Beyond the Standard Model Physics

Winter Semester 2014/15

Karlsruhe Institute of Technology (KIT)

lecture given by

Prof. Dr. M. M. Mühlleitner

Contents

Preliminary Content

This lecture will treat the Higgs sectors of the extensions beyond the Standard Model.

1. Revision of the Standard Model (SM) Higgs Sector
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4. The Next-to-Minimal Supersymmetric Extension of the SM (NMSSM)
5. Composite Higgs Model

Chapter 1

The Standard Model Higgs Sector

<u>*Literature*:</u>

1. A lot of material for this chapter can be found in my lectures TTP1 SS13, TTP2 WS 11/12 and TTP2 WS13/14.

2. Recent physics results are presented at the webpages of the LHC experiments ATLAS and CMS.

3. A. Djouadi, "The Anatomy of electro-weak symmetry breaking. I: The Higgs boson in the standard model," Phys. Rept. **457** (2008) 1 [hep-ph/0503172].

4. M. Spira, "QCD effects in Higgs physics," Fortsch. Phys. **46** (1998) 203 [hep-ph/9705337].

5. S. Dittmaier *et al.* [LHC Higgs Cross Section Working Group Collaboration], "Handbook of LHC Higgs Cross Sections: 1. Inclusive Observables," arXiv:1101.0593 [hep-ph].

6. S. Dittmaier, S. Dittmaier, C. Mariotti, G. Passarino, R. Tanaka, S. Alekhin, J. Alwall and E. A. Bagnaschi *et al.*, "Handbook of LHC Higgs Cross Sections: 2. Differential Distributions," arXiv:1201.3084 [hep-ph].

7. S. Heinemeyer *et al.* [LHC Higgs Cross Section Working Group Collaboration], "Handbook of LHC Higgs Cross Sections: 3. Higgs Properties," arXiv:1307.1347 [hep-ph].

8. H. E. Logan, "TASI 2013 lectures on Higgs physics within and beyond the Standard Model," arXiv:1406.1786 [hep-ph].

1.1 The Introduction of the Higgs Boson

There are two reasons for the introduction of the Higgs boson [1, 2] in the Standard Model (SM) of particle physics:

1. A theory of massive gauge bosons and fermions, which is weakly interacting up to very high energies, requires for unitarity reasons the existence of a Higgs particle. The Higgs particle is a scalar 0^+ particle, *i.e.* a spin 0 particle with positive parity, which couples to the other particles with a coupling strength proportional to the mass (squared) of the particles.

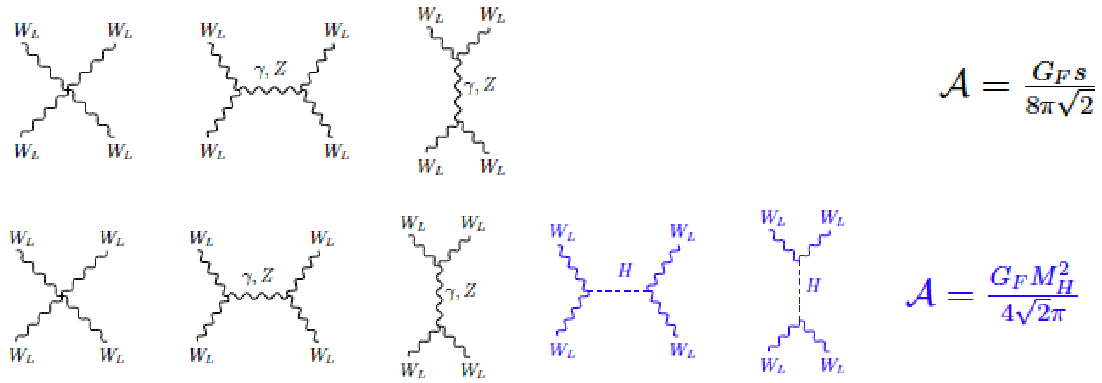


Figure 1.1: The scattering of longitudinal gauge bosons in longitudinal gauge bosons. Upper: without a Higgs boson. Lower: with a Higgs boson

Look *e.g.* at the amplitude for the scattering of longitudinal gauge bosons W_L into a pair of longitudinal gauge bosons W_L, see Fig. 1.1. Without a Higgs boson the amplitude diverges proportional to the center-of-mass (c.m) energy squared, s, *cf.* Fig. 1.1 (upper), where G_F denotes the Fermi constant. The introduction of a Higgs boson which couples proportional to the mass squared of the gauge boson, regularizes the amplitude, *cf.* Fig. 1.1 (lower), where M_H denotes the Higgs boson mass.

2. The introduction of mass terms for the gauge bosons violates the $SU(2)_L \times U(1)$ symmetry of the SM Lagrangian. The same problem arises for the introduction of mass terms for the fermions.

