 8$) $\Rightarrow$

$$\boxed{\text{SUSY-Standard Model} = SM \otimes SUSY(N=1)}$$

minimal particle content

$\longrightarrow$ Doubling of the particle spectrum: SM+SUSY partner

Minimal Supersymmetric Standard Model (MSSM)

benchmark model, compatible with experiment, for exploiting phenomenology of SUSY models

Vector multiplets

$J=1$	$J=\frac{1}{2}$
Gluon g	Gluino $\tilde{g}$
$W^\pm, W^3$	Wino $\tilde{W}^\pm, \tilde{W}^3$
B	Bino $\tilde{B}$

Chiral multiplets

$J=\frac{1}{2}$	$J=0$
Quarks q_L, q_R	Squarks $\tilde{q}_L, \tilde{q}_R$
Leptons l_L, l_R	Sleptons $\tilde{l}_L, \tilde{l}_R$
Higgsinos $\tilde{H}_1, \tilde{H}_2$	Higgs H_1, H_2

Interactions

Gauge field and matter Lagrangians adapted to $SU(3) \times SU(2) \times U(1)$ from previous basis

Lagrangian

$$\triangleright\ \mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{matter} + \mathcal{L}_D + \mathcal{L}_W + \mathcal{L}_{soft}$$

$$\triangleright\ \mathcal{L}_{gauge} = -\frac{1}{4}G^{a\mu\nu}G^a_{\mu\nu} - \frac{1}{4}W^{a\mu\nu}W^a_{\mu\nu} - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} + Tr[\bar{\tilde{g}} i\not{D}\tilde{g}] + Tr[\bar{\tilde{W}} i\not{D}\tilde{W}] + \frac{1}{2}\bar{\tilde{B}} i\not{\partial}\tilde{B}$$

covariant derivative $iD_\mu = i\partial_\mu - gV_\mu$ (minimal substraction)
field strength tensor $V_{\mu\nu} = \partial_\nu V_\mu - \partial_\mu V_\nu - ig[V_\mu, V_\nu]$ and the vector potential $V_\mu = V^a_\mu T^a$
T^a group generators, $[T^a, T^b] = if_{abc}T^c$
$SU(2): T^a = \frac{\sigma^a}{2}$ Pauli matrices, $SU(3): T^a = \frac{\lambda^a}{2}$ Gell-Mann matrices

$$\triangleright\ \mathcal{L}_{matter} = \sum_{\psi=f,\tilde{H}_i} \bar{\psi} i\not{D}\psi + \sum_{\tilde{f},H_i} |D_\mu\phi|^2 + i\sum_{\psi,\phi,V} \frac{g_V}{\sqrt{2}}[\overline{\psi_L}T^a\tilde{V}^a\phi - \tilde{V}^a T^a \psi_L \phi^*]$$

$$\triangleright\ \mathcal{L}_D = -\frac{1}{2}\sum_{a,V}|D^V_a|^2 \quad \text{with } D^V_a = -g_V\phi^*_i T^a_{ij}\phi_j$$

$$\triangleright\ \mathcal{L}_W = -\sum_i \left|\frac{\partial W}{\partial\phi_i}\right|^2 - \frac{1}{2}\sum_{ij}\overline{\psi^c_{iL}}\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\psi_{jL} + h.c.$$

with

$$W = W_R + W_{\not{R}}$$

$$W_R = -\epsilon_{ij}\mu H^i_1 H^j_2 + \epsilon_{ij}[\lambda_L H^i_1\tilde{L}_j\tilde{E}^c + \lambda_D H^i_1\tilde{Q}_j\tilde{D}^c + \lambda_U H^i_2\tilde{Q}_j\tilde{U}^c]$$

$$W_{\not{R}} = \epsilon_{ij}[\lambda\tilde{L}_i\tilde{L}_j\tilde{E}^c + \lambda'\tilde{L}_i\tilde{Q}_j\tilde{D}^c] + \lambda''\tilde{U}^c\tilde{D}^c\tilde{D}^c \qquad R-\text{parity violating}$$

with $\epsilon_{12} = -\epsilon_{21} = 1, \epsilon_{11} = \epsilon_{22} = 0$

Remark: Multiplicative quantum number R-parity

$R-$parity	$=$	$+1$	for SM particles	$\overset{\text{if conserved}}{\Rightarrow}$	• SUSY particle production in pairs
	$=$	-1	for SUSY partners		• lightest SUSY particle LSP stable

R-parity violating superpotential $\sim \epsilon_{ij}[\lambda \tilde{L}_i \tilde{L}_j \tilde{E}^c + \lambda' \tilde{L}_i \tilde{Q}_j \tilde{D}^c] + \lambda'' \tilde{U}^c \tilde{D}^c \tilde{D}^c$

potentially leading to proton decay:

$$\begin{array}{ccccccc} \lambda''\lambda' & \sim & \begin{array}{c} ud \\ u \end{array} & \rightarrow & d^c & \rightarrow & \begin{array}{c} u^c e^+ \\ u \end{array} \\ & & P & & \rightarrow & & \pi^0 e^+ \end{array}$$

$\lambda''\lambda' \sim 0$ necessarily, <u>MSSM:</u> $\lambda = \lambda' = \lambda'' = 0$

$$\begin{aligned} \triangleright\ \mathcal{L}_{soft} \ = \ & -\tfrac{1}{2}\textstyle\sum_{i=1}^{3} M_i \bar{\lambda}_i^a \lambda_i^a - m_1^2 |H_1|^2 - m_2^2 |H_2|^2 + B\mu\epsilon_{ij}(H_1^i H_2^j + h.c.) \\ & - M_{\tilde{Q}}^2 (\tilde{u}_L^\dagger \tilde{u}_L + \tilde{d}_L^\dagger \tilde{d}_L) - M_{\tilde{U}}^2 \tilde{u}_R^\dagger \tilde{u}_R - M_{\tilde{D}}^2 \tilde{d}_R^\dagger \tilde{d}_R + (\tilde{l}) \\ & + \frac{g}{\sqrt{2} M_W} \epsilon_{ij} \left[\frac{M_d}{\cos\beta} A_d H_1^i \tilde{Q}^j \tilde{d}_R^\dagger + (\tilde{u}) + (\tilde{l}) \right] \end{aligned}$$

a) Higgs sector

(i) <u>mass generation of SM particles</u> via Higgs mechanism

(ii) <u>two independent Higgs fields</u> H_1, H_2 to provide mass to down- and up-type particles in holomorphic superpotential of SUSY theories

$$\begin{array}{llll} \underline{\mathrm{SM:}} & \mathcal{L}_d \sim \bar{Q}_L \phi d_R & \phi_0 = [0, \frac{v}{\sqrt{2}}] & \Rightarrow \ m_d \bar{d}_L d_R + h.c. \\ & \mathcal{L}_u \sim \bar{Q}_L \phi^c u_R & \phi^c = i\tau_2 \phi^*,\ \phi_0^c = [\frac{v}{\sqrt{2}}, 0] & \Rightarrow \ m_u \bar{u}_L u_R + h.c. \end{array}$$

<u>SUSY :</u> $\phi^c \sim \phi^*$ not allowed in holomorphic function next to ϕ

(iii) Higgs potential:

$$\begin{aligned}
V_{Higgs} &= V_F + V_D + V_{soft} \\
V_F &= |F|^2 = \sum_{i=1,2} \left|\frac{\partial W}{\partial H_i}\right|^2 = \mu^2(|H_1|^2 + |H_2|^2) \\
V_D &= \tfrac{1}{2}D^2 = \tfrac{1}{2}\sum_{a,V} |-g_V H_i^* T^a_{ij} H_j|^2 = \frac{g^2+g'^2}{8}[|H_1|^2 - |H_2|^2]^2 + \frac{g^2}{2}|H_1^\dagger H_2|^2 \\
V_{soft} &= m_1^2|H_1|^2 + m_2^2|H_2|^2 - B\mu\epsilon_{ij}(H_1^i H_2^j + h.c.)
\end{aligned}$$

$$\boxed{\begin{aligned} V_{Higgs} = {} & (m_1^2+\mu^2)|H_1|^2 + (m_2^2+\mu^2)|H_2|^2 - B\mu\epsilon_{ij}(H_1^i H_2^j + h.c.) \\ & + \frac{g^2+g'^2}{8}[|H_1|^2 - |H_2|^2]^2 + \frac{g^2}{2}|H_1^\dagger H_2|^2 \end{aligned}}$$

with $H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} \phi_1^{0*} \\ -\phi_1^- \end{pmatrix} \qquad H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}$

quartic couplings $\sim$ gauge couplings

spontaneous symmetry breaking: minV_{Higgs} for $< H_{1,2} >_0 \neq 0$:

$$H_1 = \begin{pmatrix} \phi_1^{0*} + \frac{v_1}{\sqrt{2}} \\ -\phi_1^- \end{pmatrix} \qquad H_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 + \frac{v_2}{\sqrt{2}} \end{pmatrix}$$

8 degrees of freedom: 3 zero-mass states: Goldstone bosons absorbed by elw gauge bosons

5 physical states:

diagonalisation:

$$
\begin{aligned}
H_1^1 &= \tfrac{1}{\sqrt{2}}(v_1 + H^0\cos\alpha - h^0\sin\alpha + iA^0\sin\beta - iG^0\cos\beta)\\
H_1^2 &= H^-\sin\beta - G^-\cos\beta\\
H_2^1 &= H^+\cos\beta + G^+\sin\beta\\
H_2^2 &= \tfrac{1}{\sqrt{2}}(v_2 + H^0\sin\alpha + h^0\cos\alpha + iA^0\cos\beta + iG^0\sin\beta)
\end{aligned}
$$

$G^\pm, G^0 \Rightarrow$ Goldstone bosons

parameters:

$$
\begin{aligned}
M_Z^2 &= \tfrac{g^2+g'^2}{4}(v_1^2+v_2^2) & \qquad M_A^2 &= B\mu\tfrac{v_1^2+v_2^2}{v_1 v_2}\\
\tan\beta &= \tfrac{v_2}{v_1} & &\\
m_1^2+\mu^2 &= M_A^2\sin^2\beta - \tfrac{M_Z^2}{2}\cos 2\beta & \qquad m_2^2+\mu^2 &= M_A^2\cos^2\beta + \tfrac{M_Z^2}{2}\cos 2\beta
\end{aligned}
$$

5 physical states:

- charged: $M_{H^\pm}^2 = M_A^2 + M_W^2$
- pseudoscalar: M_A^2 $\qquad \mathcal{CP}_o$
- scalar: $M_{h,H}^2 = \frac{1}{2}\{M_A^2 + M_Z^2 \mp \sqrt{(M_A^2+M_Z^2)^2 - 4M_A^2 M_Z^2\cos^2 2\beta}\}$ $\qquad \mathcal{CP}_e$

$$\tan 2\alpha = \tan 2\beta\frac{M_A^2+M_Z^2}{M_A^2-M_Z^2} \qquad -\frac{\pi}{2} < \alpha < 0$$

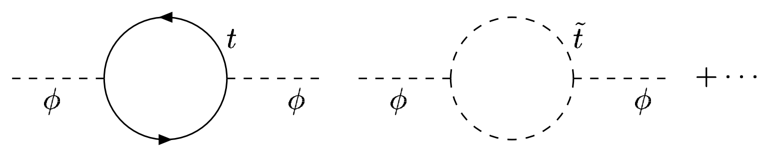
Higgs system parameterized by (at LO): M_A and $\tan\beta$

$M_A \sim M_H \sim M_{H^\pm} \gg v$ decoupling limit: $h \to$ SM-like

Inequalities:

$$\begin{aligned} M_h &< M_Z, M_A \\ M_H &> M_Z, M_A \\ M_{H^\pm} &> M_A, M_W \end{aligned}$$

modified through radiative corrections



$\epsilon = \frac{3 G_F m_t^4}{\sqrt{2}\pi^2 \sin^2\beta} \log \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}$ + stop mixing $[\mu, A_t] \Rightarrow$

$$\boxed{\begin{aligned} M_h &\lesssim 140 \text{ GeV} \\ M_{H,A,H^\pm} &= \mathcal{O}(v)\ldots 1 \text{ TeV} \end{aligned}}$$

b) Gluino, Charginos/Neutralinos

- Gluino mass: $M_3 \leftarrow$ soft mass term

- Chargino masses: Charginos are mixings of charged Winos $\tilde{W}^\pm$ and Higgsinos $\tilde{H}^\pm \Rightarrow$
  2 charginos $\tilde{\chi}_1^\pm, \tilde{\chi}_2^\pm$

$$\mathcal{L}_C = -\tfrac{1}{2}\psi_\pm^T \mathcal{M}_C \psi_\pm + h.c. \quad \text{with} \quad \psi_\pm = \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \\ \tilde{W}^- \\ \tilde{H}_d^- \end{pmatrix} \quad \text{and} \quad \mathcal{M}_C = \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix}$$

where

$$X = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix}$$

M_2 :	from $\mathcal{L}_{soft}$				
μ :	from $W(\Phi)$	$\rightarrow$	$-\frac{1}{2}\sum_{ij} \overline{\psi_{iL}^c} \frac{\partial^2 W}{\partial\phi_i \partial\phi_j} \psi_{jL}$	$\rightarrow$	higgsino term μ
non-diag:	from $\mathcal{L}_{matter}^{\lambda H \psi}$	$\rightarrow$	mixing term $\sim \tilde{W}\tilde{H}$	$\rightarrow$	$M_W \sqrt{2} \cos\beta (\sin\beta)$

diagonalisation $\rightarrow$ mass eigenstates $\tilde{\chi}_{1,2}^\pm$ with

$$m^2_{\tilde{\chi}^\pm_{1,2}} = \tfrac{1}{2}\{M_2^2 + \mu^2 + 2M_W^2 \mp \sqrt{(M_2^2 - \mu^2)^2 + 4M_W^2 (M_2^2 + \mu^2 + 2M_2 \mu \sin 2\beta + M_W^2 \cos^2 2\beta)}\}$$

Limits: $\mu, M_2 \gg M_W$

$$\begin{aligned} m_{\tilde\chi^\pm_{1,2}} &= M_2 - \frac{M_W^2(M_2+\mu\sin 2\beta)}{\mu^2-M_2^2} \\ m_{\tilde\chi^\pm_{2,1}} &= |\mu| + \frac{M_W^2(|\mu|+\epsilon M_2\sin 2\beta)}{\mu^2-M_2^2} \qquad \epsilon = \text{sign}(\mu) \end{aligned}$$

- $|\mu| \gg M_2$: $\tilde\chi_1^\pm$ gaugino-like, $\tilde\chi_2^\pm$ Higgsino-like
- $|\mu| \ll M_2$: $\tilde\chi_1^\pm$ Higgsino-like, $\tilde\chi_2^\pm$ gaugino-like
- $\mu \sim -M_2, \tan\beta \sim 1$: $m_{\tilde\chi^\pm_{1,2}} \sim \sqrt{M_2^2 + M_W^2}$ degenerate

- Neutralino masses: Neutralinos are mixings of $\tilde B$, $\tilde W^3$, $\tilde H_u^0$, $\tilde H_d^0$ $\Rightarrow$ 4 neutralinos $\tilde\chi^0_{1,2,3,4}$

$$\mathcal{L}_0 = -\frac{1}{2}\psi_0^T \mathcal{M}_0 \psi_0 + h.c. \quad \text{with} \quad \psi_0 = \begin{pmatrix} \tilde B \\ \tilde W^3 \\ \tilde H_d^0 \\ \tilde H_u^0 \end{pmatrix}$$

where

$$\mathcal{M}_0 = \begin{pmatrix} M_1 & 0 & -M_Z\sin\theta_W\cos\beta & M_Z\sin\theta_W\sin\beta \\ 0 & M_2 & M_Z\cos\theta_W\cos\beta & -M_Z\cos\theta_W\sin\beta \\ -M_Z\sin\theta_W\cos\beta & M_Z\cos\theta_W\cos\beta & 0 & -\mu \\ M_Z\sin\theta_W\sin\beta & -M_Z\cos\theta_W\sin\beta & -\mu & 0 \end{pmatrix}$$

Diagonalisation $\longrightarrow$ mass eigenstates $\tilde\chi^0_{1,2,3,4}$

Limits: $\mu, M_1, M_2 \gg M_Z$

$$
\begin{aligned}
m_{\tilde{\chi}_1^0} &= M_1 - \frac{M_Z^2 s_W^2 (M_1 + \mu s_\beta)}{\mu^2 - M_1^2} \\
m_{\tilde{\chi}_2^0} &= M_2 - \frac{M_Z^2 c_W^2 (M_1 + \mu s_\beta)}{\mu^2 - M_2^2} \\
m_{\tilde{\chi}_3^0} &= |\mu| + \frac{M_Z^2 (1 - \epsilon s_{2\beta})(|\mu| + M_1 c_W^2 + M_2 s_W^2)}{2(|\mu| + M_1)(|\mu| + M_2)} \\
m_{\tilde{\chi}_4^0} &= |\mu| + \frac{M_Z^2 (1 + \epsilon s_{2\beta})(|\mu| - M_1 c_W^2 - M_2 s_W^2)}{2(|\mu| - M_1)(|\mu| - M_2)}
\end{aligned}
$$

$$\Rightarrow \tilde{\chi}_1^0 \sim \tilde{B}^0, \tilde{\chi}_2^0 \sim \tilde{W}^3, \tilde{\chi}_{3,4}^0 \sim \frac{\tilde{H}_d^0 \pm \tilde{H}_u^0}{\sqrt{2}}, m_{\tilde{\chi}_2^0} \sim m_{\tilde{\chi}_1^\pm}$$

- $|\mu| \gg M_1, M_2$: LSP gaugino-like
- $|\mu| \ll M_1, M_2$: LSP Higgsino-like; $m_{\tilde{\chi}_1^0} \sim m_{\tilde{\chi}_2^0} \sim m_{\tilde{\chi}_1^\pm}$

c) Slepton/Squark masses

L and R states for fermions and sfermions independent degrees of freedom

LR mass matrix:

$$\mathcal{L}_{\tilde{f}} = -\tilde{f}^\dagger \mathcal{M}_{\tilde{f}}^2 \tilde{f} \qquad \text{with } \tilde{f} = \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix} \qquad \text{and}$$

$$\mathcal{M}^2_{\tilde f} = \begin{pmatrix} M^2_{\tilde f_L} + m_f^2 + \overbrace{M_Z^2 (I_{3L} - e_f \sin^2\theta_W)\cos 2\beta}^{D-term} & m_f(A_f - \mu r_f) \\ m_f(A_f - \mu r_f) & M^2_{\tilde f_R} + m_f^2 - M_Z^2 (I_{3R} - e_f \sin^2\theta_W)\cos 2\beta \end{pmatrix}$$

$$r_f = \begin{cases} \cot\beta & \text{for} \quad f = \text{up-type} \\ \tan\beta & \text{for} \quad f = \text{down-type} \end{cases}$$

$M^2_{\tilde f_{L/R}}$: from $\mathcal{L}_{soft}$

m_f^2 : from $\mathcal{L}_W \sim \left|\frac{\partial W}{\partial \phi}\right|^2$

$D - term$: from $\mathcal{L}_D$

mixing $m_f \mu r_f$: from $\mathcal{L}_W \sim \left|\frac{\partial W}{\partial \phi}\right|^2$

Mixing large only for 3rd generation: $L, R \to 1, 2$

$$m^2_{\tilde f_{1,2}} = m_f^2 + \tfrac{1}{2}\left\{ M^2_{\tilde f_L} + M^2_{\tilde f_R} \mp \sqrt{(M^2_{\tilde f_L} - M^2_{\tilde f_R})^2 + 4 m_f^2 (A_f - \mu r_f)^2} \right\} \qquad \text{(without } D - term\text{)}$$

Diagonalisation:

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_f & \sin\theta_f \\ -\sin\theta_f & \cos\theta_f \end{pmatrix} \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}$$

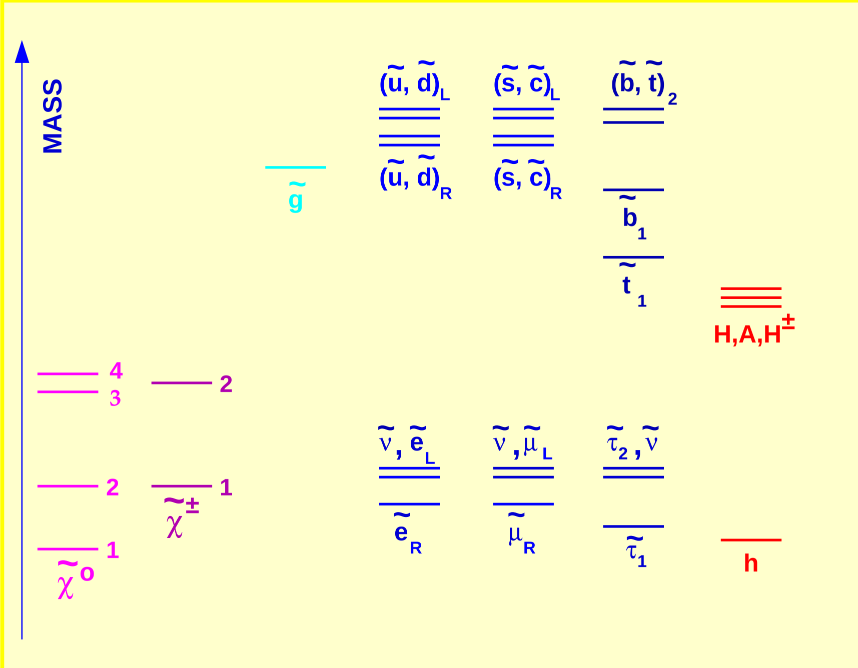
$$\sin 2\theta_f = \frac{2m_f(A_f - \mu r_f)}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}}$$

$$\cos 2\theta_f = \frac{M^2_{\tilde{f}_L} - M^2_{\tilde{f}_R}}{m^2_{\tilde{f}_1} - m^2_{\tilde{f}_2}}$$

- Splitting is largest for the heaviest fermions $\Rightarrow$ $\tilde{t}_1$ can be the lightest squark
  $\tilde{\tau}_1$ can be the lightest slepton
- large $\tan\beta$: splitting $\tilde{b}_1 \leftrightarrow \tilde{b}_2$ large $\Rightarrow$ there are regions where $\tilde{b}_1$ is the lightest squark

Example of SUSY Spectrum

Schematic Sparticle Spectrum in MSSM



(IV) SUSY-GUT

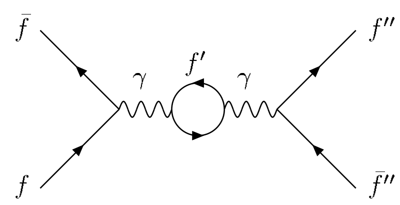
SUSY physically motivated by GUT: stability of the radiative corrections to W, Z, H masses.
Aim: resonstruction of the fundamental SUSY theory at the GUT/PLANCK scale.

MSSM: 124 free parameters $\rightsquigarrow$ constrained by unification at the GUT scale

(a) Unification of the gauge couplings

SM: $SU_3 \times SU_2 \times U_1 \overset{GUT}{\longrightarrow} SU_5 \quad g_3 \;\; g_2 \;\; g_1 \to g_5 : \sin^2\theta_W = \frac{3}{8}$ at GUT

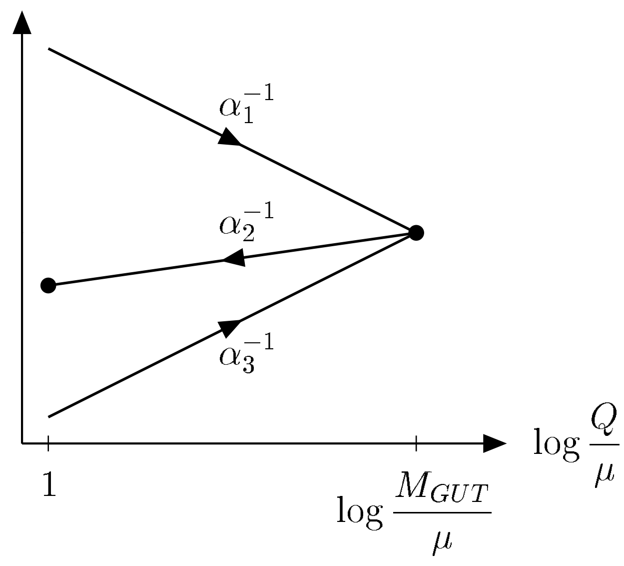
Running couplings:



$$\frac{\partial \alpha_i}{\partial \log \mu^2} = -\frac{b_i}{4\pi}\alpha_i^2 + \mathcal{O}(\alpha_i^3)$$

$$\text{SM:} \quad b_i = \begin{vmatrix} 0 \\ \frac{22}{3} \\ 11 \end{vmatrix} - N_G \begin{vmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{vmatrix} - N_H \begin{vmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{vmatrix}$$

$$\text{SUSY:} \quad b_i = \begin{vmatrix} 0 \\ 6 \\ 9 \end{vmatrix} - N_G \begin{vmatrix} 2 \\ 2 \\ 2 \end{vmatrix} - N_H \begin{vmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{vmatrix}$$



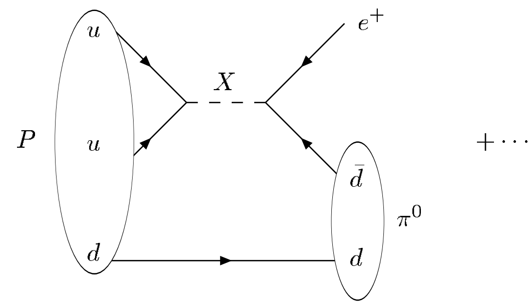
	SM:	0.2100 ± 0.0026
$\sin^2\theta_W(M_Z^2)$:	MSSM:	0.2335 ± 0.0017
	exp:	0.2316 ± 0.0002

- SM couplings cannot be unified. SUSY couplings can
- SUSY predictions of $\sin^2\theta_W$ within 2 permille. Independent of details of the $\tilde{M} \sim 1$ TeV spectrum, only sensitive to particle spectrum.

