Physics beyond the Standard Model

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Exercise Sheet 5

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Exercise 6: Exotic Higgs Multiplets

Higgs seven-plets (septets) of $SU(2)_L$ play a special role, as by coincidence these also allow to obtain the correct mass ratio for the W and Z bosons.

(a) Which hypercharge assignment is needed for the seven-plet to get the correct mass ratio? Which component(s) should therefore receive a vev?

(b) Work out the gauge boson mass-squared matrix in the W^i, B basis and show that it is indeed proportional to the SM case

$$M^2_{\rm SM} = \frac{v^2_{\rm SM}}{4}\begin{pmatrix} g^2 & 0 & 0 & 0 \\ 0 & g^2 & 0 & 0 \\ 0 & 0 & g^2 & -gg' \\ 0 & 0 & -gg' & g'^2 \end{pmatrix} .$$

The generators T^i, $i = 1, 2, 3$ are now 7×7 matrices, whose exact form we do not need to specify.
Use the relation $T^+T^- + T^-T^+ = 2\,[T(T+1) - (T^3)^2]$ (why does this hold?) to simplify the expression.
Intermediate result: Proportionality factors are in general:
$(W^{1,2})^2$*:* $T(T+1) - (T^3)^2$, $(W^3)^2$*:* $2(T^3)^2$, B^2*:* $Y^2/2$, W^3B*:* $-T^3Y$

(c) If the W and Z masses were generated entirely by the seven-plet, what value of the vev v would be needed?
($M_W = gv_{SM}/2$*,* $M_Z = \sqrt{g^2 + g'^2}v_{SM}/2$ *with* $v_{SM} \simeq 246$ *GeV)*
Could all mass generation in the SM be accomplished by the seven-plet?

Exercise 7: Higgs triplets and the Georgi-Machacek model

One can repeat the previous exercise with Higgs triplets. Two choices are possible: real scalar Ξ with $Y = 0$ and complex scalar X with $Y = 2$.

(a) The current experimental bound on the oblique parameter T is $T = 0.09 \pm 0.13$, and is related to the ρ parameter via $\rho = 1 + \alpha T$. Assume that you have the SM Higgs doublet plus a triplet. For each of the two triplet possibilities mentioned above, what is the respective limit on the triplet vev?
What changes if there are two Higgs doublets plus a triplet?
Using both triplets, how can one preserve custodial symmetry?
If you want to use the formula for the ρ parameter given in the lecture, note that an additional factor $1/2$ appears in the numerator if the field is real.

(b) One can construct a bi-doublet from the SM Φ and $\tilde{\Phi}$ as a 2×2 matrix $\mathcal{H} = \left(\tilde{\Phi}, \Phi\right)$. Rewrite the SM potential using $\mathcal{H}$ only.

(c) Analoguously, one can define a 3×3 matrix

$$\bar{X} = \begin{pmatrix} \chi^{0*} & \xi^{+} & \chi^{++} \\ -\chi^{+*} & \xi^{0} & \chi^{+} \\ -\chi^{++*} & -\xi^{+*} & \chi^{0} \end{pmatrix} ,$$

where it is now more natural that $\langle \bar{X} \rangle = v_X \cdot \mathbb{1}_3$.
Can one deduce anything about v_X?
How would you look for such an extension? Discuss some physical consequences of this model.