Physics beyond the Standard Model

L: Prof. Dr. M. Mühlleitner, T: Dr. M. Rauch

Exercise Sheet 4

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Exercise 4: Flavour-changing Neutral Currents – An Example

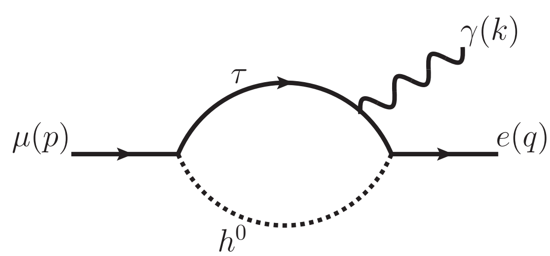
In the last exercise, we have seen how flavour-changing neutral currents (FCNC) arise in general 2HDMs. In this exercise we will calculate an example where such a vertex appears, and compare with experimental limits. The process we are going to study is

$$\mu^-(p) \to e^-(q)\gamma(k)\,.$$

(a) Let us first consider the equivalent process in the quark sector: $s(p) \to d(q)\gamma(k)$ and take as model the SM.
Draw the corresponding Feynman diagrams contributing to this process. Show then that in the limit of equal quark masses their contribution vanishes. In the case of unequal masses, what do you expect the amplitudes to be proportional to?

(b) Now consider the muon decay processes in the flavour-violating 2HDM. Which diagrams can contribute to this process?
As we have seen in the previous exercise, the intermediate fermions should be preferably heavy. Which of these diagrams would you therefore expect to give the largest contribution?

(c) As next step we want to estimate the size of the following diagram:



Take the coupling constant of both h^0ff vertices to be g_Y. A rough order of magnitude estimate of the size of loop integrals can be obtained by the fact that loop integrals are typically proportional to the largest energy scale appearing inside the loop integral, i.e.

$$\int \frac{\mathrm{d}^D\ell}{(2\pi)^D} \frac{1}{(\ell^2 - M_{h^0}^2)(\ell^2 - m_\tau^2)(\ell^2 - m_\mu^2)} \to \frac{1}{M_{h^0}^2},$$
$$\int \frac{\mathrm{d}^D\ell}{(2\pi)^D} \frac{\not{\ell}}{(\ell^2 - M_{h^0}^2)(\ell^2 - m_\tau^2)(\ell^2 - m_\mu^2)} \to \frac{1}{M_{h^0}},$$
$$\text{etc.}.$$

You may also assume $M_{h^0} \gg m_\tau \gg m_\mu \gg m_e$.
With these simplifications estimate the partial width $\Gamma(\mu \to e\gamma)$.
Solution: $\Gamma(\mu \to e\gamma) = \alpha g_Y^4 m_\mu$

(d) Now assume that the flavour-violating coupling constant $g_Y = m_\tau / M_h^0$. Given the current upper limit $\mathrm{BR}(\mu \to e\gamma) < 5.7 \cdot 10^{-13}$ ("Mu to E Gamma" experiment 2013 at PSI, Switzerland) and the total decay width of the muon $\Gamma_\mu^{\text{tot}} \simeq 3 \cdot 10^{-16}$ MeV, derive a lower bound on the Higgs mass.

Exercise 5: Yukawa Coupling Coefficients

In exercise 3 we have seen that in the Yukawa-aligned 2HDM, where $\Pi_{f,2} = \xi_f \Pi_{f,1}$, the Yukawa couplings of fermions to the physical Higgs bosons get diagonalized simultaneously when the fermions are rotated into their mass eigenstates. Thereby $f = u, d, \ell$ denotes the up-type and down-type quark sector and the charged leptons, respectively, and the ξ_f are allowed to be different from each other. Use our previous result of the Yukawa-aligned 2HDM,

$$\mathcal{L}_{\text{Yuk}} = -\bar{f}\,\frac{M_f}{v}\left[\frac{-\sin\alpha + \xi_f\cos\alpha}{\cos\beta + \xi_f\sin\beta}\,h^0 + \frac{\cos\alpha + \xi_f\sin\alpha}{\cos\beta + \xi_f\sin\beta}\,H^0 - i\gamma_5\,\frac{s_f\,(-\sin\beta + \xi_f\cos\beta)}{\cos\beta + \xi_f\sin\beta}\,A^0\right] f$$

with $s_{d,\ell} = -1$ and $s_u = +1$, to obtain the coupling factors relative to the SM(-like) expression, which are given by the fractions in the expression above. Show that, with suitable redefinitions, they can be written as

	h^0	H^0	A^0
u	$\frac{\cos\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\sin\beta}$	$\frac{1}{\tan\beta}$
d	$-\frac{\sin(\alpha-\gamma_b)}{\cos(\beta-\gamma_b)}$	$\frac{\cos(\alpha-\gamma_b)}{\cos(\beta-\gamma_b)}$	$\tan(\beta-\gamma_b)$
ℓ	$-\frac{\sin(\alpha-\gamma_\tau)}{\cos(\beta-\gamma_\tau)}$	$\frac{\cos(\alpha-\gamma_\tau)}{\cos(\beta-\gamma_\tau)}$	$\tan(\beta-\gamma_\tau)$

What are the values for γ_b and γ_τ to reproduce the four discrete models, type-I, type-II, lepton-specific and flipped 2HDM?