(b) Proton decay:

SM-SU(5): gauge interactions between leptons and quarks:

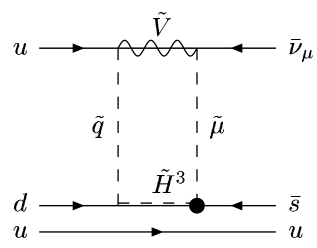
$$\tau(P \to \pi^0 + e^+) \sim \frac{M_X^4}{\alpha_5^2 M_P^5}$$



SM: $M_X \sim 10^{15}$ GeV: $\sim 10^{30}$ a   SUSY: $M_X > 10^{16}$ GeV: $\gtrsim 10^{35}$ a (exp: $> 10^{33}$ a)

However: additional dim 5 contribution

$$\tau(P \to K^+\bar{\nu}_\mu) \sim \left(\frac{16\pi^2}{\lambda^2 g^2}\right)^2 \frac{m_{\tilde{H}^3}^2 \tilde{m}^2}{M_P^5}$$ near experimental limit



(c) Radiative symmetry breaking:

universal sGUT masses (mSUGRA): Gauginos: $M_1, M_2, M_3 = M_{1/2}$

Scalars: $M_H^2 - \mu^2 = m_{\tilde{l}}^2 = m_{\tilde{q}}^2 = M_0^2$

RGE: GUT $\rightarrow$ ELW

$$\frac{\partial}{\partial \log Q} \begin{pmatrix} M_{H_2}^2 \\ m_{\tilde{t}_R}^2 \\ m_{\tilde{t}_L}^2 \end{pmatrix} = \frac{g_t^2}{8\pi^2} \begin{pmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} M_{H_2}^2 \\ m_{\tilde{t}_R}^2 \\ m_{\tilde{t}_L}^2 \end{pmatrix} + \frac{g_t^2 A_t^2}{8\pi^2} \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

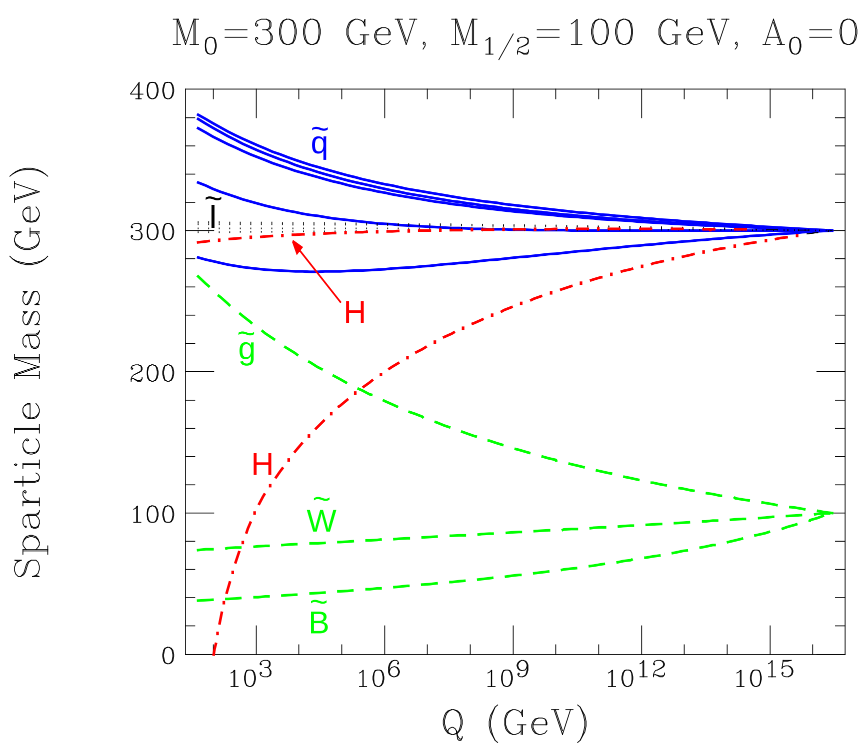
Evolution: $M_{H_2}^2(Q^2) \approx M_0^2 + \mu^2 - \frac{3g_t^2}{8\pi^2}(3M_0^2 + \mu^2)\log\frac{M_{GUT}}{Q}$

Bagger

$M_{H_2}^2(M_Z^2) < 0$ possible for $m_t \sim 100 - 200$ GeV

$\rightarrow$ radiative symmetry breaking

$SU_3 \times SU_2 \times U_1 \rightarrow SU_3 \times U_1^{em}$



(V) SUSY Breaking Mechanisms

Ferrara sum rule: no spontaneous symmetry breaking in eigenworld: $\sum(-1)^{2J}(2J+1)M_J^2 \neq 0$.

Mechanisms:		
	Supergravitation	mSUGRA
	Gauge-mediated SUSY breaking	GMSB
	Anomaly-mediated SUSY breaking	AMSB
	Scherk-Schwarz SUSY breaking	SSSB ...

Minimal SUPERGRAVITATION:

$$\boxed{\begin{array}{l}\text{Hidden sector}\\ \text{SUSY: } \langle\bar{\lambda}\lambda\rangle \neq 0\end{array}} \xrightarrow[\rightsquigarrow]{\rightsquigarrow \; GRAV.} \boxed{\text{Eigenworld}} \qquad \Rightarrow \boxed{M_{SUSY\,break} \sim 10^{11}\ \text{GeV}}$$

Soft parameters "universal": 5 parameters: $M_0, M_{1/2}, A_0, \tan\beta, \text{sgn}\mu$

Evolution: GUT $\rightarrow$ ELW:

$$\begin{aligned} \frac{\partial M_i}{\partial \log\mu^2} &= -\frac{b_i}{4\pi}\alpha_i M_i \\ \frac{\partial \alpha_i}{\partial \log\mu^2} &= -\frac{b_i}{4\pi}\alpha_i^2 \\ \Rightarrow \frac{M_i}{\alpha_i} &= \text{const} \end{aligned}$$

Gaugino masses run like gauge couplings $\Rightarrow$

$$M_1 = M_2\frac{5}{3}\cdot\tan^2\theta_W \sim \frac{1}{2}M_2 \qquad (M_i(M_{GUT}) = M_{1/2})$$

$$\frac{M_3}{M_2} = \frac{\alpha_3}{\alpha_2} \gg 1 \Rightarrow \qquad m_{\tilde{g}} \gg m_{\tilde{\chi}}$$

Reconstruction of the fundamental theory at GUT/Planck scale:

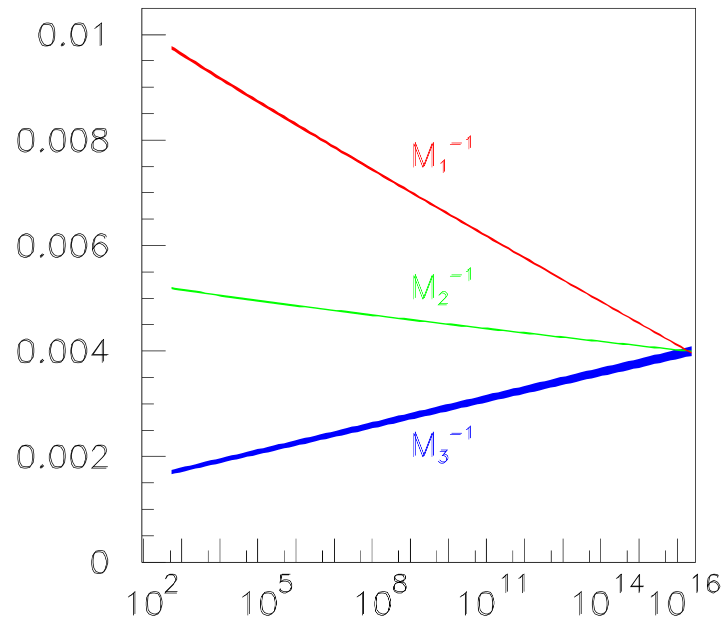
high-precision measurements of low-energy Lagrangian parameters (LHC+ILC)

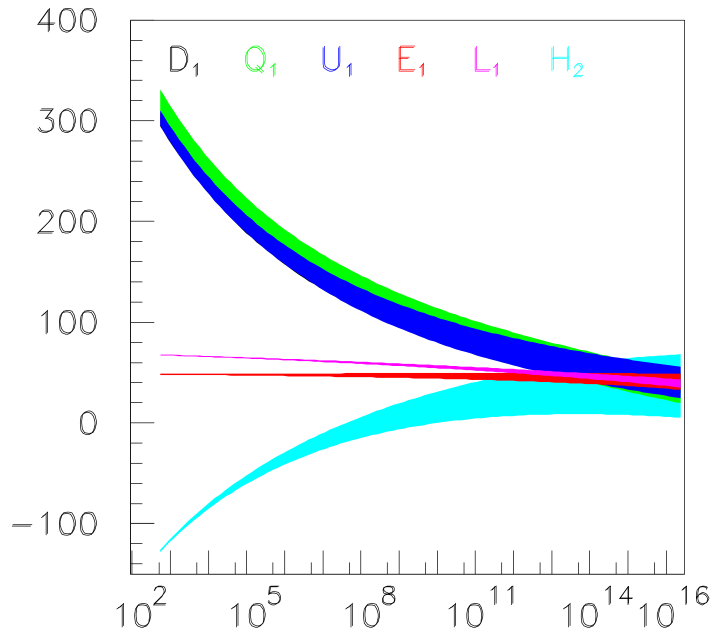
$\Rightarrow$ extrapolate to high scale: - symmetries/universal behaviour?
- impact of high-scale physics?

Evolution: RG equations

Evolution: gaugino and scalar mass parameters

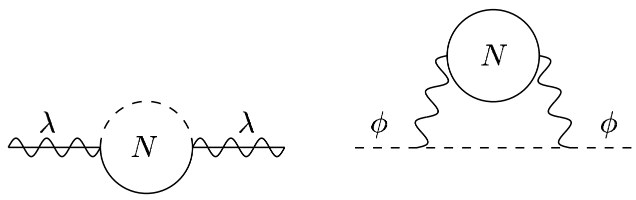
Blair, Porod, Zerwas



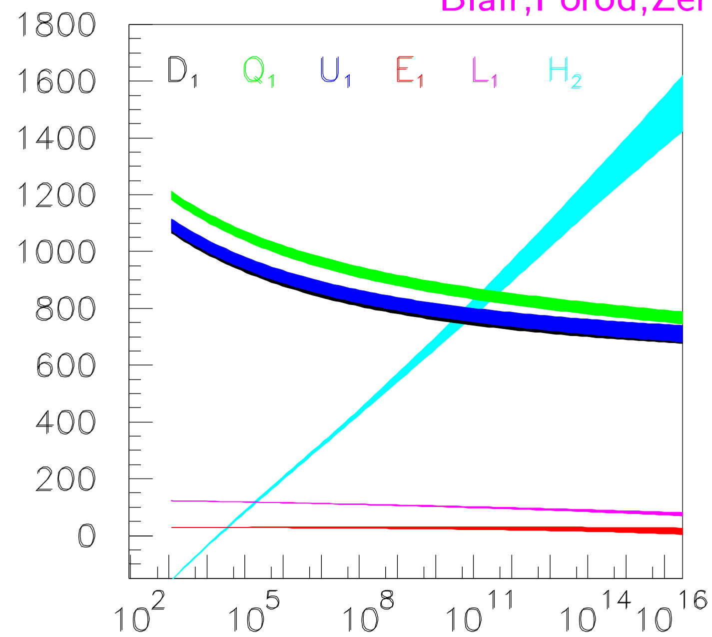


Gauge-mediated SUSY breaking (GMSB):

- flavor-blind SUSY-breaking mediating interactions are ordinary EW and QCD gauge interactions
- MSSM soft terms come from loop diagrams involving messenger particles
- messengers are new chiral supermultiplets coupling to a SUSY breaking VEV (F) and also have $SU(3)_C \times SU(2)_L \times U(1)_Y$ interactions providing the necessary connection to the MSSM



- gaugino masses evolve like in mSUGRA but without unification
- scalar masses significantly different, determined by F, the messenger mass M_{mess} and the number N of messenger fields
- 5 parameters: $F, M_{mess}, N, \tan\beta, \text{sgn}\mu$
- $m_{\tilde{e}_L} = M_{H_u}$ at $\mu = \sqrt{F} \rightarrow$ exp. reconstr. of $\sqrt{F} = M_{SUSY\,break}$
- lower limit to M_{mess}, SUSY masses $\Rightarrow \sqrt{F} \gtrsim 10^5$ GeV
- Gravitino is LSP, $m_{\tilde{G}} \sim \frac{<F>}{M_{Planck}} \ll M_{weak}$
- NLSP: (i) N small $\rightarrow$ neutralino
  (ii) N large $\rightarrow$ slepton



(VI) $\mathcal{SUSY}$ $\mathcal{P}$article $\mathcal{P}$roduction

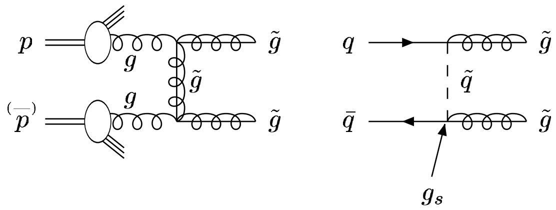
(i) Hadron Collider

Large production cross sections for moderate squark/gluino masses through strong interactions in $pp/p\bar{p}$ collisions

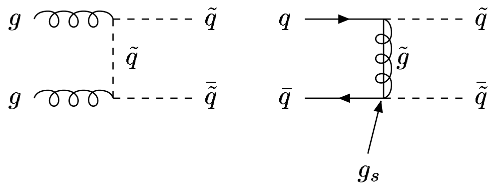
3 classes of SUSY pair production processes:

(i) Strongly interacting particle pairs

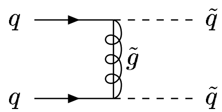
$p\overset{(-)}{p} \longrightarrow \tilde{g}\tilde{g}$



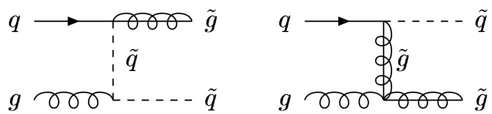
$p\overset{(-)}{p} \longrightarrow \tilde{q}\bar{\tilde{q}}$



$p\overset{(-)}{p} \longrightarrow \tilde{q}\tilde{q}$



$p\overset{(-)}{p} \longrightarrow \tilde{q}\tilde{g}$



$$\sigma_{gg} = \int_{\tau_0}^1 dx_1 \int_{\tau_0/x_1}^1 dx_2\, g(x_1,\mu_F^2) g(x_2,\mu_F^2)\, \hat\sigma_{gg}(\hat s = x_1 x_2 s)$$

with $\tau_0 = 4m^2/s$ $[m = (m_1+m_2)/2]$ and

$$\hat\sigma_{gg}(\hat s = x_1 x_2 s) = \alpha_s^2(\mu_R) f_0(\hat s = x_1 x_2 s)$$

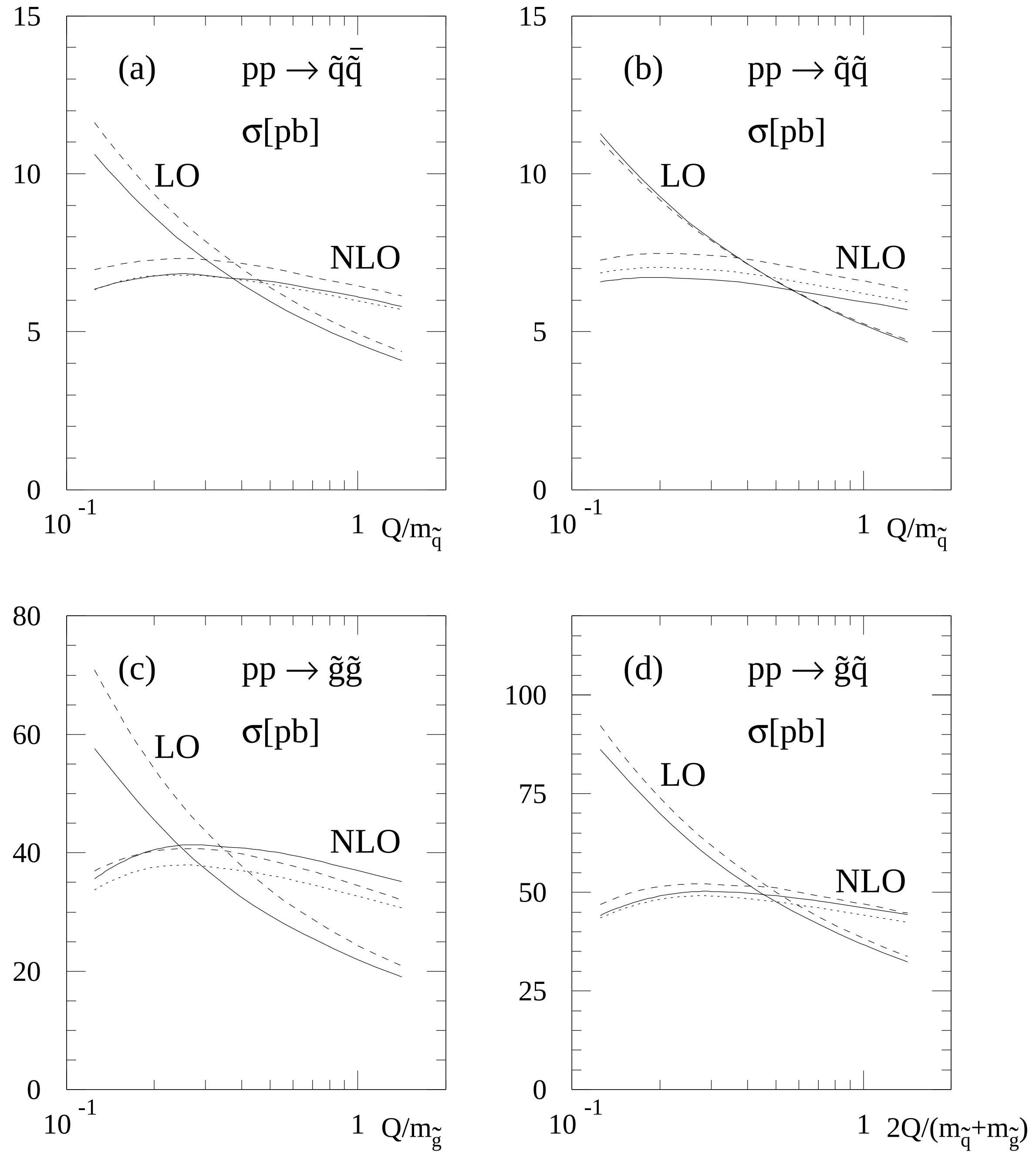
- leading order: large theoretical uncertainties (due to undefined renormalization scale μ_R, factorisation scale μ_F)
- natural scale: $\mu_R = \mu_F = m$; scale variation $\rightarrow$ estimate of theoretical uncertainties with respect to scale choice (large logarithms in higher order):

$$\begin{aligned} \alpha_s(Q^2) &= \frac{\alpha_s(\mu^2)}{1+\frac{33-2N_F}{12}\frac{\alpha_s(\mu^2)}{\pi}\log\frac{Q^2}{\mu^2}} \\ &= \alpha_s(\mu^2)\left[1 - \frac{33-2N_F}{12}\frac{\alpha_s}{\pi}\log\frac{Q^2}{\mu^2} + \mathcal{O}(\alpha_s^2)\right] \end{aligned}$$

[parton densities in analogy]

$$\tfrac{1}{2}m < \mu_R = \mu_F < 2m : \delta\sigma \sim \pm 50\ \%$$

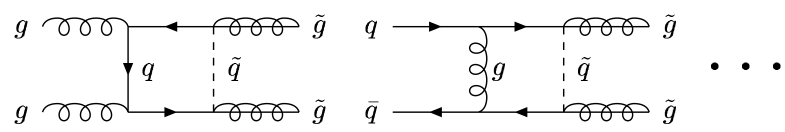
<u>LHC</u>



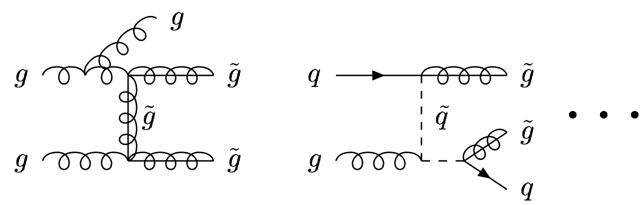
Beenakker,Höpker,Spira,Zerwas

Effect reduced through SUSY QCD corrections:

- virtual 1-loop contributions



- real contributions through gluon radiation/crossing



$$\begin{aligned}\hat{\sigma}_{gg} &= \alpha_s^2(\mu_R)\left\{f_0 + f_0\frac{33-3N_F}{6}\frac{\alpha_s}{\pi}\log\frac{\mu_R^2}{m^2} + \frac{\alpha_s}{\pi}f_1\right\} \\ &= \alpha_s^2(m^2)\left\{f_0 + \frac{\alpha_s}{\pi}f_1 + \mathcal{O}(\alpha_s^2)\right\}\end{aligned}$$

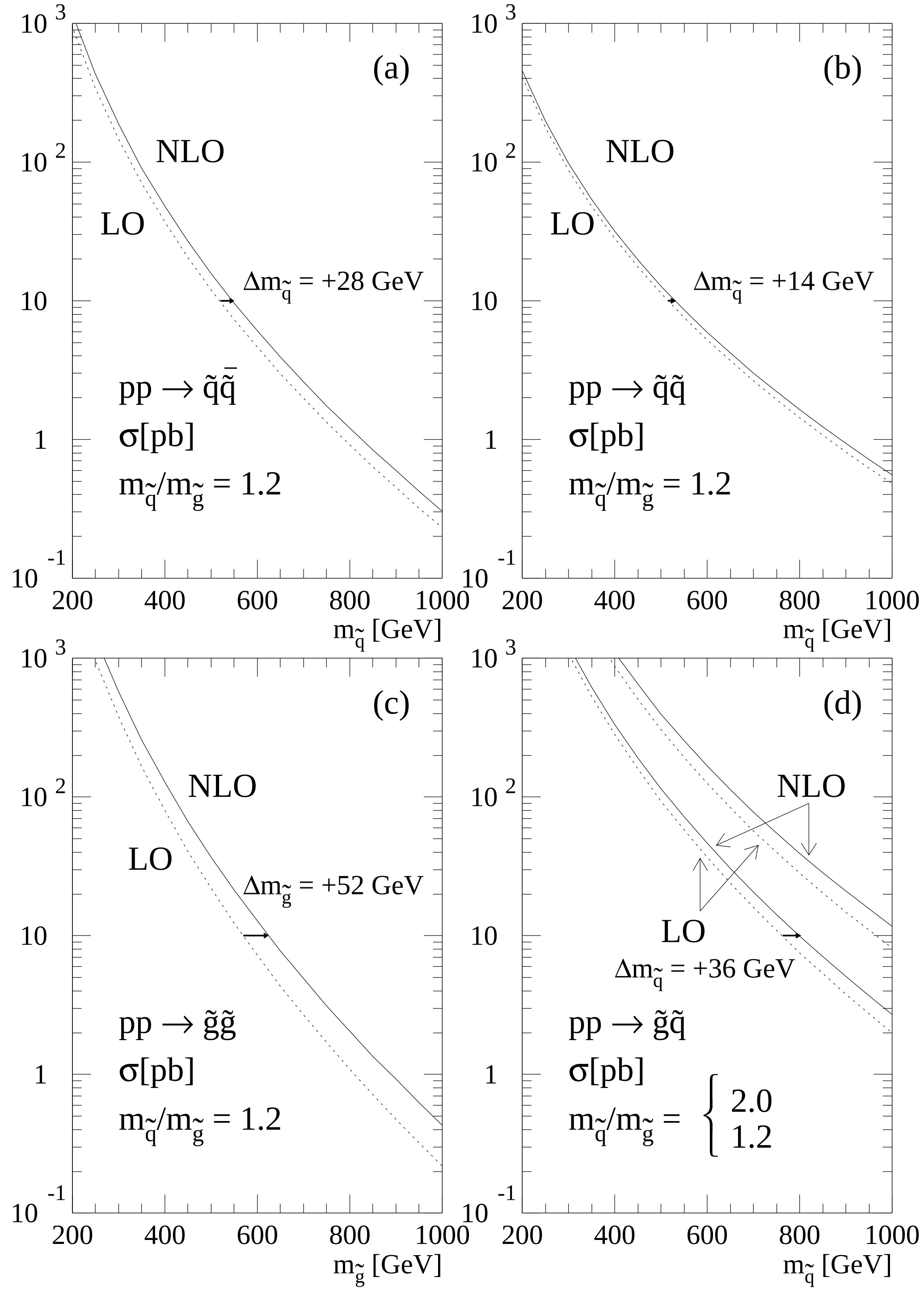
[parton densities in analogy]

$$\frac{1}{2}m < \mu_R = \mu_F < 2m : \delta\sigma \sim \pm 10\ \%$$

$\Rightarrow$ NLO corrections provide reliable predictions of cross sections at hadron colliders

central scale: $K = \frac{\sigma_{NLO}}{\sigma_{LO}} \sim 1.1 - 2.0$

LHC



Beenakker,Höpker,Spira,Zerwas

Implications for exp. searches:

