Physics beyond the Standard Model

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Exercise Sheet 10

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Exercise 13: Divergences and the Hierarchy Problem

Consider the Lagrangian of QED with a single fermion field, e.g. an electron,

$$\mathcal{L} = i\bar{\psi}\not{D}\psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

with the covariant derivative $D^\mu = \partial^\mu - ieA^\mu$ and the field strength tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$.

(a) Calculate the photon self-energy $\Pi^{\mu\nu}_{\gamma\gamma}(q)$. Show that it vanishes in the limit $q^2 \to 0$, i.e. for an on-shell photon quantum corrections do not generate a mass term.

(b) Calculate the fermion self-energy $\Pi_{ff}(q^2 = 0)$ for vanishing external momentum. (The dependence on the fermion mass in internal lines shall be kept.)
Assume that the model is valid up to the Planck scale $M_{\text{Planck}} = 10^{19}$ GeV. How large would one expect the corrections to the fermion mass to be at this scale?

Now add a scalar field Φ to the Lagrangian with mass M_H and coupling $\lambda_f = \frac{\sqrt{2}m_f}{v}$ to the fermion,

$$\mathcal{L}_{\bar{f}f\Phi} = -\lambda_f \bar{f}_L f_R \Phi + \text{ h.c.}\,,$$

which obtains a vacuum expectation value $\text{Re}\Phi = \frac{1}{\sqrt{2}}(v + H)$. The term with v of course gives the fermion mass term already introduced by hand in the original Lagrangian.

(c) Calculate the Higgs self-energy $\Pi_{HH}(q^2 = 0)$ at zero momentum transfer. What happens when going up to the Planck scale?

As last step we now add a scalar partner $\tilde{f}$ to the fermion, called sfermion. As we want to add the same number of degrees of freedom, we actually need two of them (plus their respective antiparticles marked with an asterisk), one for the left-handed and one for the right-handed fermion: $\tilde{f}_L$, $\tilde{f}_R$. The additional terms in the Lagrangian take the form

$$\mathcal{L}_{\tilde{f}^*\tilde{f}\Phi} = \left(\lambda_f A_f \Phi \tilde{f}_L \tilde{f}_R^* + \text{ h.c.}\right) + \lambda_{\tilde{f}} \Phi^\dagger \Phi \left(\tilde{f}_L^* \tilde{f}_L + \tilde{f}_R^* \tilde{f}_R\right) .$$

with new constants A_f and $\lambda_{\tilde{f}}$.

(d) Calculate the sfermion contribution to the Higgs self-energy. What are the relevant Feynman diagrams?
Postulate that all quadratic divergencies in the full Higgs self-energy (fermion and sfermion contribution) should cancel. What follows for the coupling constants?