Physics beyond the Standard Model

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Exercise Sheet 9

Discussion: Fr, 16.01.15

Exercise 11: Coupling Unification

The one-loop equations of the running coupling constants are given by

$$\frac{1}{\alpha_j(Q^2)} = \frac{1}{\alpha_j(\mu^2)} + \frac{b_j}{4\pi} \ln\left(\frac{Q^2}{\mu^2}\right) \ .$$

In the SM one obtains for the b_j

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{22}{3} \\ 11 \end{pmatrix} - N_F \begin{pmatrix} \frac{4}{3} \\ \frac{4}{3} \\ \frac{4}{3} \end{pmatrix} - N_H \begin{pmatrix} \frac{1}{10} \\ \frac{1}{6} \\ 0 \end{pmatrix} \ ,$$

and in the Minimal Supersymmetric Standard Model (MSSM)

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 6 \\ 9 \end{pmatrix} - N_F \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} - N_H \begin{pmatrix} \frac{3}{10} \\ \frac{1}{2} \\ 0 \end{pmatrix} \ ,$$

where $N_F = 3$ denotes the number of particle families and N_H the number of Higgs doublets ($N_H = 2$ for the MSSM).

(a) Calculate the GUT unification scale M_{GUT}, where $\alpha_1(M_{\text{GUT}}^2) = \alpha_2(M_{\text{GUT}}^2) = \alpha_3(M_{\text{GUT}}^2)$, for both SM and MSSM from the equations given above by comparing α and α_s. Start at an energy scale equal to the Z boson mass, with $g_1 = \cot \vartheta_W^{\text{GUT}} g'$ and $g_2 = g$.

(b) Determine the Weinberg angle at the Z boson mass as function of $\alpha(m_Z)$ and $\alpha_s(m_Z)$. How does this compare with the measured value in both models?

Input parameters:

$$\sin^2 \vartheta_W(M_{\text{GUT}}) = \frac{3}{8} \qquad \alpha(m_Z) = \frac{1}{127.9}$$

$$m_Z = 91.188 \text{ GeV} \qquad \alpha_s(m_Z) = 0.119 \pm 0.004$$

Exercise 12: Kinematic Edges

The decay of SUSY particles typically happens via cascades. First, a heavy SUSY particle is produced, e.g. a gluino. This then decays in a series of $1 \to 2$ decays into a SUSY and a SM particle, until finally the lightest supersymmetric particle (LSP) remains, which is stable. While the LSP manifests itself only as missing transverse momentum, the emerging SM particles can be measured in the detector. Invariant-mass distributions of combinations of those partly show sharp edges. Their position allows to reconstruct mass differences of the SUSY particles and hence to measure the mass spectrum.
The left-handed squark $\tilde{q}_L$ shall decay via the following cascade

$$\tilde{q}_L \to q_L \tilde{\chi}_2^0 \to q_L \ell_R^- \tilde{\ell}_R^+ \to q_L \ell^- \ell^+ \tilde{\chi}_1^0$$

with the second-lightest neutralino $\tilde{\chi}_2^0$ and a right-handed selectron $\tilde{\ell}_R^+$ as intermediate SUSY particles and the lightest neutralino $\tilde{\chi}_1^0$ as LSP.
Consider the invariant-mass spectrum of the lepton pair. What is the position of the edge? What mass difference does it correspond to?
For the distribution only kinematic information is used; matrix elements or spin correlations are not considered. The masses of the SM particles can be neglected. It may be helpful to perform the calculation in the rest frame of the selectron.