(i) Renorm./factor. scale dep. reduced by $\sim 2.5 - 4 \rightsquigarrow$ stable theor. predictions for σ

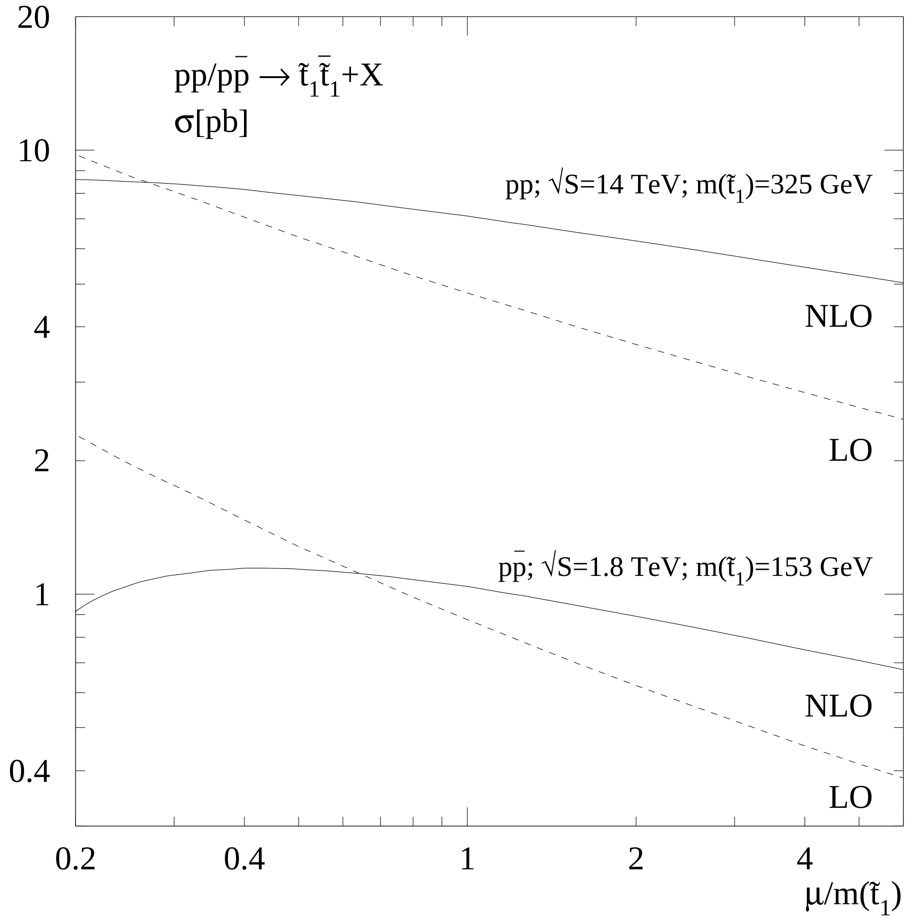
(ii) NLO corrs. large & positive $\rightsquigarrow$ to be included in analyses ($\longrightarrow$ masses)

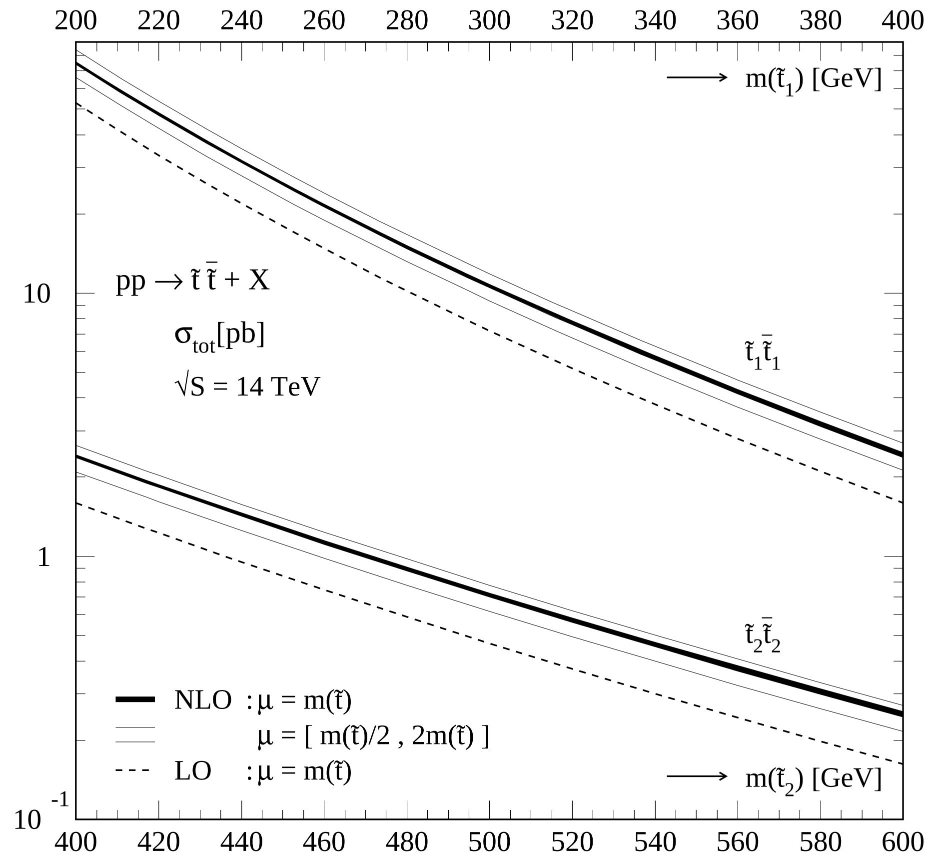
(iii) p_T and y distributions hardly affected by NLO

(iv) NLO $\rightsquigarrow$ raise of Tev lower $\tilde{q}, \tilde{g}$ mass bounds by +10–30 GeV

Stop-antistop pairs

Beenakker, Krämer, Plehn, Spira, Zerwas





Classical signatures (R-parity conserving SUSY, i.e. pair production/LSP stable):

- gluino $>$ squark:

$$\tilde{q} \rightarrow q\tilde{\chi}_1^0 = q + E_T^{miss}$$
$$\tilde{g} \rightarrow q\tilde{q} \rightarrow qq\tilde{\chi}_1^0 = qq + E_T^{miss}$$

- squark $>$ gluino:

$$\tilde{g} \rightarrow q\tilde{q}_{virt} \rightarrow qq\tilde{\chi}_1^0 = qqE_T^{miss}$$
$$\tilde{q} \rightarrow q\tilde{g} \rightarrow qqq\tilde{\chi}_1^0 = qqq + E_T^{miss}$$

$$\boxed{pp \rightarrow n\ jets + E_T^{miss}}$$

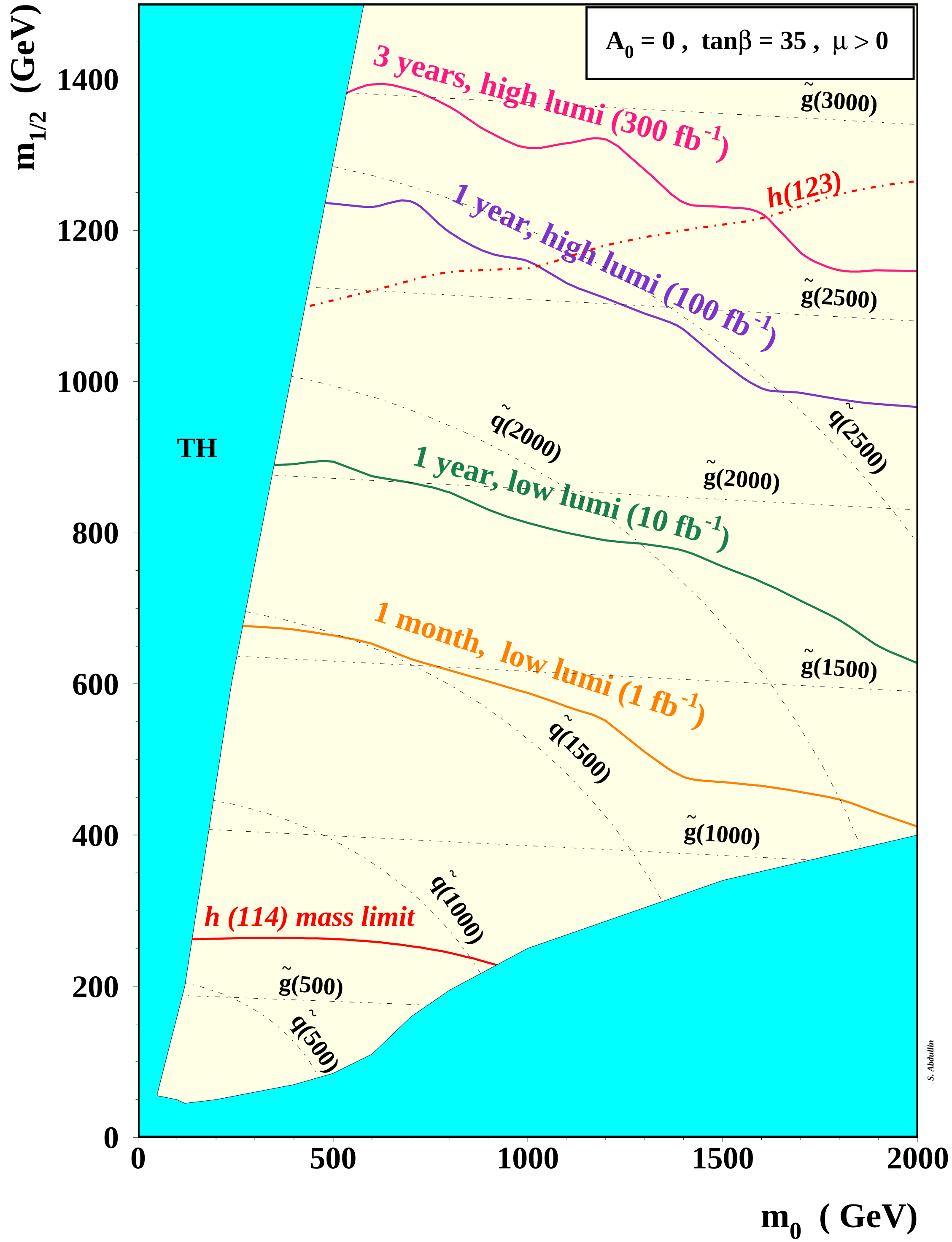
Discovery range:

Tevatron	$\lesssim$	500 GeV
LHC	$\lesssim$	$2.5...3$ TeV

Mass reach of the LHC

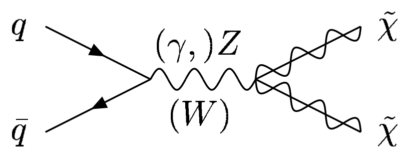
$\boxed{M_{\tilde{q},\tilde{g}} \leq 2.5 \text{ to } 3 \text{ GeV}}$

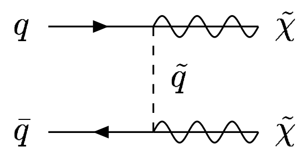
Signature: $E_T^{\text{miss}} +$ jets



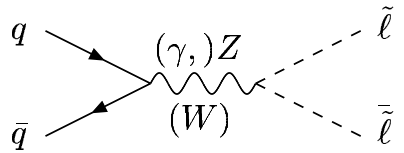
(ii) Weakly interacting particle pairs

$$p \overset{(-)}{p} \to \tilde{\chi}\tilde{\chi}$$





$$p \overset{(-)}{p} \to \tilde{l}\bar{\tilde{l}}$$



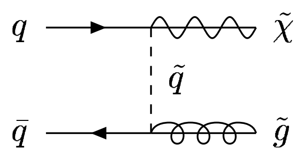
Signatures

$\tilde{l} \quad \to \quad l\tilde{\chi}_1^0$: $\quad pp \to l^+l^- + E_T^{miss}$

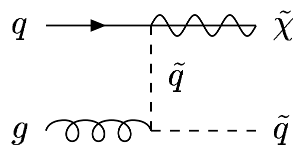
$\tilde{\chi}_2^0 \quad \to \quad l^+l^-\tilde{\chi}_1^0$ etc. : $\quad pp \to l^+l^+l^-l^- + E_T^{miss}$ etc.

(iii) Associated production

$$p \overset{(-)}{p} \to \tilde{g}\tilde{\chi}$$



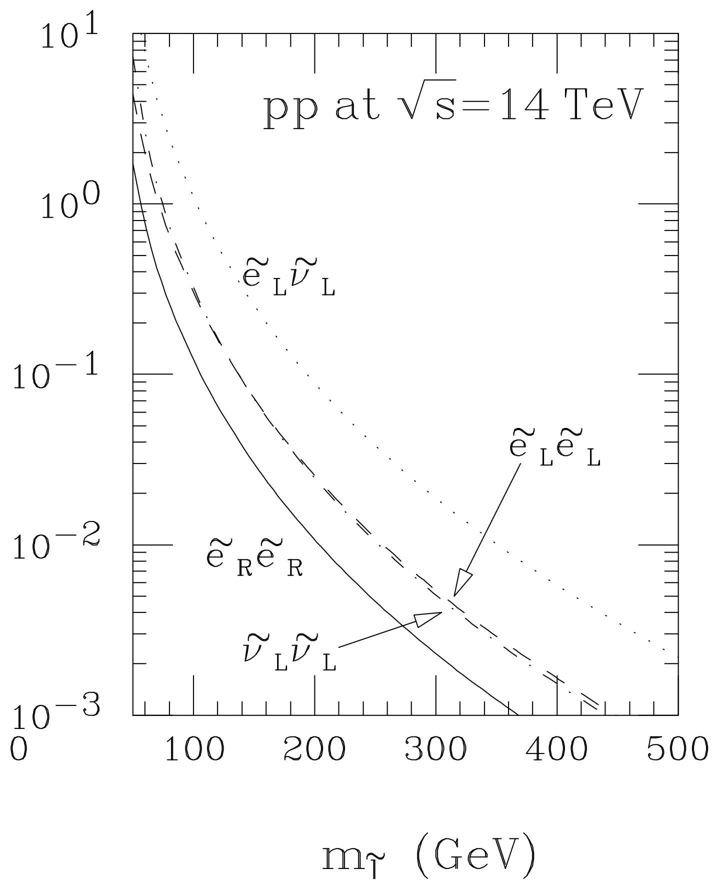
$$p \overset{(-)}{p} \to \tilde{q}\tilde{\chi}$$



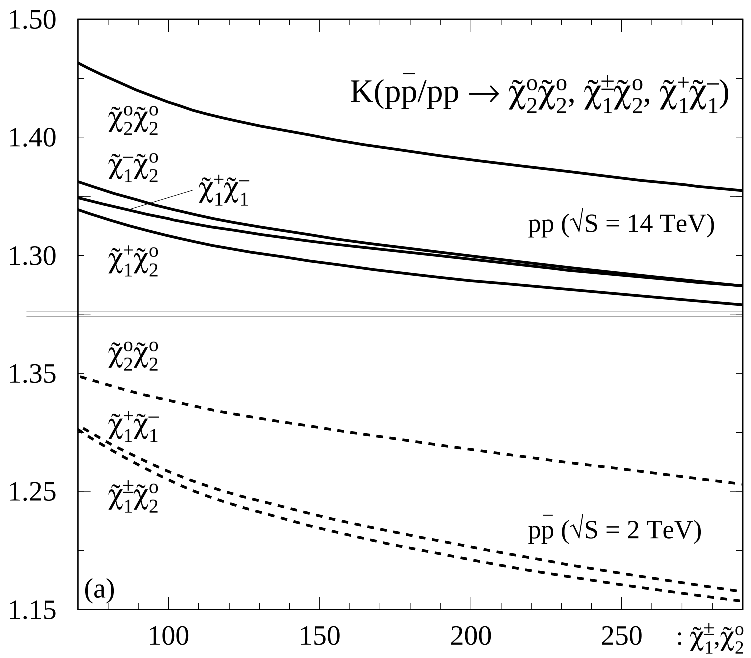
Slepton pair production at NLO

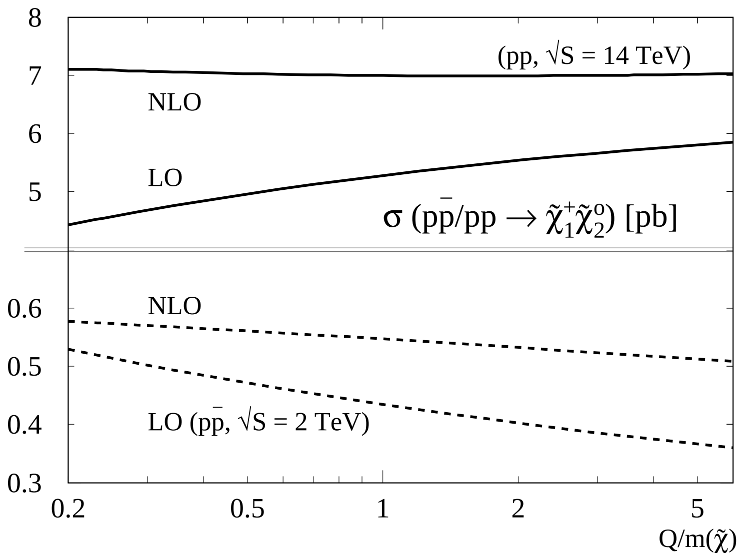
- Drell-Yan processes mediated by Z^* or W^*
- With QCD corrections at LHC $\sigma_{NLO} \sim (1.25 - 1.35)\sigma_{LO}$
- Cross section is small < 1 fb at $m_{\tilde{l}} \approx 500$ GeV

Baer et al., hep-ph/9712315: Cross section in [pb]



Gaugino Pairs

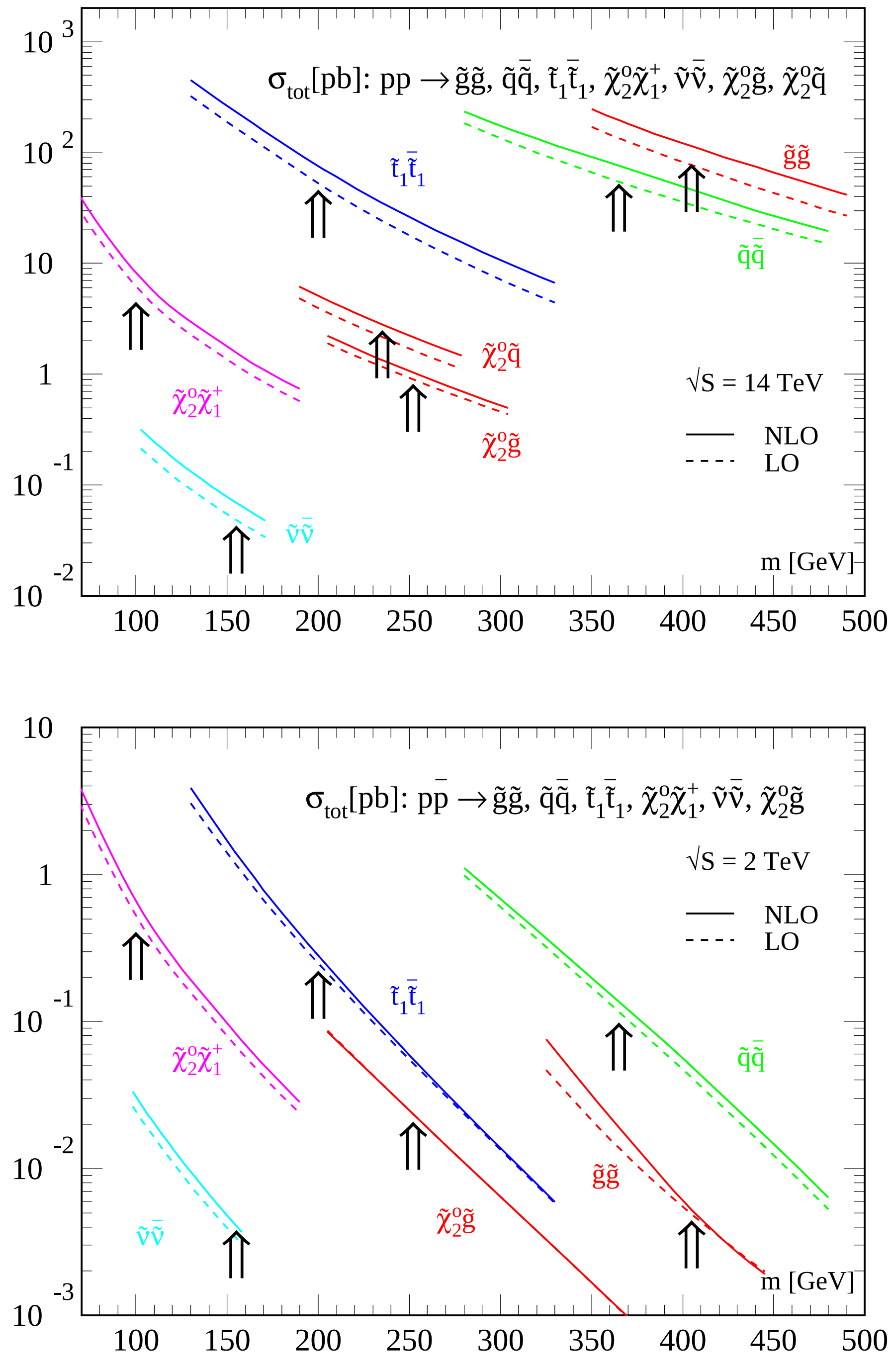




Beenakker,Klasen,Krämer,Plehn,Spira,Zerwas

Prospino

Beenakker, Krämer, Plehn, Spira, Zerwas



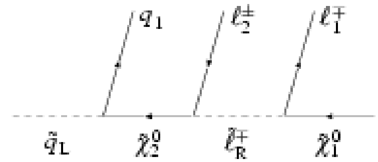
Cascade decays

gluino/squark decays = rich source for non-colored (supersymmetric) particles

Mass determination: kinematic endpoint technique

Contruct lepton/quark upper/lower endpoints and relate them to the masses in the decay chain

E.g.: $\tilde{q} \to \tilde{\chi}_2^0 q \to \tilde{l}^{\pm} l^{\mp} q \to \tilde{\chi}_1^0 l^+ l^- q = j + l^+ + l^- + E_T^{miss}$



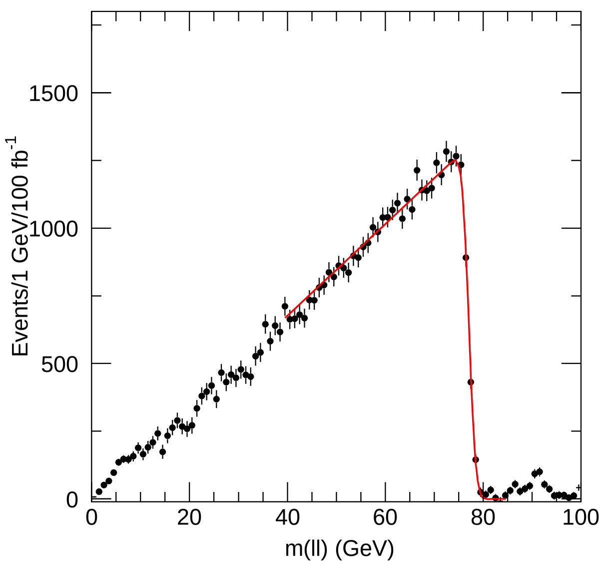
4 unknown masses: $M_{\tilde{q}}, M_{\tilde{\chi}_2^0}, M_{\tilde{l}}, M_{\tilde{\chi}_1^0}$

