Physics beyond the Standard Model

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Exercise Sheet 11

Discussion: Fr, 06.02.15

Exercise 14: Sfermion sector of the MSSM

In the MSSM, under the assumption of "minimal flavour violation", the following mass terms for the sfermions appear:

$$\mathcal{L}_{\tilde{f},\text{mass}} = -\tilde{f}^\dagger M^2_{\tilde{f}} \tilde{f} \quad \text{with} \quad \tilde{f} = \begin{pmatrix} \tilde{f}_L \\ \tilde{f}_R \end{pmatrix}, \qquad M^2_{\tilde{f}} = \begin{pmatrix} M^{LL}_{\tilde{f}} + m_f^2 & m_f \left(M^{LR}_{\tilde{f}}\right)^* \\ m_f M^{LR}_{\tilde{f}} & M^{RR}_{\tilde{f}} + m_f^2 \end{pmatrix} \quad \text{as well as}$$

$$M^{LL}_{\tilde{f}} = M_Z^2 \left(I_3^f - Q_f s_W^2\right) \cos 2\beta + \begin{cases} M^2_{\tilde{L}} & \text{for left-handed sleptons} \\ M^2_{\tilde{Q}} & \text{for left-handed squarks} \end{cases}$$

$$M^{RR}_{\tilde{f}} = M_Z^2 \left(Q_f s_W^2\right) \cos 2\beta + \begin{cases} M^2_{\tilde{E}} & \text{for right-handed electron-type sleptons} \\ M^2_{\tilde{U}} & \text{for right-handed up-type squarks} \\ M^2_{\tilde{D}} & \text{for right-handed down-type squarks} \end{cases}$$

$$M^{LR}_{\tilde{f}} = A_f - \mu^* \begin{cases} \frac{1}{\tan\beta} & \text{for up-type squarks} \\ \tan\beta & \text{for electron-type sleptons and down-type squarks.} \end{cases}$$

Thereby, Q_f denotes the electromagnetic charge of the sfermion and I_3^f the quantum number of the third component of the weak isospin operator.

(a) How do you obtain the mass eigenstates from there? What are the corresponding masses?

(b) What happens in the limit of
  (i) small fermion mass m_f,
  (ii) large SUSY parameters $M_{\tilde{L},\tilde{E},\tilde{Q},\tilde{D},\tilde{U}} = A_f = \mu =: M_{\text{SUSY}}$?

Exercise 15: Radiative electroweak symmetry breaking

Given are the following simplified ($A_t = 0$, $\mu = 0$) renormalization group equations for the up-like Higgs mass and the two stop masses

$$\frac{\partial}{\partial \log(Q^2/M_{\text{GUT}}^2)} \begin{pmatrix} M_{H_u}^2 \\ M_{\tilde{t}_R}^2 \\ M_{\tilde{t}_L}^2 \end{pmatrix} = \frac{\alpha_t}{\pi} \begin{pmatrix} 3 & 3 & 3 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} M_{H_u}^2 \\ M_{\tilde{t}_R}^2 \\ M_{\tilde{t}_L}^2 \end{pmatrix} ,$$

where $\alpha_t = G_F m_t^2/(\sqrt{2}\pi)$.
Solve these equations under the condition $M_{H_u} = M_{\tilde{t}_R} = M_{\tilde{t}_L} = M_0$ at $Q = M_{\text{GUT}}$. Show that the Higgs mass squared $M_{H_u}^2$ gets negative at $Q = M_Z$, if the top mass m_t is sufficiently large, whereas the stop masses squared remain positive.