Physics beyond the Standard Model

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Exercise Sheet 7

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Exercise 9: Renormalization Group

Higher-order corrections induce a dependence on the renormalization scale, when ultraviolet divergences appear and the poles are absorbed into a minimal counter term. This can be used to study the running of the (masses and) coupling constants between different scales.
In this exercise, we will consider only a toy model. The procedure is completely equivalent in the 2HDM, but much more cumbersome due to the longer expressions.
Consider the pseudoscalar massless Yukawa theory

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu\varphi\right)^2 - \frac{\lambda}{4!}\varphi^4 + \bar{\psi} i \partial\!\!\!/ \psi - ig\bar{\psi}\gamma^5\psi\varphi$$

where φ is a real scalar field and ψ is a Dirac fermion.

(a) Calculate the divergent parts, as prefactors proportional to the corresponding Born expression, of
  (i) fermion self energy δ_ψ,
  (ii) scalar self energy δ_φ,
  (iii) fermion-fermion-scalar vertex δ_g and
  (iv) four-scalar vertex at one loop δ_λ.

(b) From the results of the previous exercise, compute the Callan-Symanzik β functions for λ and g, $\beta_\lambda(\lambda, g)$ and $\beta_g(\lambda, g)$, assuming that λ and g are of the same order. Use the relation between bare and renormalized coupling, $g_0 = g(1 + \delta_g - 2\delta_\psi/2 - \delta_\varphi/2)$ and analoguously for λ. Take into account that together with the $1/\epsilon$ terms logarithms $\ln\frac{\mu^2}{Q^2}$ appear, i.e. $1/\epsilon \to 1/\epsilon + \ln\frac{\mu^2}{Q^2}$.

Consider now the one-loop renormalization group equation (RGE) of the SM Higgs field as given in the lecture

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\ln Q} = \beta = \frac{1}{16\pi^2}\left(24\lambda^2 - (3g'^2 + 9g^2 - 12y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2g^2 + \frac{9}{8}g^4 - 6y_t^2\right)\,.$$

(c) We will first look at the region where λ becomes large. Therefore, ignore any subleading terms in the RGE. Solve the RGE with the boundary condition $\lambda(Q = v) = \lambda_0 = \frac{M_H^2}{2v^2}$. Where is the position of the (Landau) pole? Assuming that the theory should be valid up to the Planck scale, 10^{19} GeV, what is the limit on the Higgs mass?

(d) The next topic to investigate is the stability of the Higgs potential. For this we go back to the full RGE. The terms in the RGE with negative sign can cause λ to become negative, which would mean that the potential is no longer bounded from below. Calculate the limit on the Higgs mass to avoid this fate for $Q = 10^3$ GeV and 10^{16} GeV.

Note that we are in the small-λ regime. Simplify the RGE accordingly.
($y_t = \sqrt{2}\frac{m_t}{v}$, $e = g' \cos\vartheta_W = g \sin\vartheta_W$).