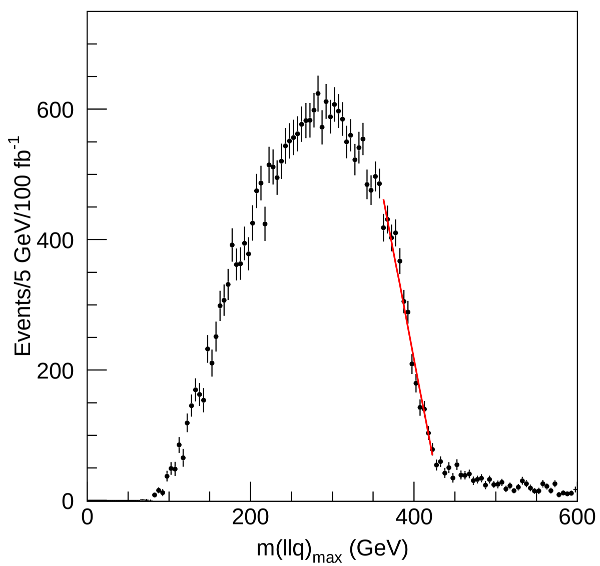
4 endpoints: $M(ll)^{max}, M(l_1 q)^{max}, M(l_2 q)^{max}, M(llq)^{max}$

$\Rightarrow$ all masses can be determined

$$\max M^2(ll) = M_{\tilde{\chi}_2^0}^2 \left[1 - \frac{M_{\tilde{l}}^2}{M_{\tilde{\chi}_2^0}^2}\right] \left[1 - \frac{M_{\tilde{\chi}_1^0}^2}{M_{\tilde{l}}^2}\right]$$

$$\max M^2(llq) = M_{\tilde{q}}^2 \left[1 - \frac{M_{\tilde{\chi}_2^0}^2}{M_{\tilde{q}}^2}\right] \left[1 - \frac{M_{\tilde{\chi}_1^0}^2}{M_{\tilde{\chi}_2^0}^2}\right]$$





Spin:

particle chain in SUSY equivalent to UED

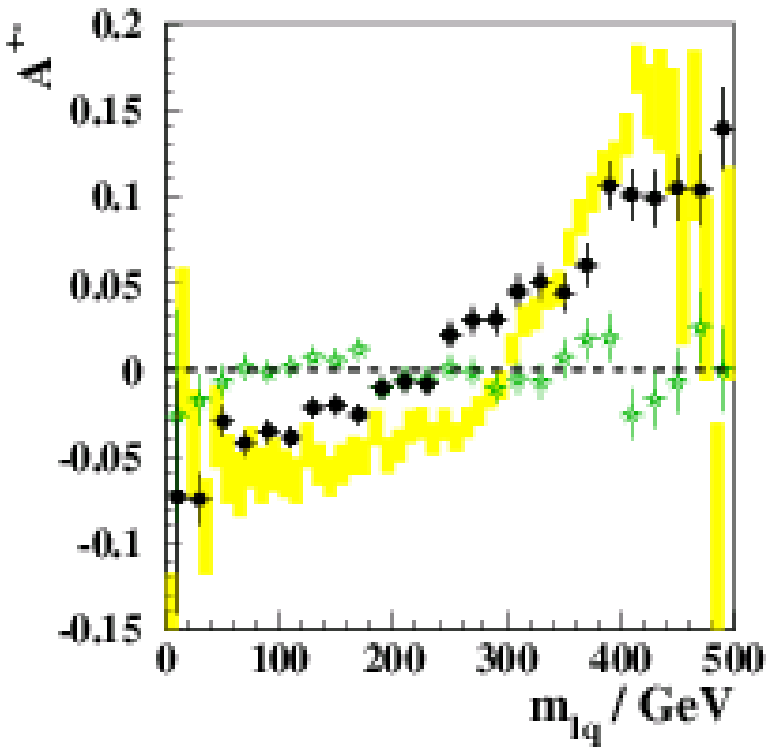
SUSY: $\tilde{q}_L \to q + \tilde{\chi}_2^0 \to q + (\tilde{l}l) \to q + ll + \tilde{\chi}_1^0$

UED: $q_1 \to q + Z_1 \to q + (l_1 l) \to q + ll + \gamma_1$

distinction by spin: $\sim$ angular distributions / invariant masses

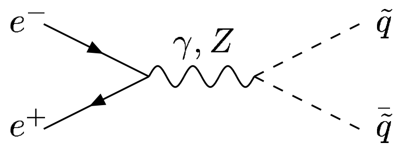
charge asymmetry in $[ql^+]$ vs $[ql^-]$:

difficult analysis $\to$ A.J.Barr, hep-ph/0405052.



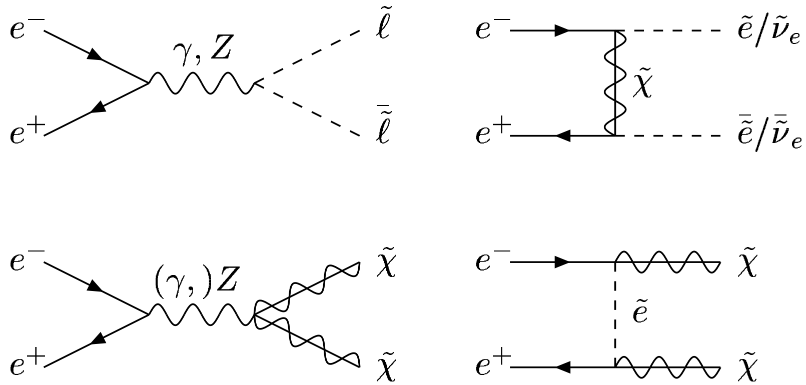
(ii) e^+e^- Collider 2 classes of SUSY particle pair production processes:

(i) Strongly interacting particle pairs



NLO QCD & SUSY-QCD corrections known, $\mathcal{O}$(several 10%)

(ii) Weakly interacting particle pairs



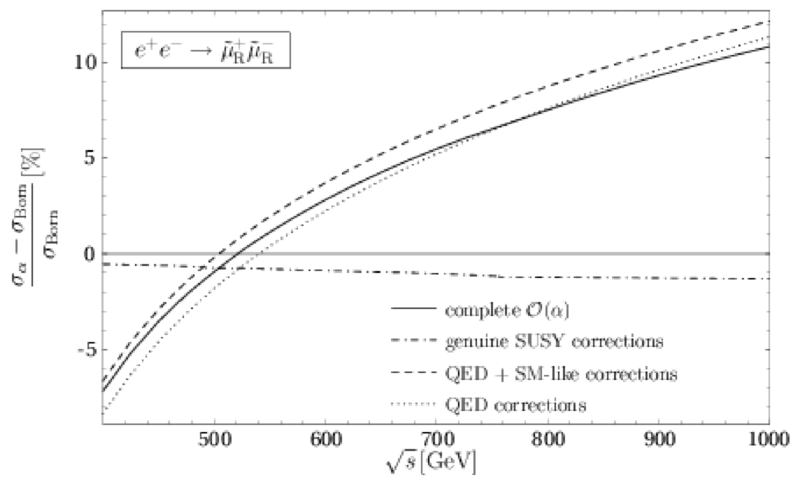
1-loop analysis

[Freitas,Manteuffel,Zerwas]:

dominating QED, but $\Rightarrow$

genuine SUSY $\sim$ few percent

experimentally relevant

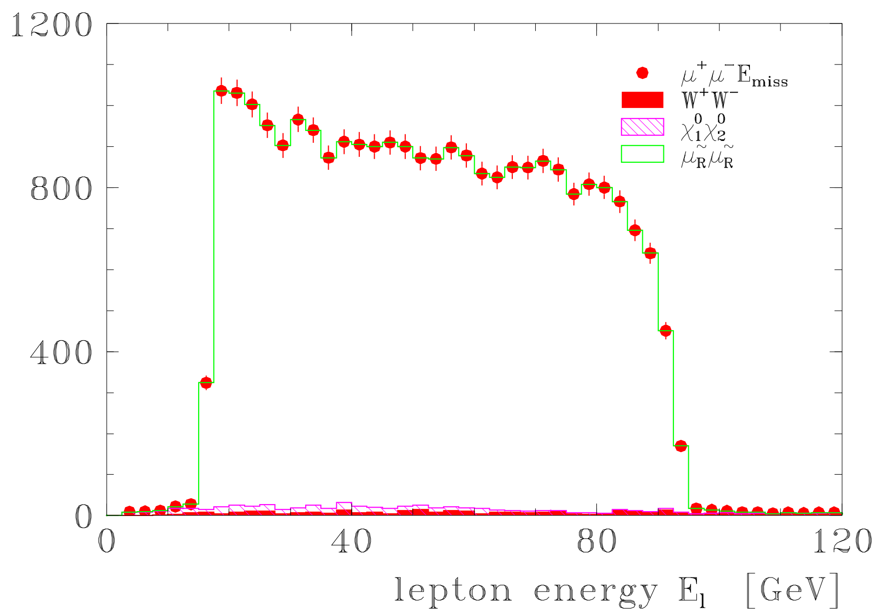


Masses at the ILC:

- Edge effects: $\tilde{\mu}_R \to \mu \tilde{\chi}_1^0$

$$m_{\tilde{l}} = \sqrt{s}[E_+ E_-]^{\frac{1}{2}}/(E_+ + E_-)$$
$$m_{\tilde{\chi}_1^0} = m_{\tilde{l}}[1 - 2(E_+ + E_-)/\sqrt{s}]^{\frac{1}{2}}$$

precision on $\tilde{\chi}_1^0$ improved by $\sim 10^2$



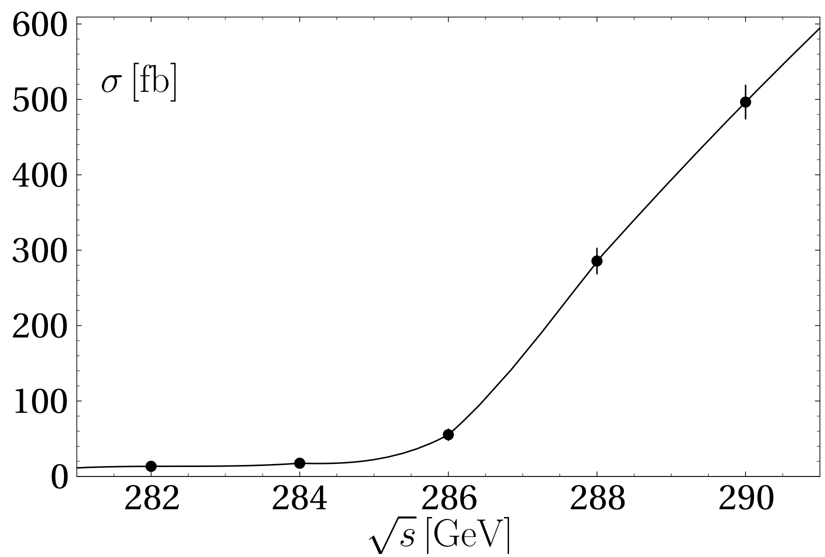
- Threshold excitations:

$$e^+e^- \to \tilde{\mu}_R^+\tilde{\mu}_R^- \to \mu^+\mu^- + E_{miss}$$

P-wave: slow $\beta^3 \sim [s - 4m_{\tilde{\mu}}^2]^{\frac{3}{2}}$ rise

$$e^-e^- \to \tilde{e}_R^-\tilde{e}_R^- \to e^-e^- + E_{miss}$$

S-wave: fast $\beta \sim [s - 4m_{\tilde{e}}^2]^{\frac{1}{2}}$ rise



- voids filled: LHC and LC complementarity
- accuracy increased by 1 to 2 orders of magnitude: $\Delta\tilde{m} \sim 50$ MeV ~ 0.2 per mille
- coherent [LHC⊕LC] analysis superior to incoherent sum of individual analyses

Summary:

$\boxed{\text{LHC+ILC}}$

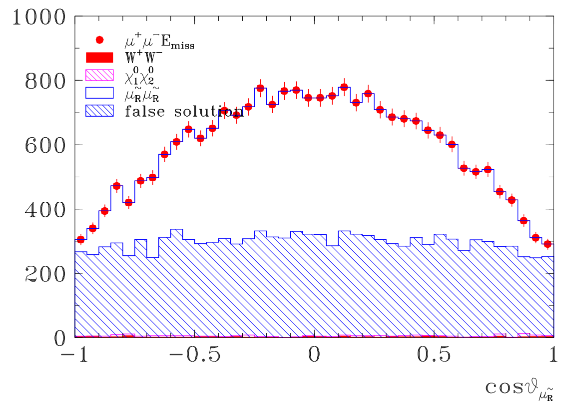
Coherent LHC+ILC analyses complete and increase resolution of SUSY picture significantly

	Mass, ideal	"LHC"	"ILC"	"LHC+ILC"
$\tilde{\chi}_1^\pm$	179.7		0.55	0.55
$\tilde{\chi}_2^\pm$	382.3	–	3.0	3.0
$\tilde{\chi}_1^0$	97.2	4.8	0.05	0.05
$\tilde{\chi}_2^0$	180.7	4.7	1.2	0.08
$\tilde{e}_R$	143.9	4.8	0.05	0.05
$\tilde{e}_L$	207.1	5.0	0.2	0.2
$\tilde{\nu}_e$	191.3	–	1.2	1.2
$\tilde{\mu}_R$	143.9	4.8	0.2	0.2
$\tilde{\tau}_1$	134.8	5-8	0.3	0.3
$\tilde{\tau}_2$	210.7	–	1.1	1.1
$\tilde{q}_L$	570.6	8.7	–	4.9
$\tilde{t}_1$	399.5		2.0	2.0
$\tilde{t}_2$	586.3		–	
$\tilde{g}$	604.0	8.0	–	6.5
h^0	110.8	0.25	0.05	0.05
A^0	399.4		1.5	1.5

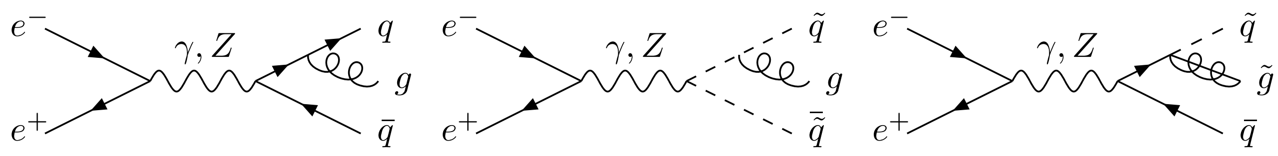
Spin at the ILC:

$$e^+e^- \to \tilde{\mu}^+\tilde{\mu}^- \to \mu^+\mu^- + \tilde{\chi}_1^0\tilde{\chi}_1^0$$

production axis can be reconstructed up to 2-fold ambiguity



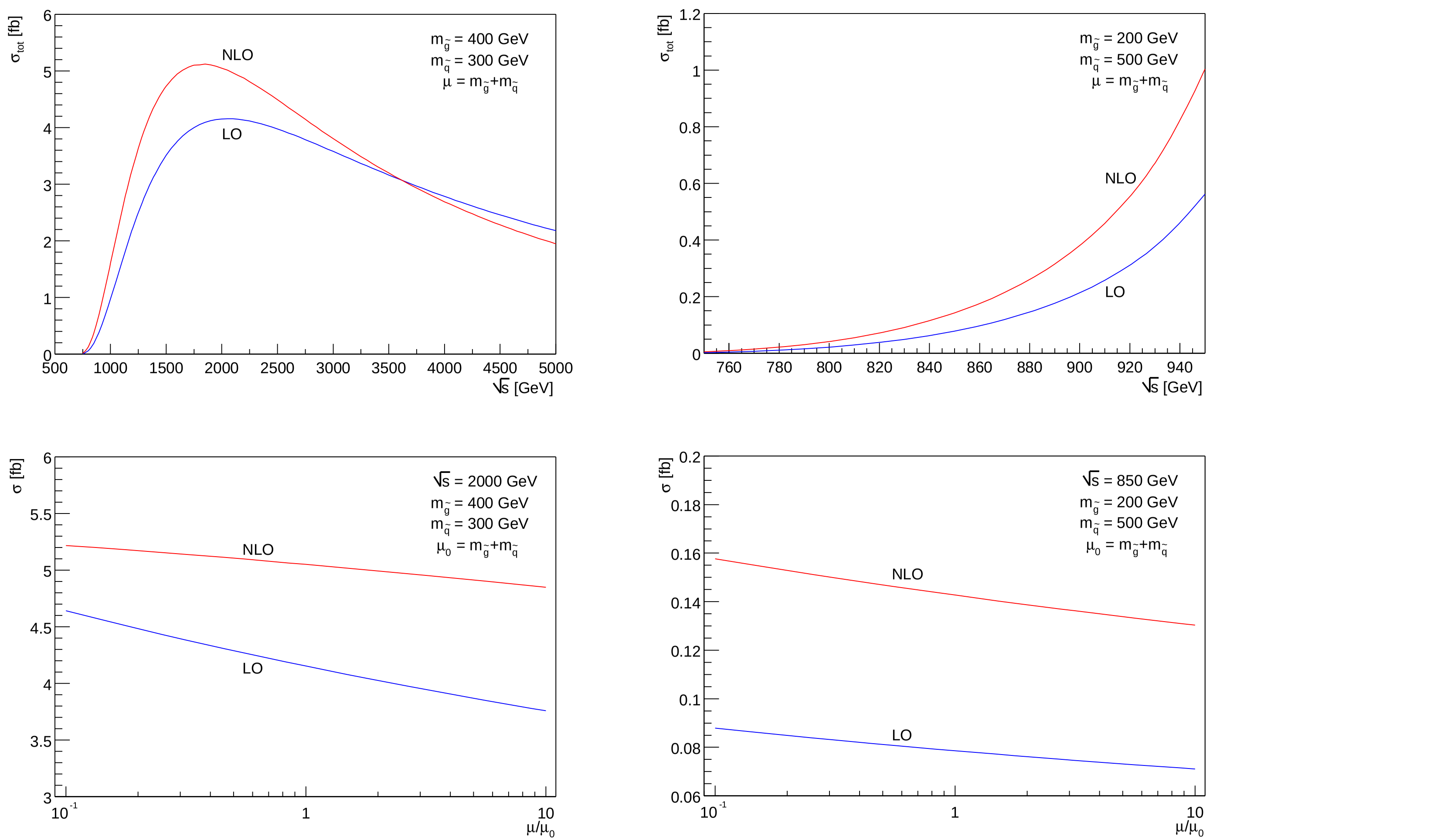
$\underline{e^+e^- \to q\bar{q}g, \tilde{q}\bar{\tilde{q}}g, \tilde{q}\bar{q}\tilde{g}}$



Possibility to test equality of couplings, needs NLO!

$\boxed{e^+e^- \to \tilde{q}\bar{q}\tilde{g}}$

Brandenburg, Maniatis, Weber, Zerwas



(VII) $\mathcal{MSSM}$ $\mathcal{H}$iggs $\mathcal{B}$osons

MSSM Higgs sector – supersymmetry & anomaly free theory $\Rightarrow$ 2 complex Higgs doublets

$\xrightarrow{\text{EWSB}}$ neutral, CP-even h, H neutral, CP-odd A charged H^+, H^-

Higgs masses

$$M_h \lesssim 140 \text{ GeV}$$
$$M_{A,H,H^\pm} \sim \mathcal{O}(v)...1 \text{ TeV}$$

Ellis et al;Okada et al;Haber,Hempfling;
Hoang et al;Carena et al;Heinemeyer et al;
Zhang et al;Brignole et al;...

Decoupling limit:

$M_A \sim M_H \sim M_{H^\pm} \gg v$

$M_h \longrightarrow$ max. value, $\tan\beta$ fixed; h becomes SM-like $M_h \sim \sqrt{M_Z^2 \cos^2 2\beta + \epsilon \sin^2\beta}$, $\alpha \sim \beta - \frac{\pi}{2}$

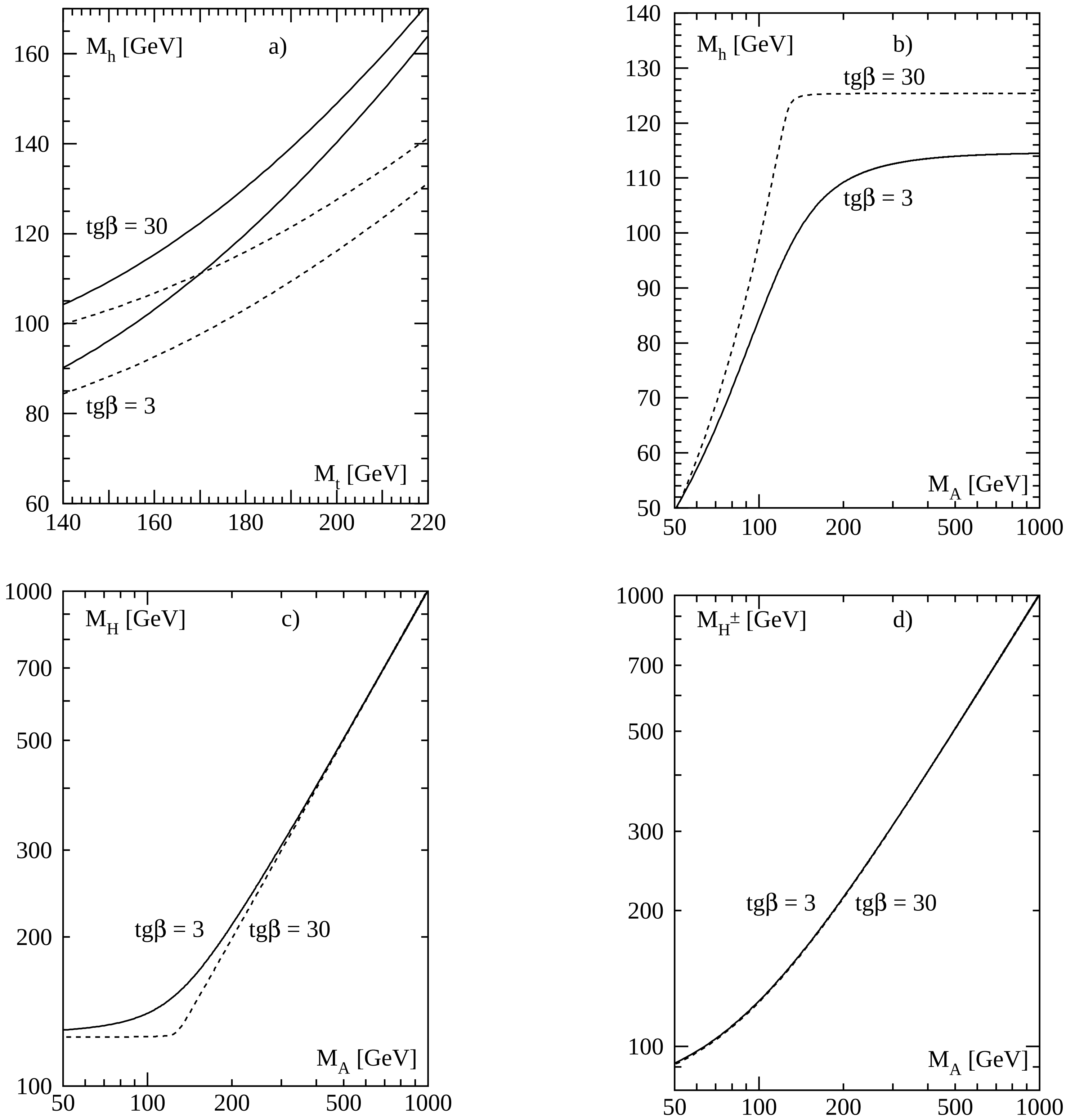
Modified couplings with respect to the SM: (decoupling limit Gunion,Haber)

Φ	$g_{\Phi u\bar{u}}$	$g_{\phi d\bar{d}}$	$g_{\Phi VV}$
h	$c_\alpha/s_\beta \longrightarrow 1$	$-s_\alpha/c_\beta \longrightarrow 1$	$s_{\beta-\alpha} \longrightarrow 1$
H	$s_\alpha/s_\beta \longrightarrow 1/\text{tg}\beta$	$c_\alpha/c_\beta \longrightarrow \text{tg}\beta$	$c_{\beta-\alpha} \longrightarrow 0$
A	$1/\text{tg}\beta$	$\text{tg}\beta$	0

$\tan\beta \uparrow \quad \Rightarrow g_{\Phi uu} \downarrow$

$g_{\Phi dd} \uparrow$

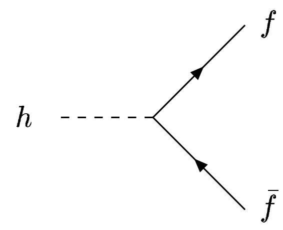
$g_{\Phi VV}^{MSSM} \lesssim g_{\Phi VV}^{SM}$



M_h [GeV]
a)
tgβ = 30
tgβ = 3
M_t [GeV]
M_h [GeV]
b)
tgβ = 30
tgβ = 3
M_A [GeV]
M_H [GeV]
c)
tgβ = 3
tgβ = 30
M_A [GeV]
M_H± [GeV]
d)
tgβ = 3
tgβ = 30
M_A [GeV]

Higgs decays

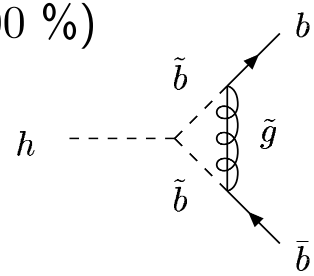
- $h \to b\bar{b}, \tau^+\tau^-, c\bar{c}$



large QCD corrections: $\sim -50 ... -80$ %

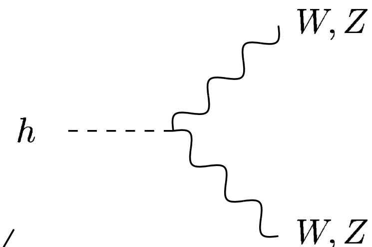
large SUSY-QCD corrections: $\mathcal{O}(10...100$ %)

$$\propto \frac{\alpha_s}{\pi} \frac{m_{\tilde{g}} \mu \tan\beta}{m_{\tilde{b}}^2}$$



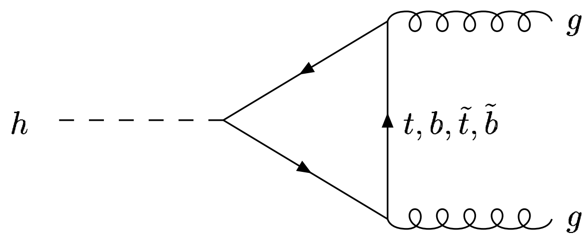
$+ \ldots$

- $h \to WW^*, ZZ^*$

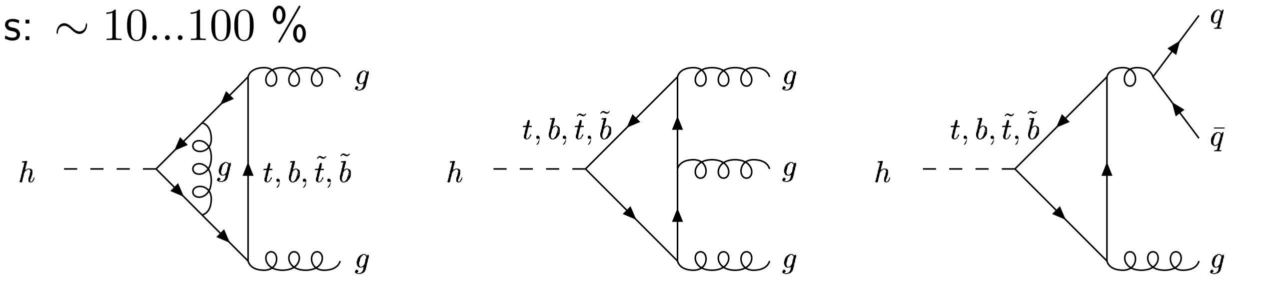


electroweak corrections: $\sim 5...10$ %

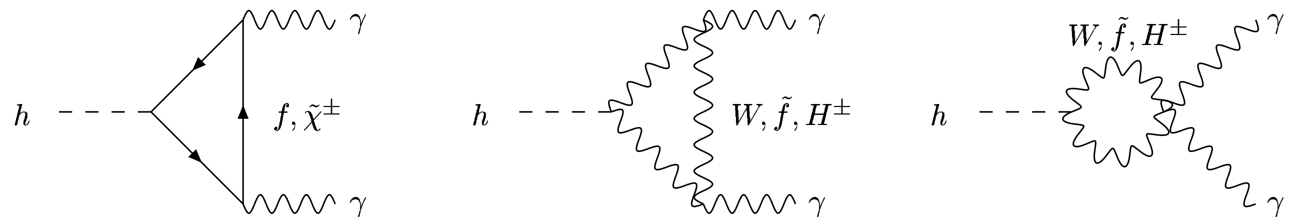
- $h \to gg$



large QCD corrections: $\sim 10...100$ %

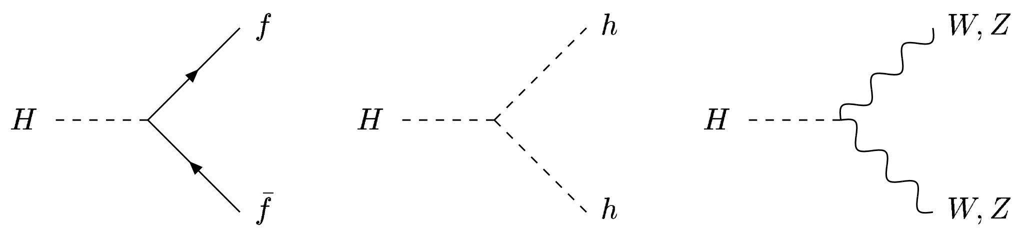


- $h \to \gamma\gamma$

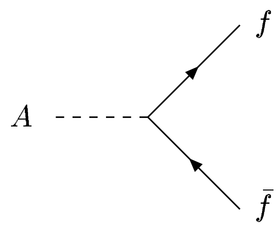


Extremely important decay channel for the LHC

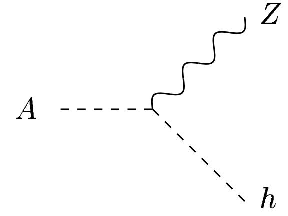
- $H \to b\bar{b}, \tau^+\tau^-$: dominant for large $\tan\beta$
- $H \to hh, WW, ZZ, t\bar{t}$

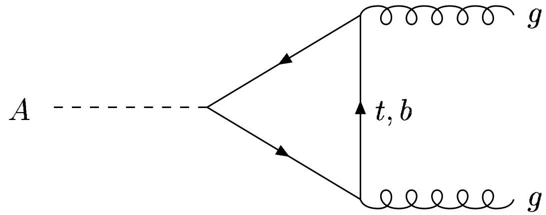


- $A \to b\bar{b}, \tau^+\tau^-$: dominant for large $\tan\beta$
- $A \to t\bar{t}$: dominant above the $t\bar{t}$ threshold for small and moderate $\tan\beta$

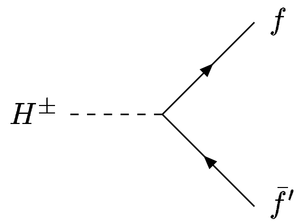


- $A \to Zh, gg$

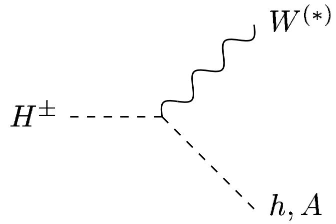


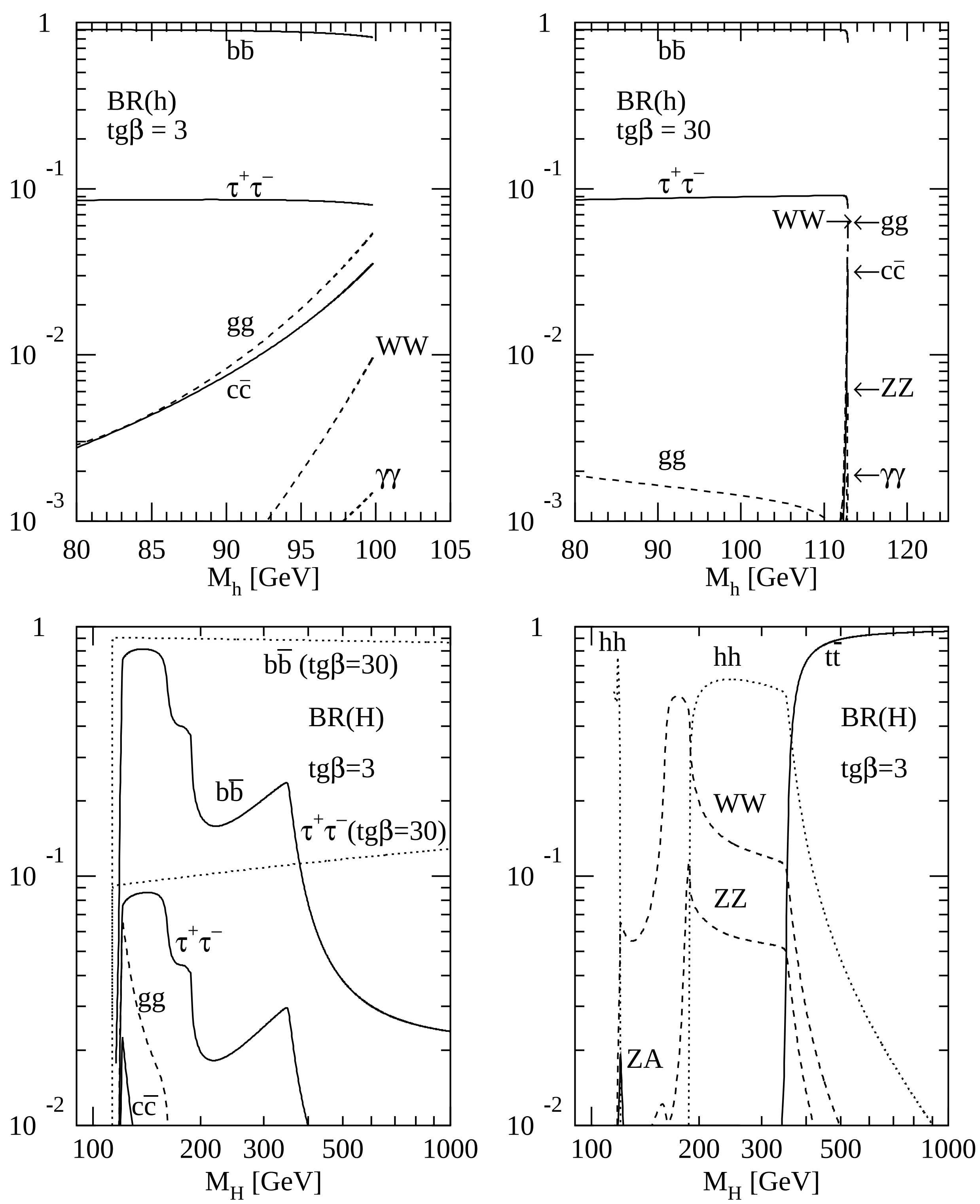


- $H^+ \to \tau^+\nu_\tau, t\bar{b}$

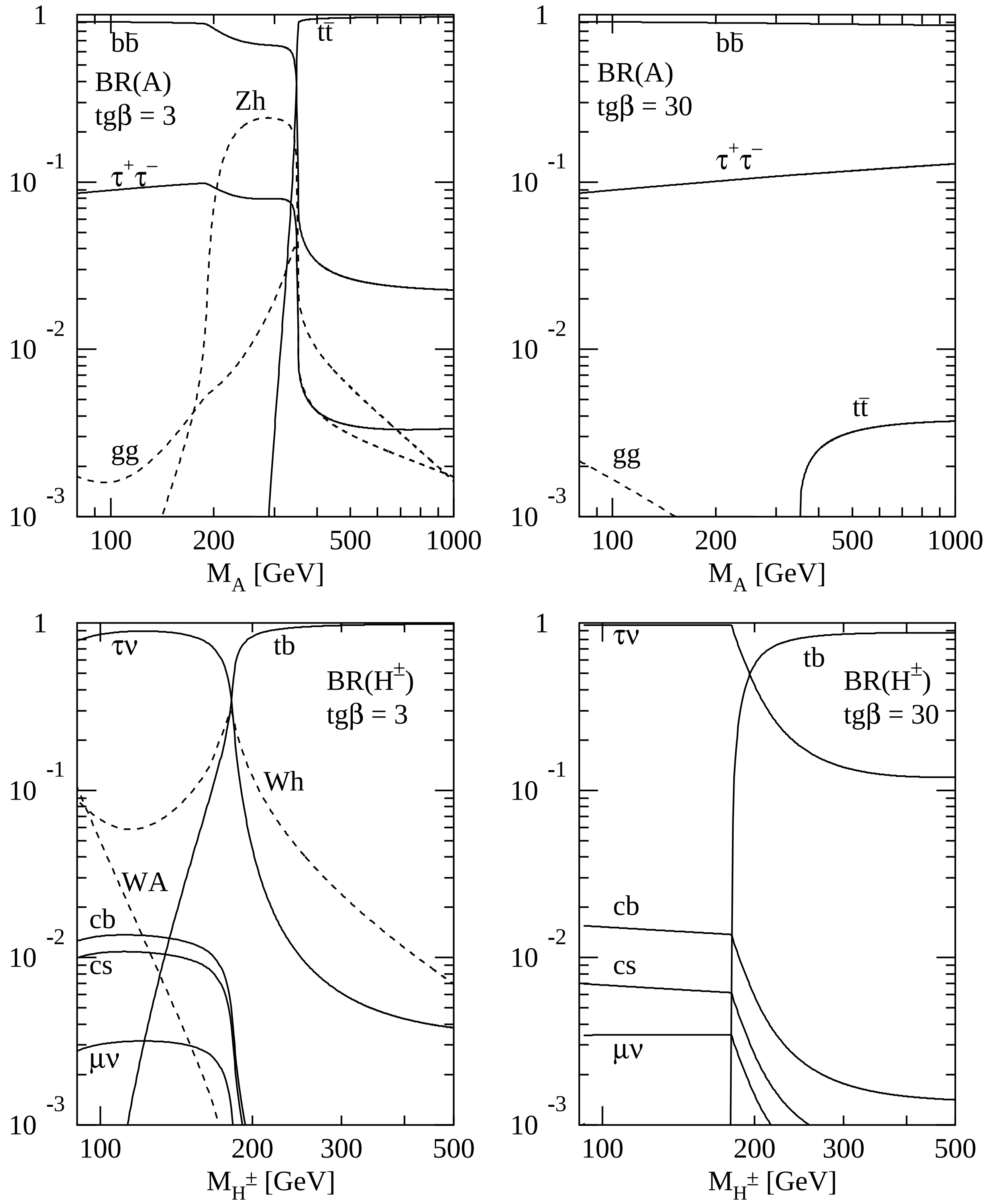


- $H^+ \to W^*A, Wh$



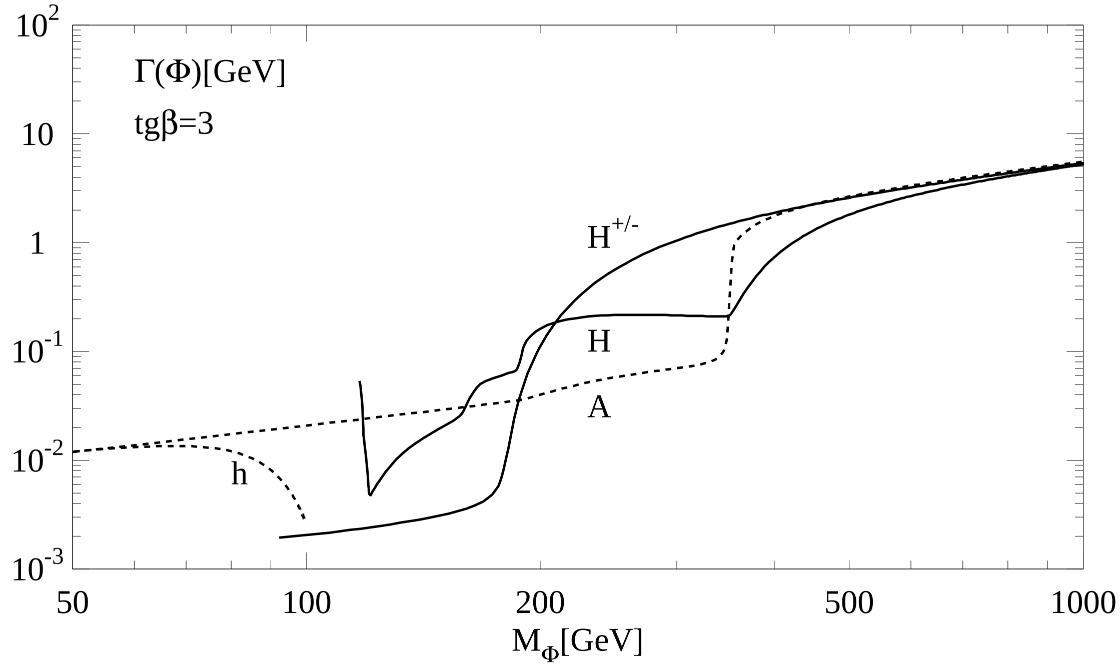


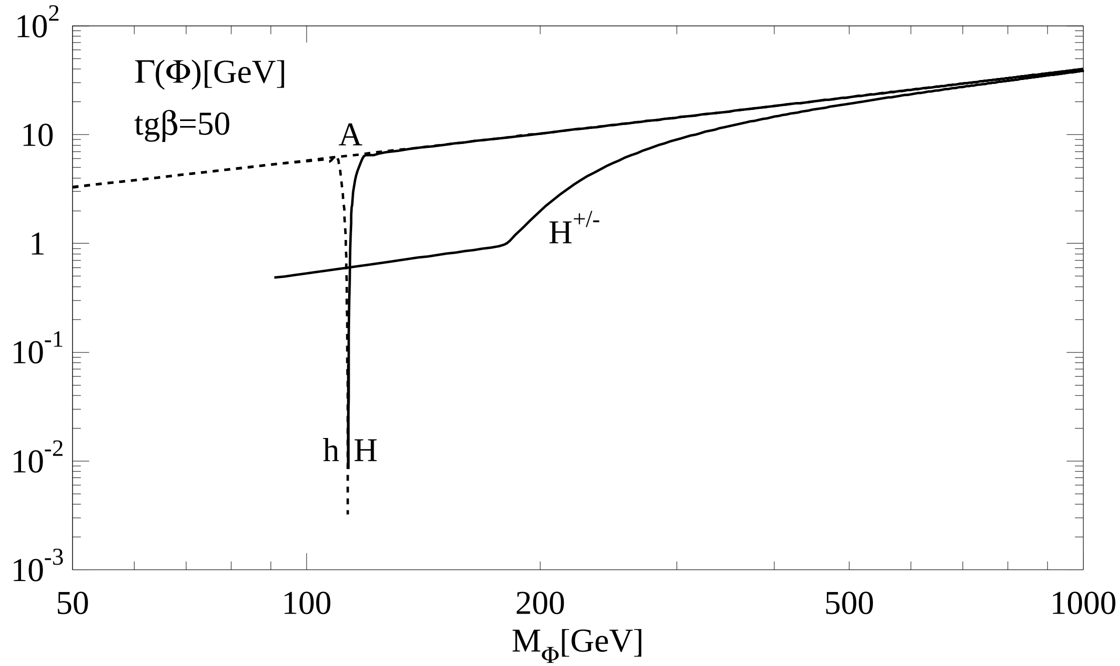
BR(h)
tgβ = 3
bb̄
τ+τ−
gg
cc̄
WW
γγ
Mh [GeV]
BR(h)
tgβ = 30
bb̄
τ+τ−
WW→←gg
←cc̄
←ZZ
←γγ
gg
Mh [GeV]
bb̄ (tgβ=30)
BR(H)
tgβ=3
bb̄
τ+τ−(tgβ=30)
τ+τ−
gg
cc̄
MH [GeV]
hh
hh
tt̄
BR(H)
tgβ=3
WW
ZZ
ZA
MH [GeV]



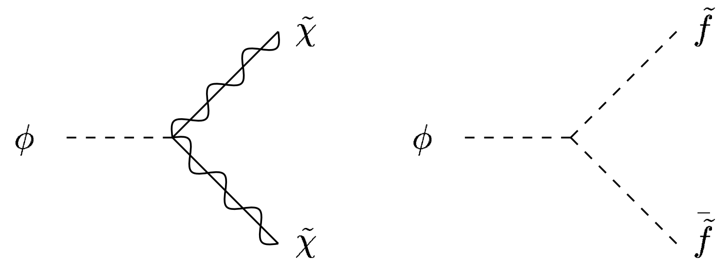
BR(A)
tgβ = 3
bb̄
tt̄
Zh
τ+τ−
gg
M_A [GeV]
BR(A)
tgβ = 30
bb̄
τ+τ−
tt̄
gg
M_A [GeV]
BR(H±)
tgβ = 3
τν
tb
Wh
WA
cb
cs
μν
M_H± [GeV]
BR(H±)
tgβ = 30
τν
tb
cb
cs
μν
M_H± [GeV]

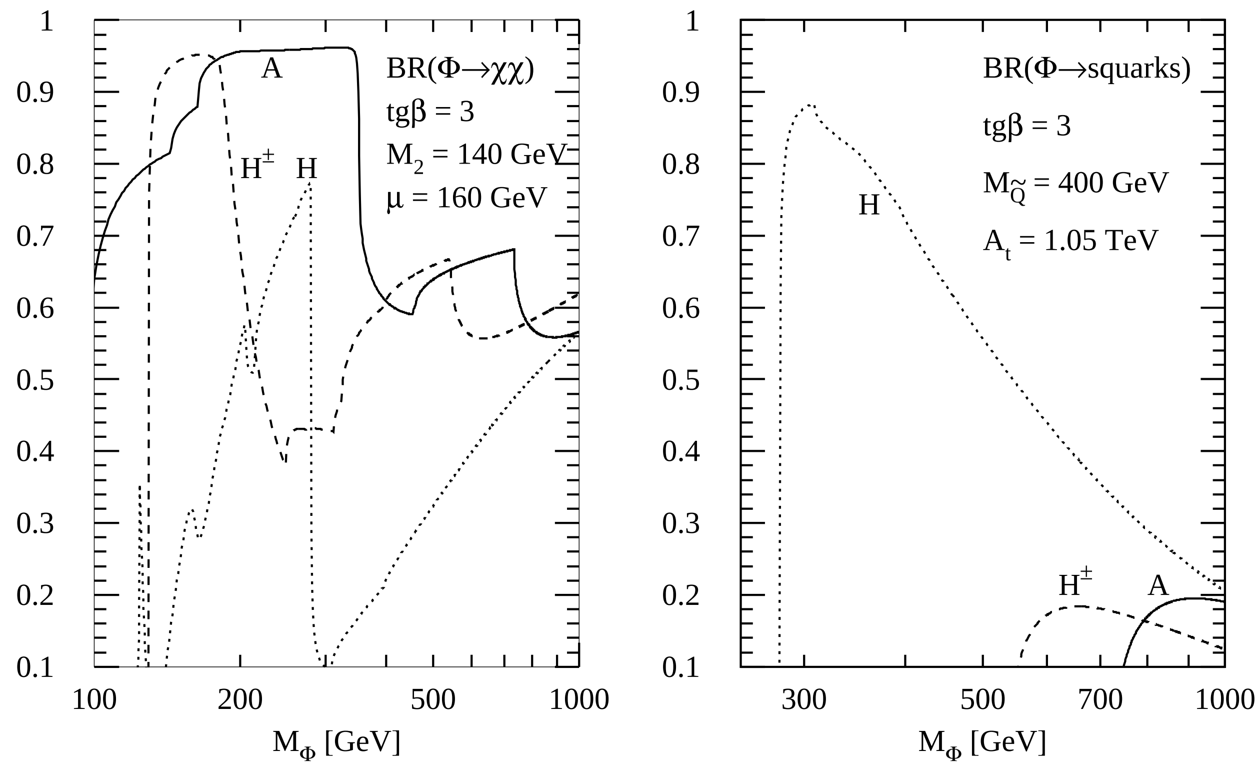
- total widths: $\Gamma_\phi \lesssim 10...30$ GeV narrow





- Decays in gauginos and sfermions (3rd generation) important, if kinematically allowed

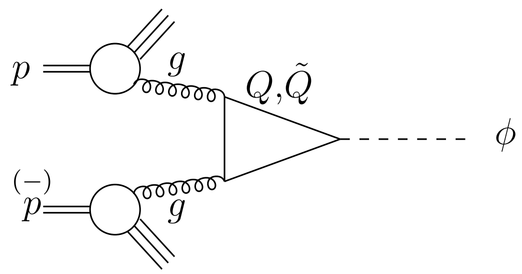




Higgs production

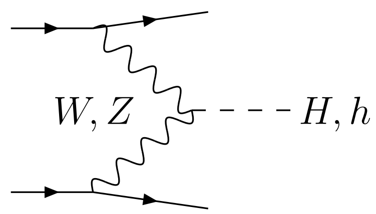
a) Hadron Collider

- Gluon fusion: $pp \to gg \to h, H, A$



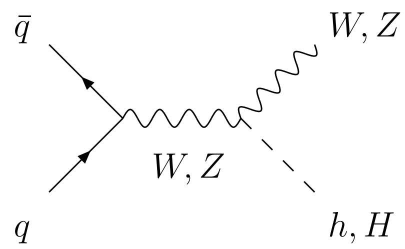
QCD corrections $\sim 10...100$ %

- W/Z fusion: $pp \to W^*W^*/Z^*Z^* \to h/H$



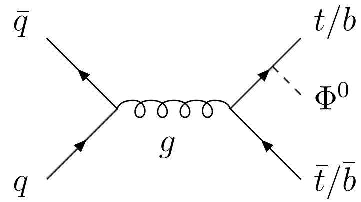
QCD corrections ~ 10 %

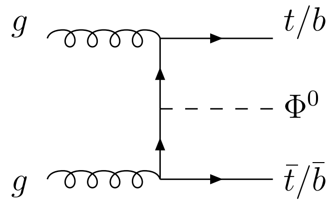
- Higgs-strahlung: $pp \to W^*/Z^* \to W/Z + h/H$



QCD corrections ~ 30 %

- Associated production with $t\bar{t}/b\bar{b}$: $pp \to t\bar{t}/b\bar{b} + h/H/A$





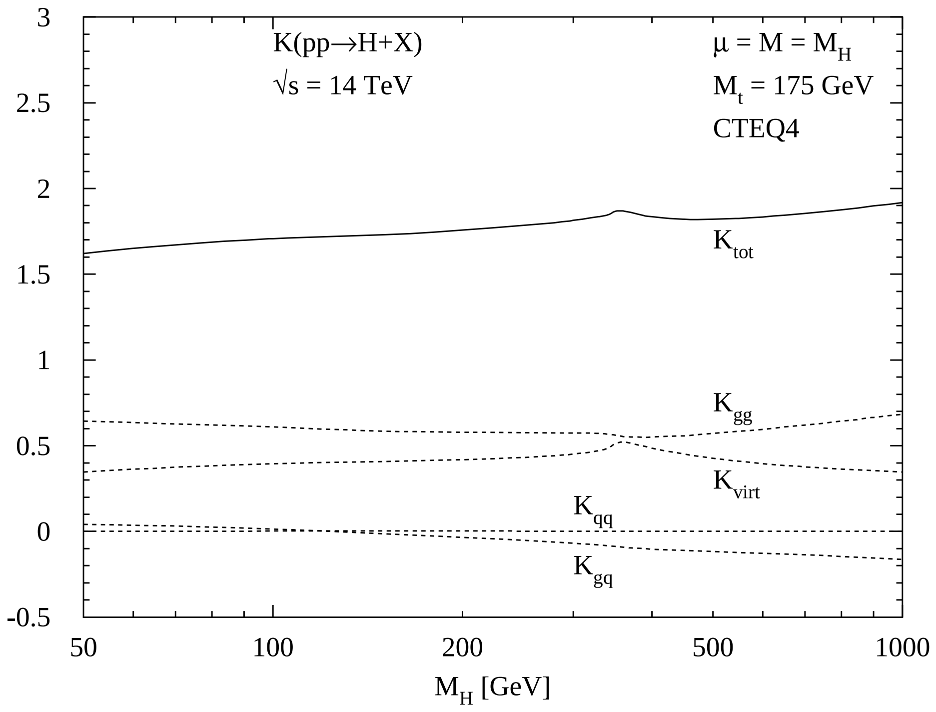
QCD corrections

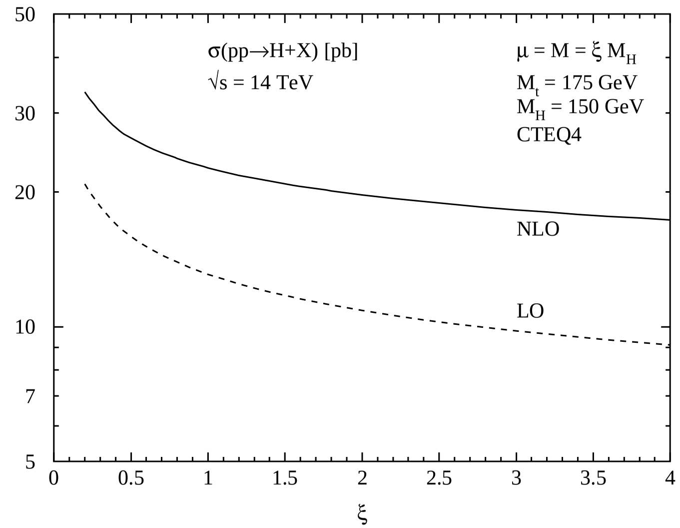
$t\bar{t}\Phi^0 : \quad \sim \pm 20\%$

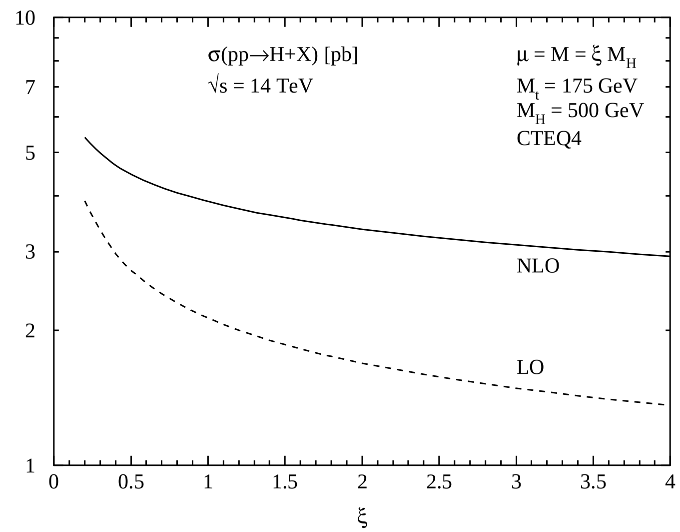
$b\bar{b}\Phi^0 : \quad \sim +(50 - 100)\%$

$gg \to H$ at NLO

Spira, Djouadi, Graudenz, Zerwas

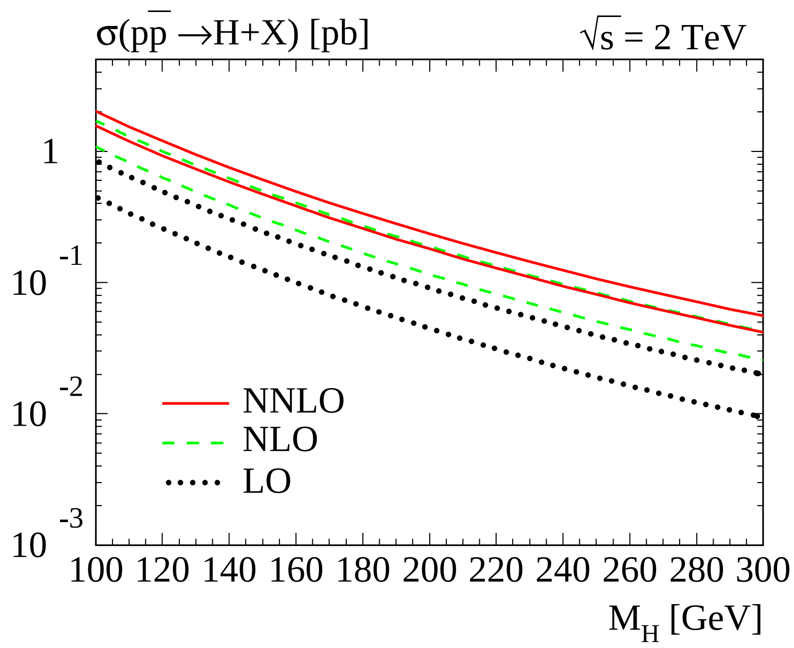




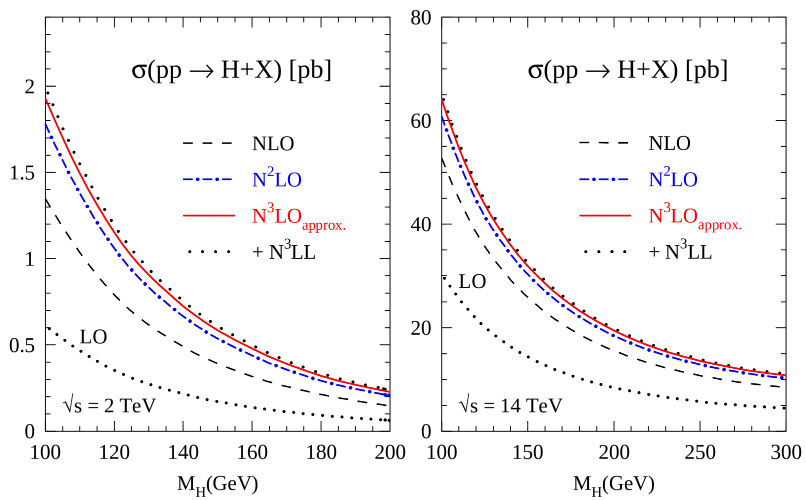


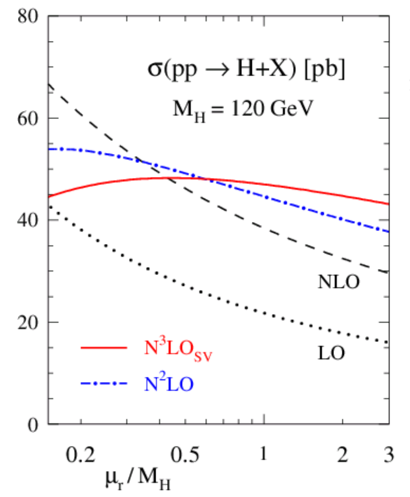
$gg \to H$ at NNLO and beyond

Harlander, Kilgore



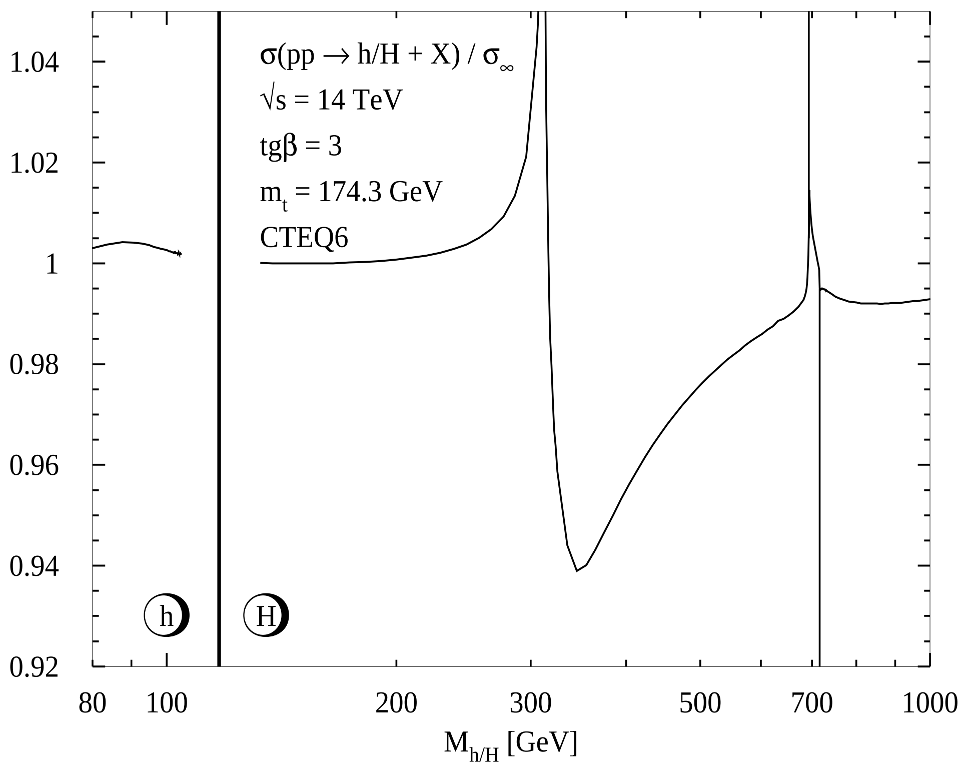
Moch, Vogt

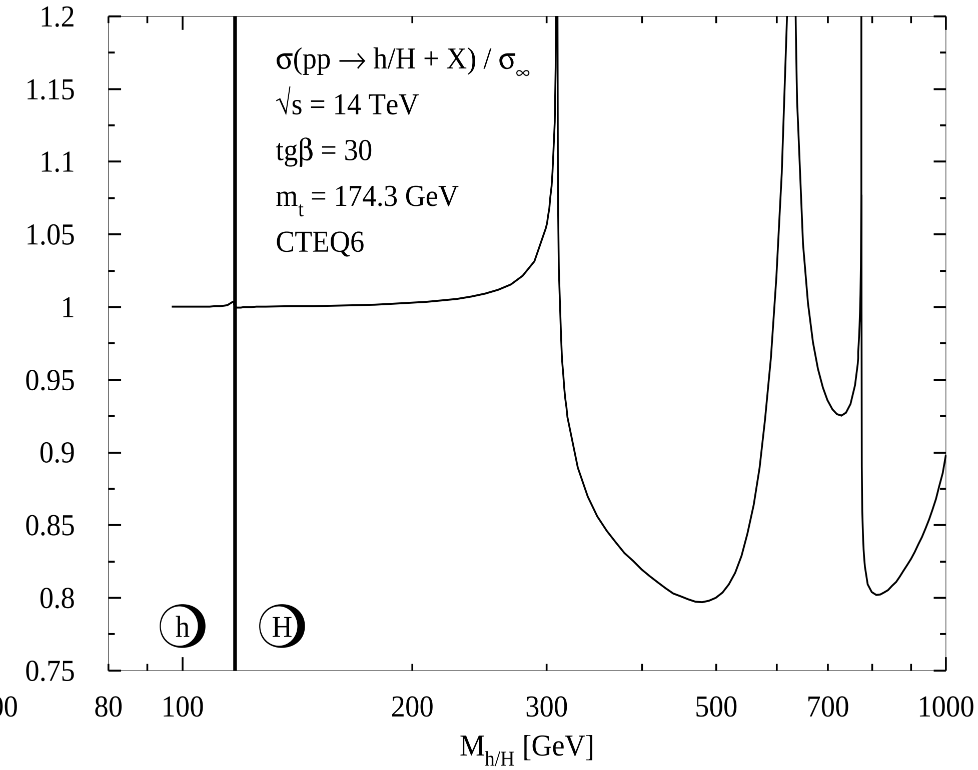




σ_{NLO} w/ full squark mass dependence / σ_{NLO} in the heavy squark limit

MMM,Spira



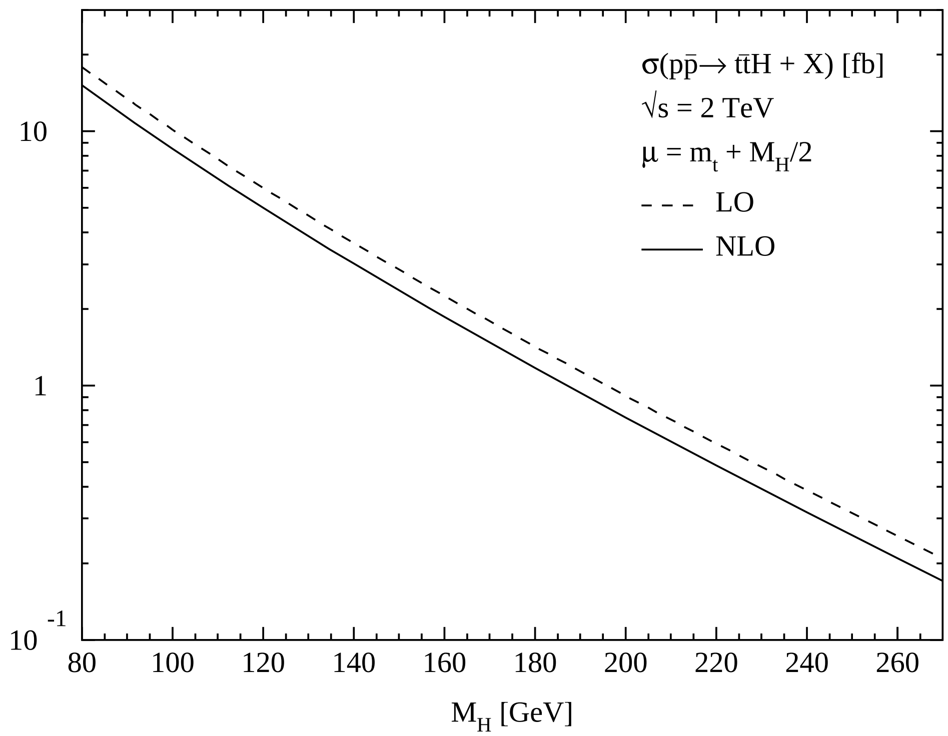


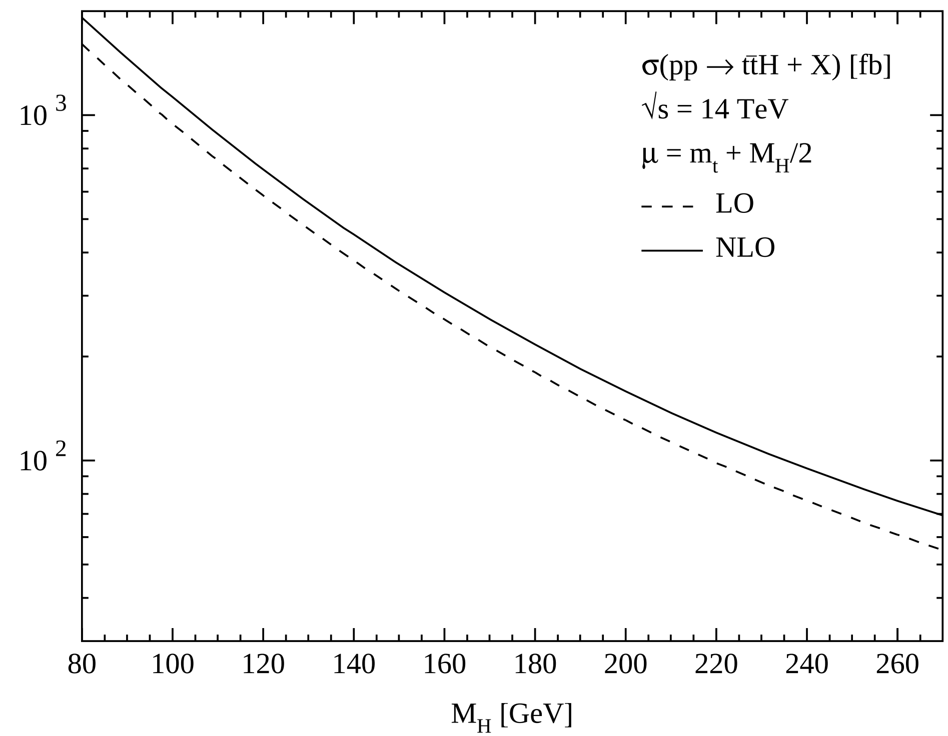
no $\tilde{g}$ loops, but bulk of expected SUSY QCD corrections

$\sigma(pp \to h/H + X)/\sigma_\infty$ up to 20%

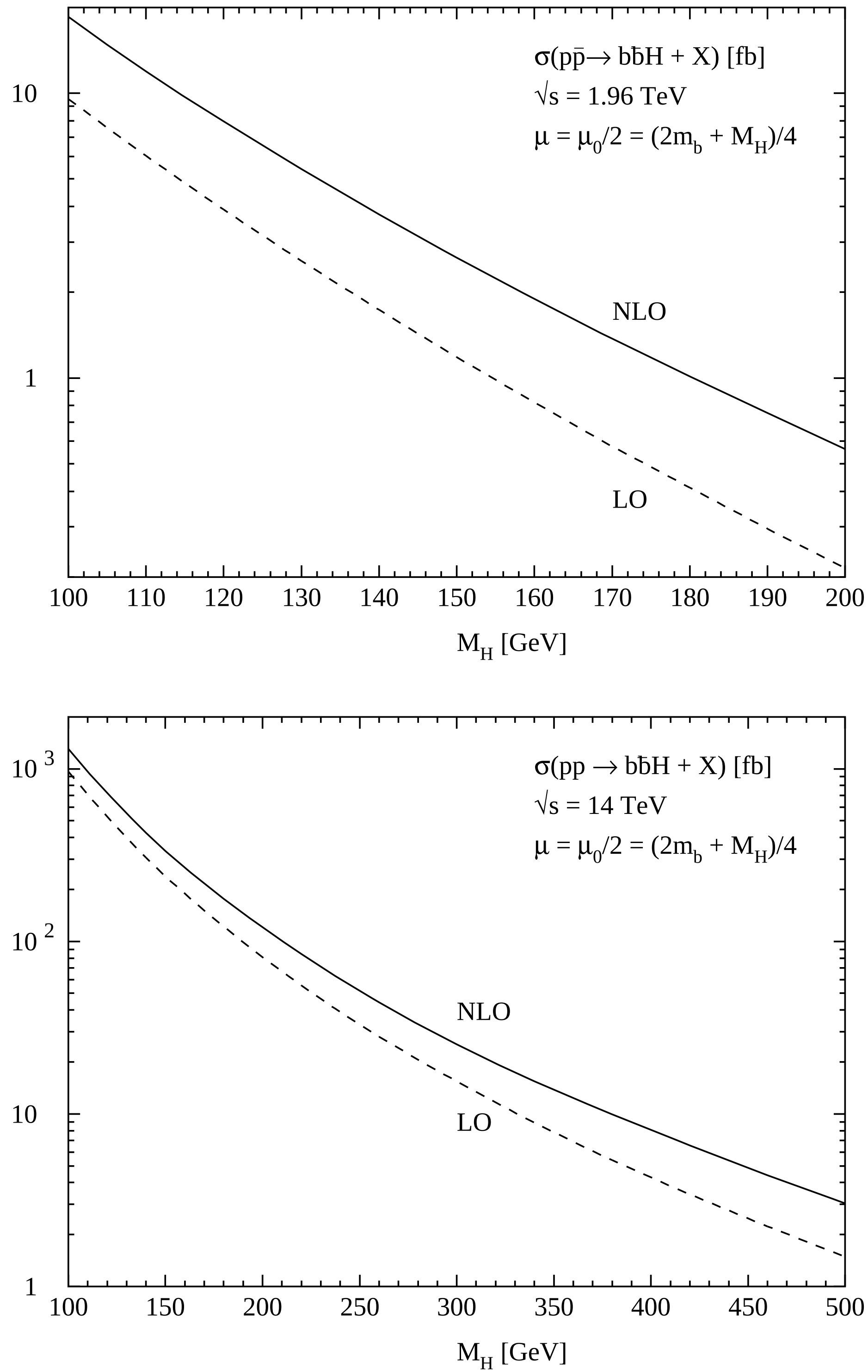
Kinks, bumps, spikes: $\tilde{t}_1\bar{\tilde{t}}_1, \tilde{b}_1\bar{\tilde{b}}_1, \tilde{b}_2\bar{\tilde{b}}_2$ thresholds in consecutive order with rising Higgs mass.

(analytically: Harlander,Kant
Anastasiou,Beerli,Bucherer,Daleo,Kunszt
Aglietti,Bonciani,Degrassi,Vicini → Comparison: full agreement)

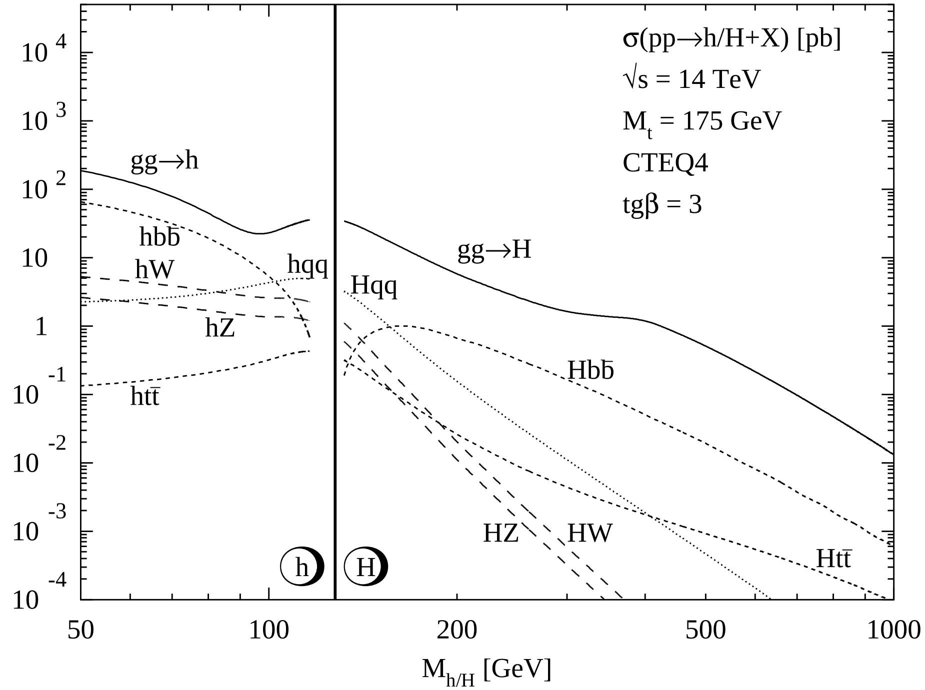


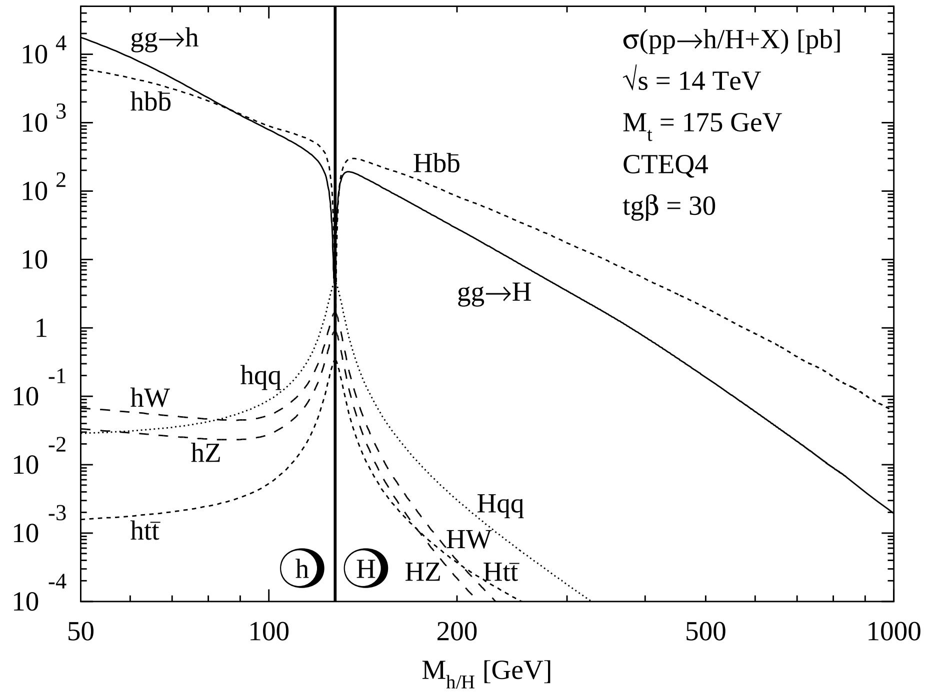


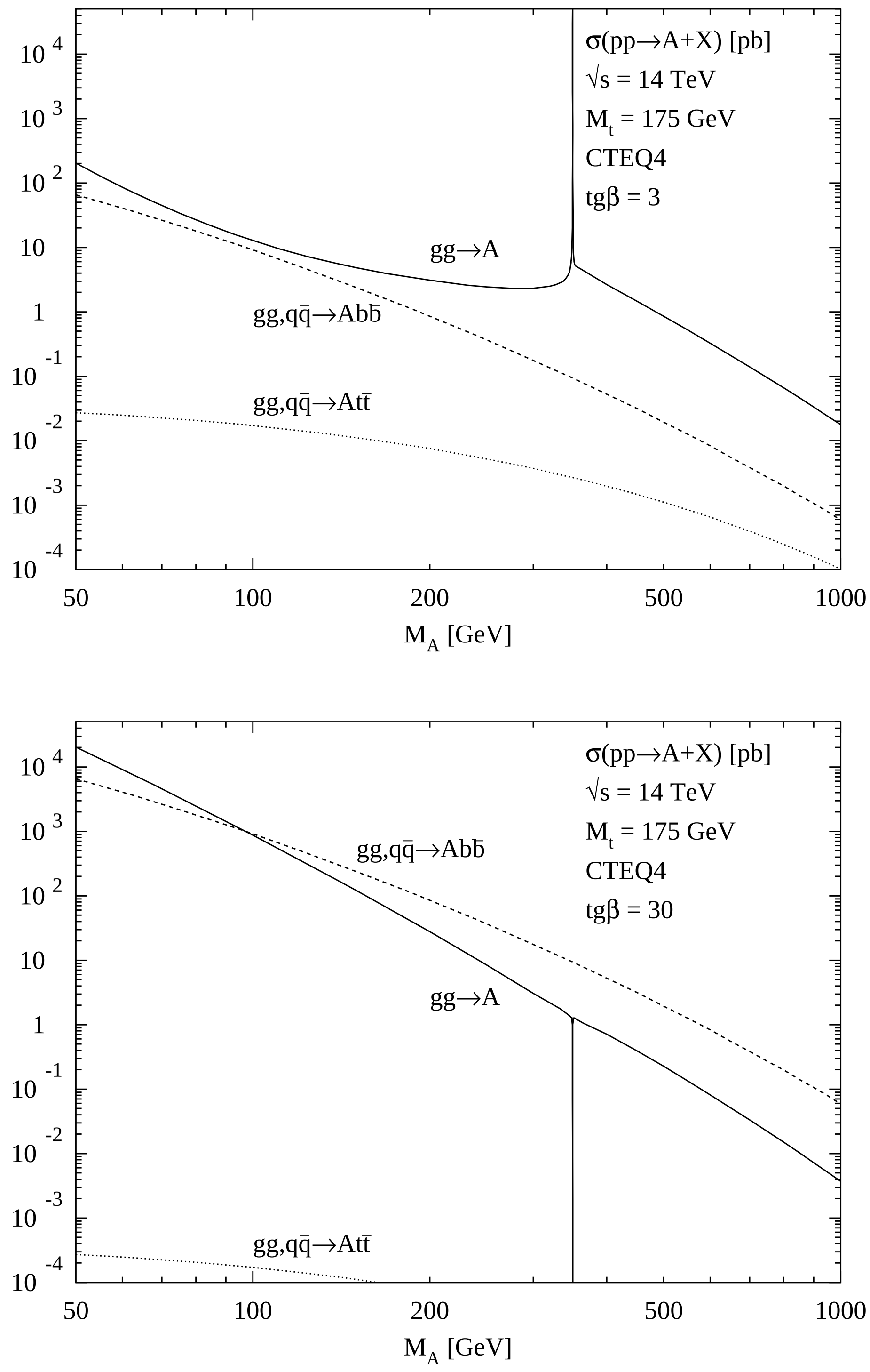
Beenakker,Dittmaier,Krämer,Plümper,Spira,Zerwas



Dittmaier,Krämer,Spira

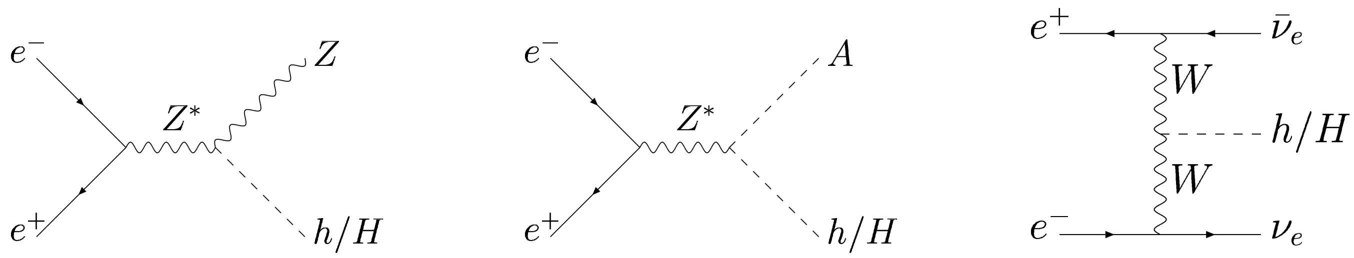

σ(pp→h/H+X) [pb]
√s = 14 TeV
M_t = 175 GeV
CTEQ4
tgβ = 3
gg→h
hbb̄
hW
hqq
hZ
htt̄
gg→H
Hqq
Hbb̄
HZ
HW
Htt̄
h
H
M_h/H [GeV]
50
100
200
500
1000
10^4
10^3
10^2
10
1
10^-1
10^-2
10^-3
10^-4

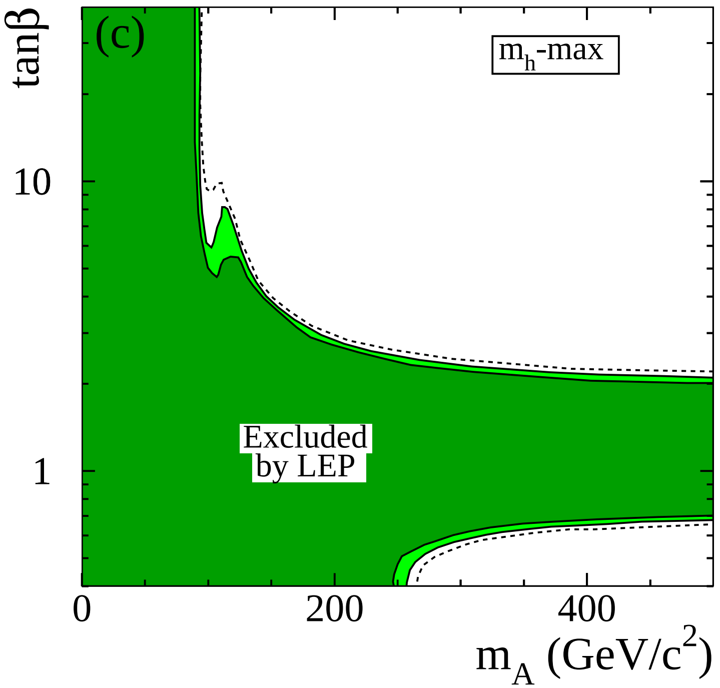

σ(pp→h/H+X) [pb]
√s = 14 TeV
M_t = 175 GeV
CTEQ4
tgβ = 30
gg→h
hbb̄
Hbb̄
gg→H
hqq
hW
hZ
htt̄
Hqq
HW
HZ
Htt̄
h
H
M_h/H [GeV]
50
100
200
500
1000
10^4
10^3
10^2
10
1
10^-1
10^-2
10^-3
10^-4


σ(pp→A+X) [pb]
√s = 14 TeV
M_t = 175 GeV
CTEQ4
tgβ = 3
gg→A
gg,qq̄→Abb̄
gg,qq̄→Att̄
M_A [GeV]
σ(pp→A+X) [pb]
√s = 14 TeV
M_t = 175 GeV
CTEQ4
tgβ = 30
gg,qq̄→Abb̄
gg→A
gg,qq̄→Att̄
M_A [GeV]

Higgs boson search: MSSM Higgs mass limits

▷ Direct Search at LEP: $e^+e^- \to Z + h/H, A + h/H, \nu_e\bar{\nu}_e + h/H$





$M_{h/H} \gtrsim 91$ GeV

$M_A \gtrsim 91.9$ GeV

$M_{H^\pm} > 78.6$ GeV

$0.5 < \tan\beta < 2.4$ excluded

(only in this scenario, $m_t = 174.3$ GeV!)

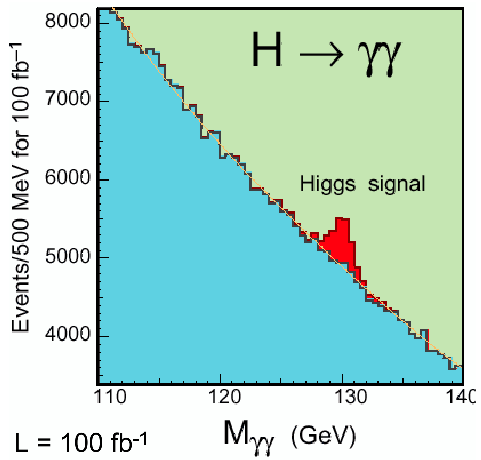
Search channels and discovery reach

Higgs Search in gg fusion

CMS

- $pp \to \gamma\gamma + X$: $M_\Phi^{SM} \lesssim$ 140 GeV

▷ bkg measurable from sideband interpolation
accurate theor. predictions ⇝ exp. acc., detector performance

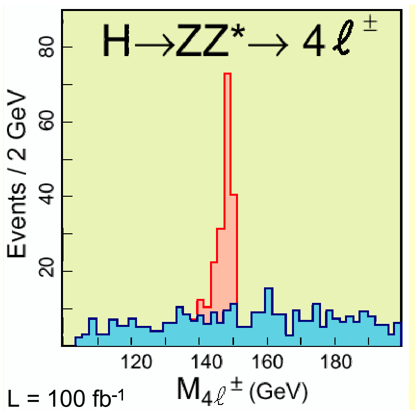


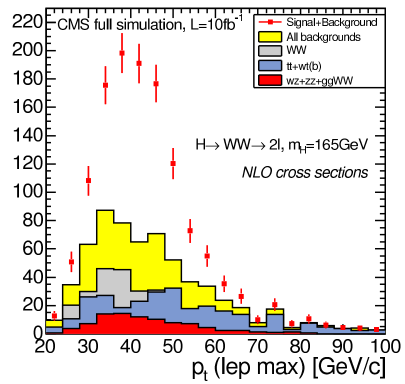
- $H \to W^+W^- \to l^+l^-\nu\bar{\nu}$: 140 GeV $\lesssim M_H \lesssim$ 180 GeV

early discovery
no bkg from sidebands

- $H \to ZZ \to 4l$: 140 GeV $\lesssim M_H \lesssim$ 180 GeV

CMS



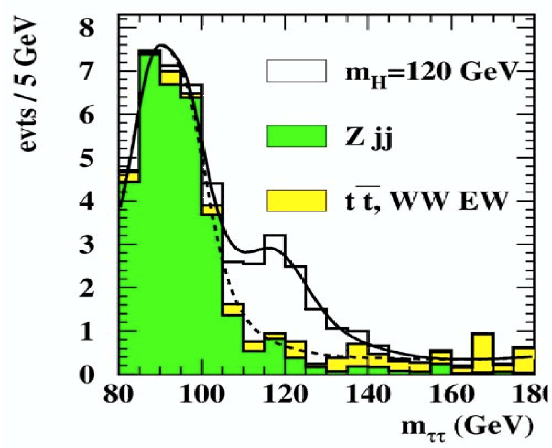


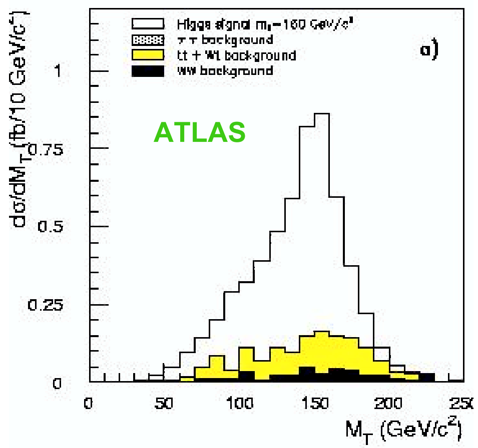
Search channels and discovery reach

Higgs Search in Vector Boson Fusion

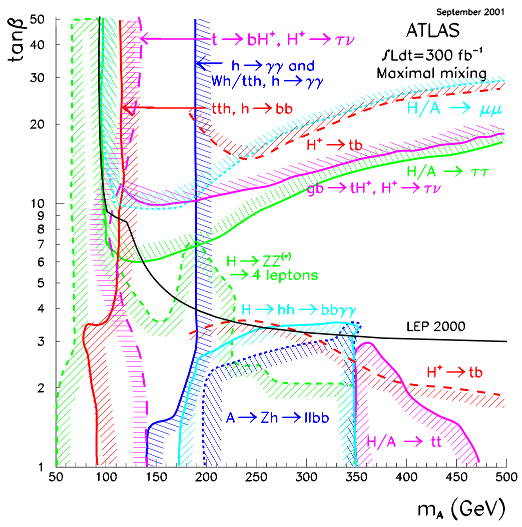
- $\Phi \longrightarrow \gamma\gamma$
- $\Phi \longrightarrow \tau^+\tau^-$: important MSSM discovery channel
- $\Phi \longrightarrow W^+W^- \longrightarrow l^+l^-\bar{\nu}\nu$: most challenging!

CMS/ATLAS

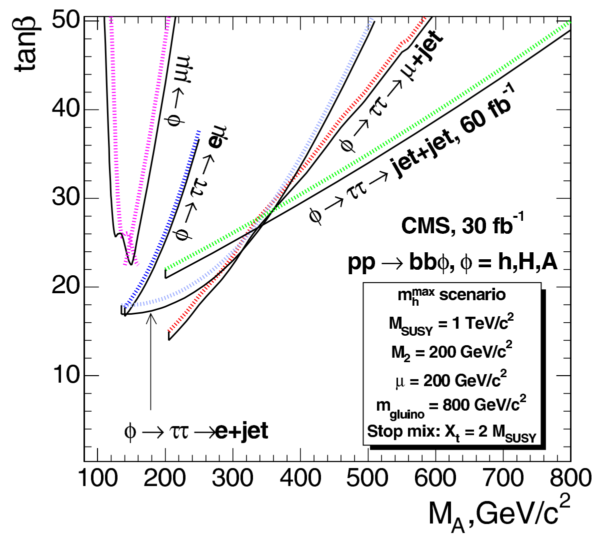




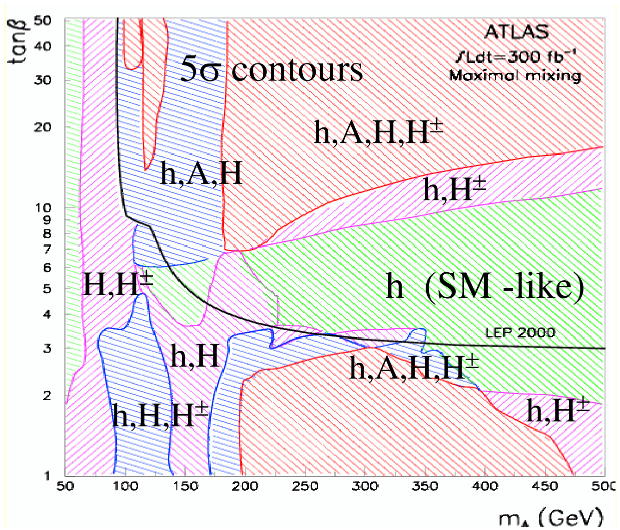
ATLAS



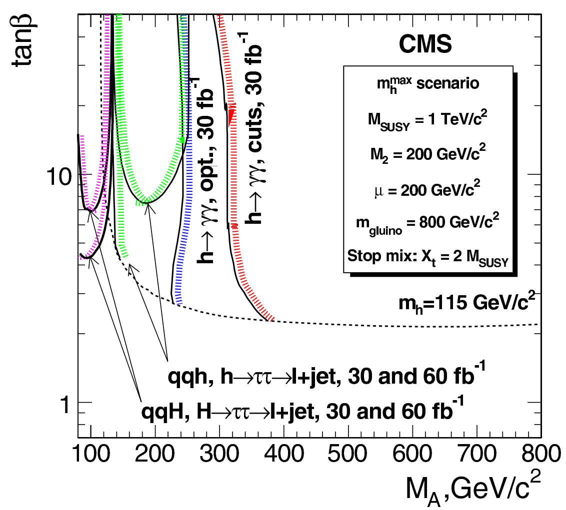
CMS



ATLAS

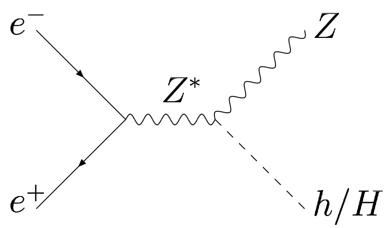


CMS



Higgs production b) e^+e^- Collider

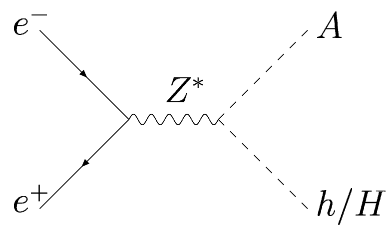
- Higgs strahlung: $e^+e^- \to Z + h/H$



$$\sigma_h \sim \sin^2(\beta - \alpha)$$
$$\sigma_H \sim \cos^2(\beta - \alpha)$$

ELW corrections $\sim -5... - 10$ %

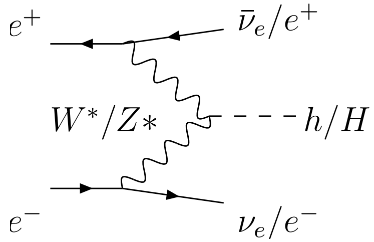
- Pair production: $e^+e^- \to A + h/H$



$$\sigma_h \sim \cos^2(\beta - \alpha)$$
$$\sigma_H \sim \sin^2(\beta - \alpha)$$

Complementarity!

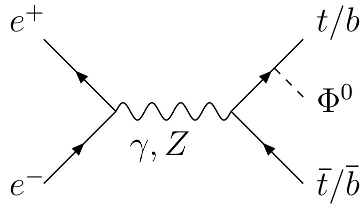
- W/Z fusion: $e^+e^- \to \nu_e\bar{\nu}_e/e^+e^- + h/H$



important at high energies

ELW corrections: $\sim -2... - 10$ %

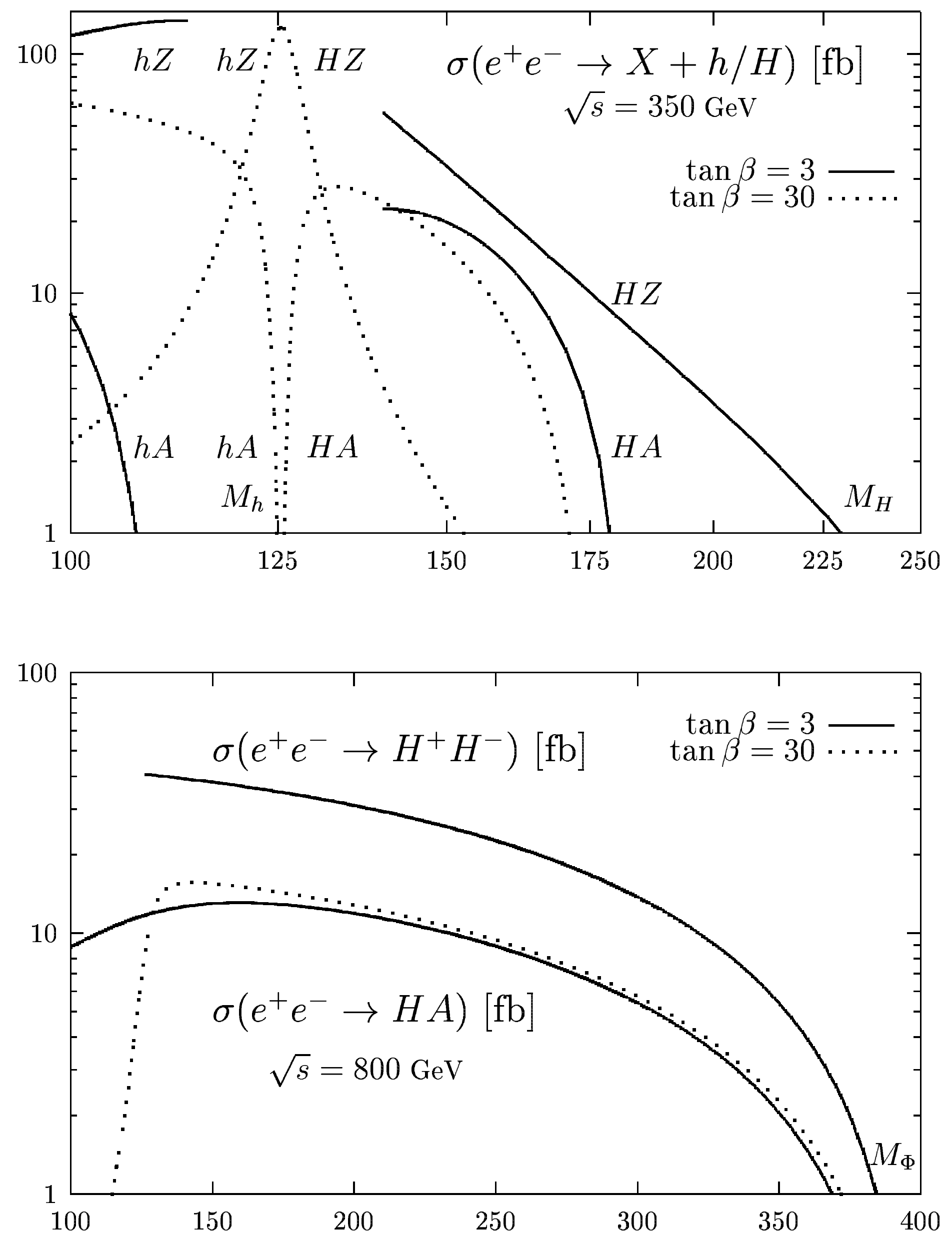
- Assoc prod w/ $q\bar{q}$: $e^+e^- \to t\bar{t}/b\bar{b} + h/H/A$



measurement of the Yukawa couplings

QCD corrections: $\sim -10... + 50$ %

ELW corrections: $\sim +10$ %



Djouadi, Kalinowski, Zerwas

* h can be discovered in the entire MSSM parameter range: Zh or Ah

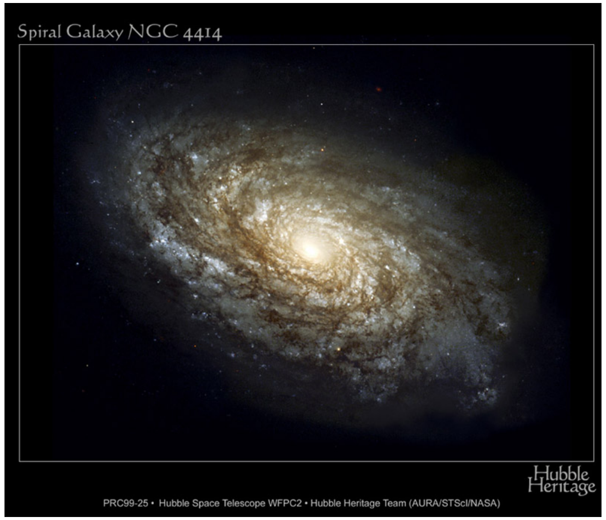
* All SUSY Higgs bosons can be discovered @ 500 GeV if $M_A, M_H, M_{H^\pm} \lesssim 230$ GeV

* If decay modes are very complicated $\rightsquigarrow$ missing mass tech. $\rightsquigarrow$ detection

(VIII) Supersymmetric Dark Matter

Why Dark Matter?

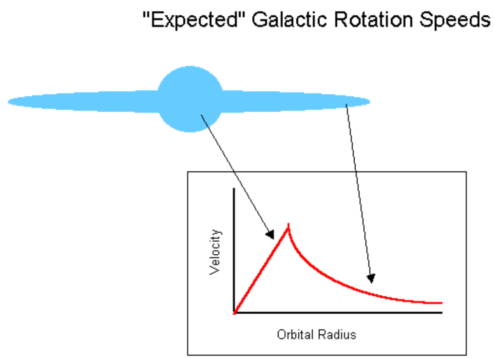
(i) Spiral galaxies:

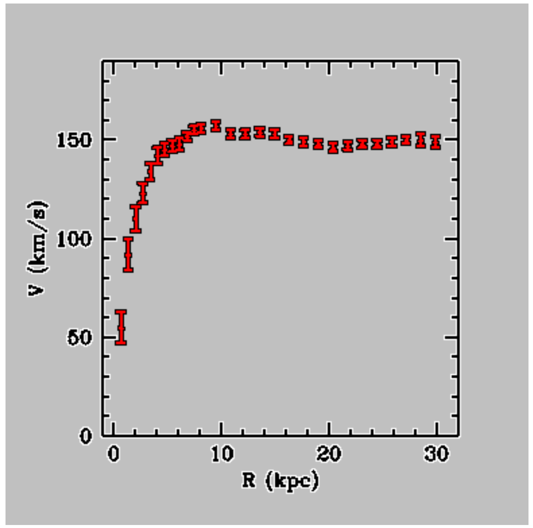


$$\frac{G_N M(r)}{r^2} \propto \frac{v^2}{r}$$

$$\Rightarrow r > r_{core} : v^2 \sim \frac{G_N M_{tot}}{r}$$

Rotation curves: 21 cm emission line of neutral hydrogen





$\Rightarrow$ $\boxed{\text{Dark Matter}}$

(ii) Cosmological background radiation (CMB):

Friedmann Equation: (homogenous universe)

$$\boxed{H^2 = \frac{\dot{R}}{R} = \frac{8\pi G_N}{3}\rho - \frac{k}{R^2} + \frac{\Lambda}{3}}$$

H : Hubble constant

ρ : matter density

k : 3-dimensional curvature

Λ : cosmological constant

$$k = \begin{cases} 0 & \text{flat metric} \\ > 0 & \text{closed universe} \\ < 0 & \text{open universe} \end{cases}$$

$k = \Lambda = 0 \Rightarrow \rho = \rho_c = \frac{3H^2}{8\pi G_N}$ critical density

Friedmann equation:

$$\begin{aligned} (\Omega - 1)H^2 &= \frac{k}{R^2} \\ \Omega &= \Omega_m + \Omega_\Lambda \\ \Omega_m &= \frac{\rho}{\rho_c} \\ \Omega_\Lambda &= \frac{\Lambda}{3H^2} \end{aligned}$$

CMB measures Ω

(iii) X-rays emitted by hot gases in elliptic galaxies:

(M = total mass, L = mass which radiates light)

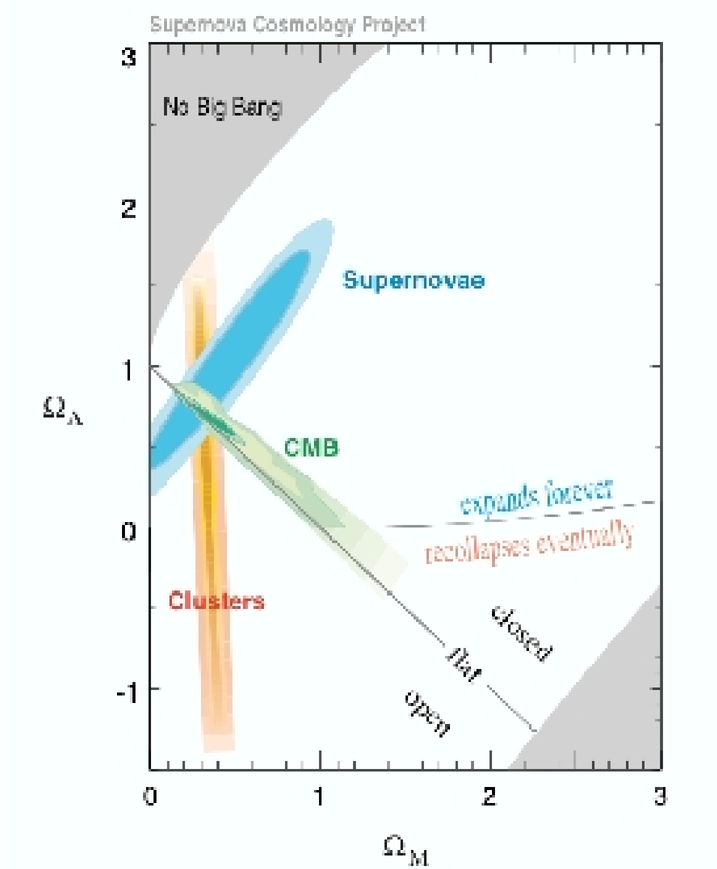
$\Omega \approx 1 \Rightarrow \quad \frac{M}{L} \sim 1400$ (in solar units)

measured: $\frac{M}{L} \lesssim 200...300 \quad \Rightarrow \Omega_m \lesssim 0.3 - 0.4$

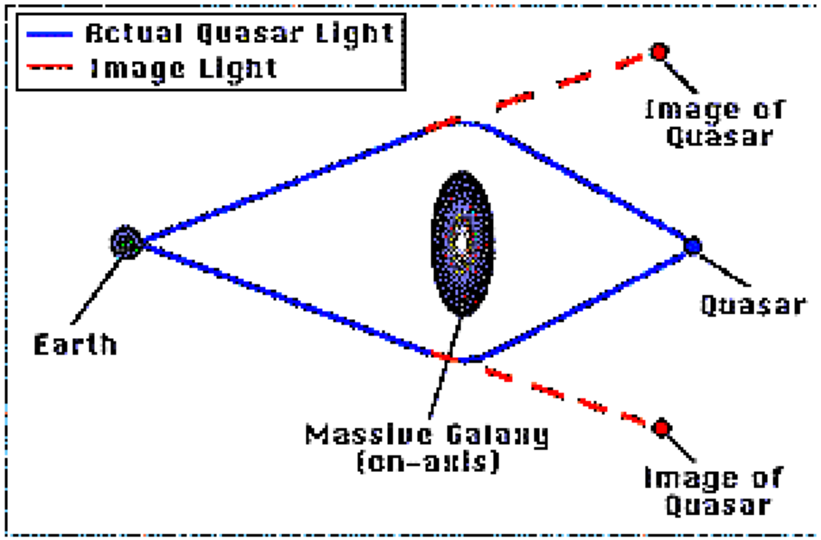
$\Rightarrow$ indication for DM (compatible with CMB)

WMAP+Supernovae+Cluster:

$$\Omega = 1.02 \pm 0.02$$
$$\Omega_\Lambda = 0.73 \pm 0.04$$
$$\Omega_B = 0.044 \pm 0.004$$
$$\Omega_{CDM} = 0.23 \pm 0.04$$



(iv) Gravitational lenses:



$\Rightarrow$ $\boxed{\text{Dark Matter}}$

Why $\Omega \approx 1$? ($\rightarrow k \approx 0$)

• Big Bang theory: adiabatic expansion $\Rightarrow R \sim T_\gamma^{-1}$

$$\Rightarrow \hat{k} = \frac{k}{R^2 T_\gamma^2} < \frac{8\pi G_N \rho}{3 T_{CMB}^2} < 2 \cdot 10^{-58} \qquad \text{constant} \qquad (T_{CMB} = 2.725\ \text{K})$$

$\rightarrow$ curvature unchanged ($\leftarrow$ contradicion)

• Solution: theory of inflation

before inflation: $R = R_i,\ T = T_i,\ R \sim T^{-1}$

inflation: $R \sim e^{Ht}$ (H =const) ($\leftarrow$ e.g. cosmological constant)

after inflation: $R = R_f \gg R_i,\ T = T_f \lesssim T_i$

$\Rightarrow \hat{k} \rightarrow 0 \Rightarrow \boxed{\Omega \approx 1}$ (prediction)

What is Dark Matter?

Hot Dark Matter (HDM): $m_\chi \lesssim 100$ eV

$\Rightarrow$ large structures form first, galaxies later

cannot reproduce our universe of today

<u>Cold Dark Matter (CDM):</u> $m_\chi \gtrsim 1$ MeV

$\Rightarrow$ small structures form first, large structures later

compatible with our universe of today

$\Rightarrow \boxed{\text{CDM}}$

Supersymmetry: R-parity converved $\Rightarrow$ LSP stable (colour charge and electrically neutral)

candidates:

	$\tilde{\nu}$	excluded in the MSSM (LEP)
	$\tilde{G}$	not excluded
	$\tilde{\chi}^0$	best candidate

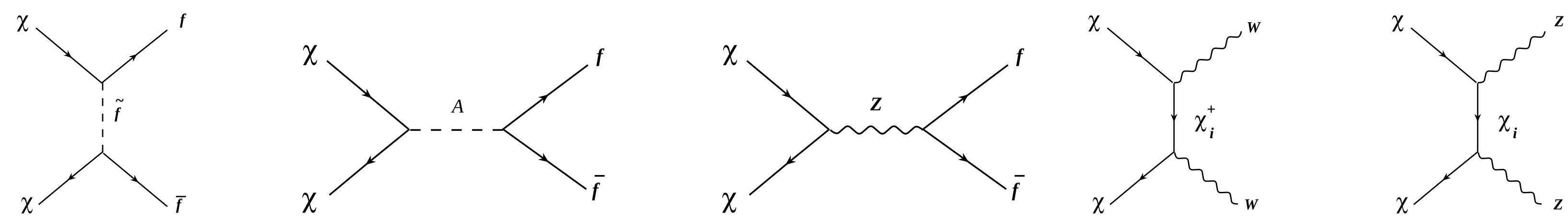
Relic density Boltzmann equation for matter in expanding universe $\leftarrow$ annihilation cross section $\sigma_{ann}(\tilde{\chi}\tilde{\chi} \to X)$

(i) <u>thermal equilibrium</u> for $T \to \infty$ $\quad \chi + \chi \leftrightarrow p_{SM}\ \bar{p}_{SM}$

(ii) <u>cooling universe</u> for decreasing T $\quad N \sim exp(-m_\chi/T)$

(iii) <u>χ's freezing out:</u> universe expanding $\to \chi$ not colliding any more

$N_{eq} \sim$ const. of size $1/ < v\sigma_{ann} >$



$A_{f\bar{f}}^{\tilde{f}} \propto (\frac{m_\chi}{m_{\tilde{f}}})^2$ $\quad A_{f\bar{f}}^{A} \propto \frac{\tan\beta m_{f_d}}{m_W}(\frac{m_\chi}{m_A})^2 N_1 N_{3(4)}$ $\quad A_{f\bar{f}}^{Z} \propto (\frac{m_f m_\chi}{m_Z^2}) N_{3(4)}^2$ $\quad A_{W^+W^-/ZZ}^{\chi_i/\chi_i^+} \propto \frac{1}{1+(m_{\chi_i^+}/m_\chi)^2-(m_W/m_\chi)^2}$

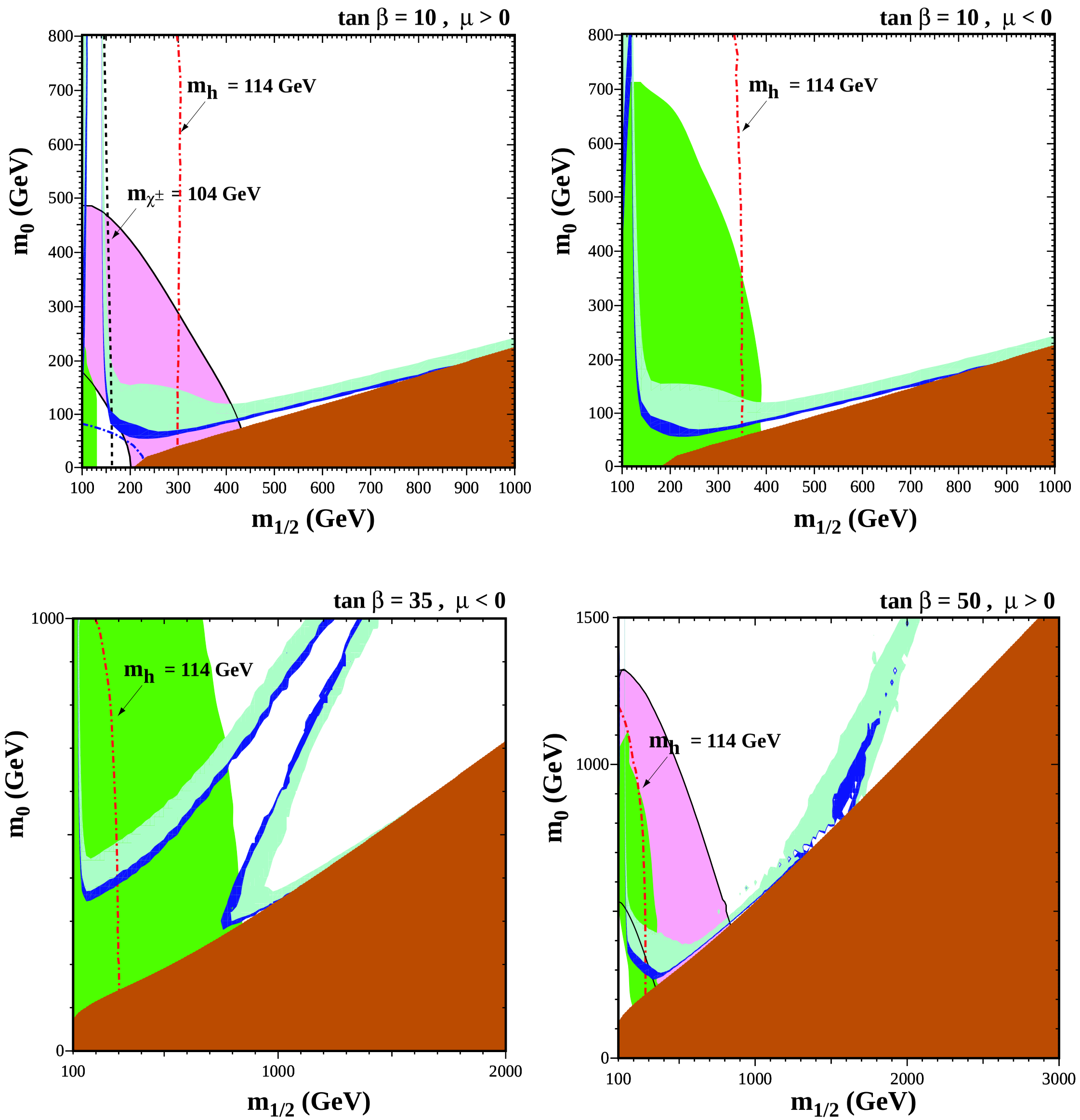
$\tan\beta m_{f_d} \leftrightarrow \frac{m_{f_u}}{\tan\beta}$ $\quad (m_{\chi_i^+}, m_W) \leftrightarrow (m_{\chi_i}, m_Z)$

$$\rho_\chi = m_\chi n_\chi \text{ with } n_\chi \sim \frac{1}{\sigma_{ann}(\tilde{\chi}\tilde{\chi}\to X)} \sim m_{\tilde{\chi}}^2$$

$\Rightarrow$ relic density increases with increasing neutralino mass $m_{\tilde{\chi}}$

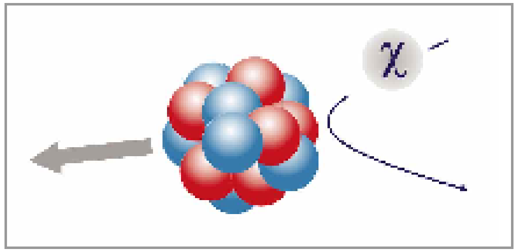
before WMAP: $0.1 \lesssim \Omega_\chi H^2 \lesssim 0.3$

after WMAP: $0.094 \lesssim \Omega_\chi H^2 \lesssim 0.129$



small $m_0, m_{1/2}$:	bulk region
large $m_{1/2}$:	coannihilation tail
large m_0 :	focus point
	(LSP=Higgsino)
large $\tan\beta$:	funnel regions
	($2m_\chi \sim M_A \Rightarrow$
	resonant $s-$channel)

Direct detection: Elastic scattering $\chi\mathcal{N} \to \chi\mathcal{N}$



effective Lagrangian

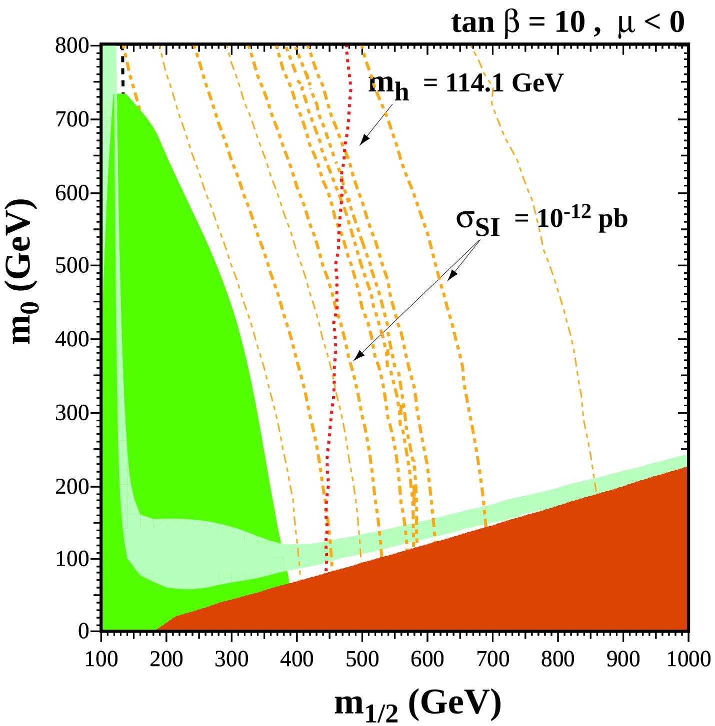
$$\begin{aligned}\mathcal{L} &= \bar{\chi}\gamma^\mu\gamma^5\chi\bar{q}_i\gamma_\mu(\alpha_{1i} + \alpha_{2i}\gamma^5)q_i \\ &+ \alpha_{3i}\bar{\chi}\chi\bar{q}_i q_i + \alpha_{4i}\bar{\chi}\gamma^5\chi\bar{q}_i\gamma^5 q_i \\ &+ \alpha_{5i}\bar{\chi}\chi\bar{q}_i\gamma^5 q_i + \alpha_{6i}\bar{\chi}\gamma^5\chi\bar{q}_i q_i\end{aligned}$$

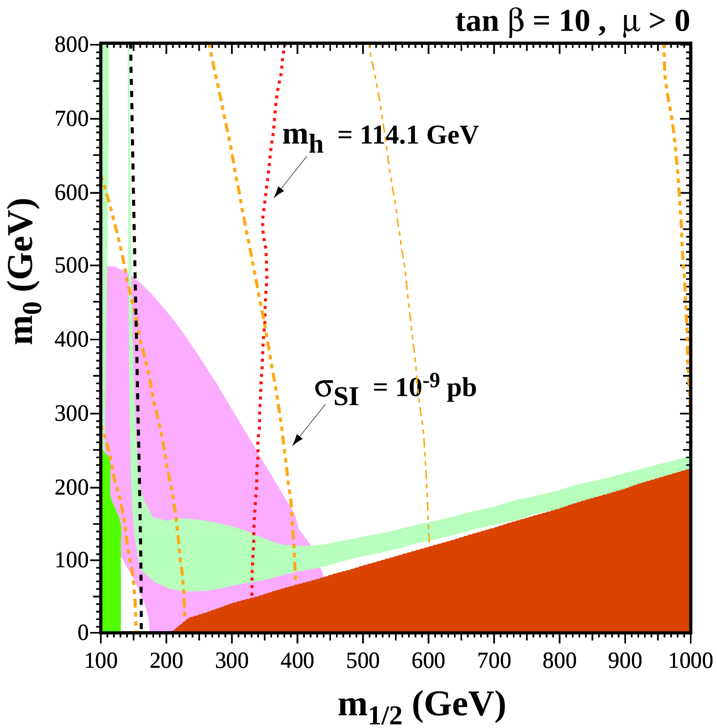
$\alpha_{1i}, \alpha_{4i}, \alpha_{5i}, \alpha_{6i} \longrightarrow$ velocity dependent elastic scattering cross section

$\alpha_{2i} \longrightarrow$ spin-dependent cross section (dominated by light $\tilde{q}$ exchange)

$\alpha_{3i} \longrightarrow$ scalar coefficient

Spin-independent cross sections constrain $m_0, m_{1/2}$:





Constraints from $b \to s\gamma$ and $g_\mu - 2$

$$\Rightarrow \quad 2\times 10^{-10}\ \text{pb} \lesssim \sigma_{SI} \lesssim 6\times 10^{-8}\ \text{pb}$$
$$2\times 10^{-7}\ \text{pb} \lesssim \sigma_{SD} \lesssim 10^{-5}\ \text{pb}$$

Global analysis: spin-independent elastic scattering seems to be better accessible

Indirect Detection: χ's are captured gravitationally by heavy objects, χ's can annihilate and thus be observed via their annihilation products

$\Rightarrow$ several annihilation processes are possible: $\chi\chi \to \gamma + X, \nu + X, e^+ + X$

(i) γ's from annihilation in galactic center:

$\chi\chi \to \gamma\gamma, Z\gamma$ loop suppressed $\Rightarrow$ continuum γ's

γ flux: $\Phi_\gamma(E_{thr}) = 5.6 \times 10^{-10}\ \text{cm}^{-2}\text{s}^{-1} \times \sum_i \int_{E_{thr}}^{m_\chi} dE\ \frac{dN_\gamma^i}{dE}\ \left(\frac{\sigma_i v}{\text{pb}}\right)\left(\frac{100\ \text{GeV}}{m_\chi}\right)^2\ \bar{J}(\Delta\Omega)\Delta\Omega$

Sum over all annihilation channels
$\Delta\Omega$: solid angle, which is covered by the γ telescope
$\bar{J}$: measure for clumpiness of the density profile of the galactic halo (huge uncertainties: $\bar{J} \sim 3...10^5$!)

(ii) Neutrinos from annihilation in sun and earth: ν's propagate and convert to μ's via charged current reactions; μ rates depend on the primary annihilation process: $\chi\chi \to b\bar{b}, t\bar{t}, W^+W^-, ZZ$

(iii) e^+'s from annihilation in galactic halo:

positron flux: $\frac{d\Phi_{e^+}}{d\Omega dE} = \frac{\rho^2}{m_\chi^2} \sum_i \sigma_i v B_{e^+}^i \int\ dE_0\ f_i(E_0)\, G(E_0, E)$

Sum over all annihilation channels
ρ : local neutralino mass density, $B_{e^+}^i$: branching ratio of channel i in e^+
$f(E_0)$: initial positron energy distribution
$G(E_0, E)$: Green function for propagation of the positrons in the galaxy

Large experimental effort to detect

Cold Dark Matter

Particle Physics $\leftrightarrow$ Astrophysics

$$\downarrow$$

Direct reaches

Indirect reaches

$$\uparrow$$

Existence proof of CDM particle

$$\uparrow$$

Discovery of SUSY at LHC