Let us have a closer look at point 2. We look at the Lagrangian

$$\mathcal{L}_f = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi \ . \tag{1.1}$$

In the chiral respresentation the 4×4 γ matrices are given by

$$\gamma^\mu = \left(\begin{pmatrix} \mathbf{0} & \mathbb{1} \\ \mathbb{1} & \mathbf{0} \end{pmatrix}, \begin{pmatrix} \mathbf{0} & -\vec{\sigma} \\ \vec{\sigma} & \mathbf{0} \end{pmatrix} \right) = \begin{pmatrix} 0 & \sigma_-^\mu \\ \sigma_+^\mu & 0 \end{pmatrix} \tag{1.2}$$

$$\gamma^5 = \begin{pmatrix} \mathbb{1} & \mathbf{0} \\ \mathbf{0} & -\mathbb{1} \end{pmatrix} , \tag{1.3}$$

where σ_i $(i = 1, 2, 3)$ are the Pauli matrices. With

$$\Psi = \begin{pmatrix} \chi \\ \varphi \end{pmatrix} \quad \text{und} \quad \bar{\Psi} = \Psi^\dagger \gamma^0 = (\chi^\dagger, \varphi^\dagger) \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} = (\varphi^\dagger, \chi^\dagger) \tag{1.4}$$

we get

$$\bar{\Psi} i\gamma^\mu D_\mu \Psi = i(\varphi^\dagger, \chi^\dagger) \underbrace{\begin{pmatrix} 0 & \sigma_-^\mu \\ \sigma_+^\mu & 0 \end{pmatrix} \begin{pmatrix} D_\mu \chi \\ D_\mu \varphi \end{pmatrix}}_{\begin{pmatrix} \sigma_-^\mu D_\mu \varphi \\ \sigma_+^\mu D_\mu \chi \end{pmatrix}} = \varphi^\dagger i\sigma_-^\mu D_\mu \varphi + \chi^\dagger i\sigma_+^\mu D_\mu \chi \ . \tag{1.5}$$

The gauge interaction holds independently for

$$\Psi_L = \begin{pmatrix} 0 \\ \varphi \end{pmatrix} = \frac{1}{2}(\mathbb{1} - \gamma_5)\Psi \quad \text{and} \quad \Psi_R = \begin{pmatrix} \chi \\ 0 \end{pmatrix} = \frac{1}{2}(\mathbb{1} + \gamma_5)\Psi \ . \tag{1.6}$$

The Ψ_L and Ψ_R can transform differently under gauge transformations,

$$\Psi'_L = U_L \Psi_L \quad \text{and} \quad \Psi'_R = U_R \Psi_R \ . \tag{1.7}$$

But

$$m\bar{\Psi}\Psi = m(\varphi^\dagger, \chi^\dagger) \begin{pmatrix} \chi \\ \varphi \end{pmatrix} = m(\varphi^\dagger \chi + \chi^\dagger \varphi) = m(\bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L) \ . \tag{1.8}$$

The mass term mixes Ψ_L and Ψ_R. From this follows *symmetry breaking* if Ψ_L and Ψ_R transform differently.

What about the mass term for gauge bosons? We have the Lagrangian

$$\mathcal{L} = -\frac{1}{4} \underbrace{F^{a\mu\nu} F^a_{\mu\nu}}_{\text{gauge invariant}} + \frac{m^2}{2} \underbrace{A^{a\mu} A^a_\mu}_{\text{not gauge invariant}} \ . \tag{1.9}$$

For example for the $U(1)$ we get

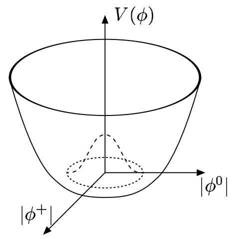
$$(A_\mu A^\mu)' = (A_\mu + \partial_\mu \theta)(A^\mu + \partial^\mu \theta) = A_\mu A^\mu + 2A_\mu \partial^\mu \theta + (\partial_\mu \theta)(\partial^\mu \theta) \ . \tag{1.10}$$

The mass term A^μ breaks the gauge symmetry.

1.2 The Standard Model Higgs sector

The problem of mass generation without violating gauge symmetries can be solved by introducing an $SU(2)_L$ Higgs doublet with weak isospin $I = 1/2$ and hypercharge $Y = 1$ and the SM Higgs potential given by

$$V(\Phi) = \lambda[\Phi^\dagger \Phi - \frac{v^2}{2}]^2 \ . \tag{1.11}$$



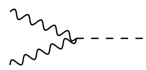
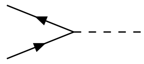
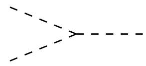
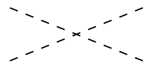
Introducing the Higgs field in a physical gauge,

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix} , \tag{1.12}$$

the Higgs potential can be written as

$$V(H) = \frac{1}{2} M_H^2 H^2 + \frac{M_H^2}{2v} H^3 + \frac{M_H^2}{8v^2} H^4 \ . \tag{1.13}$$

Here we can read off directly the mass of the Higgs boson and the Higgs trilinear and quartic self-interactions. Adding the couplings to gauge bosons and fermions we have

$\mathcal{M}$ass of the Higgs boson	$M_H = \sqrt{2\lambda}v$	
$\mathcal{C}$ouplings to gauge bosons	$g_{VVH} = \frac{2M_V^2}{v}$	
$\mathcal{Y}$ukawa couplings	$g_{ffH} = \frac{m_f}{v}$	
$\mathcal{T}$rilinear coupling [units $\lambda_0 = 33.8$ GeV]	$\lambda_{HHH} = 3\frac{M_H^2}{M_Z^2}$	
$\mathcal{Q}$uartic coupling [units λ_0^2]	$\lambda_{HHHH} = 3\frac{M_H^2}{M_Z^4}$	

In the SM the trilinear and quartic Higgs couplings are uniquely determined by the mass of the Higgs boson.

The Higgs potential with its typical form leads to a non-vanishing vacuum expectation value (VEV) v in the ground state

$$v = \frac{1}{\sqrt{\sqrt{2}G_F}} \approx 246 \text{ GeV}. \tag{1.14}$$

Expansion of Φ around the minimum of the Higgs potential leads to one massive scalar particle, the Higgs boson, and three massless Goldstone bosons, that are absorbed to give masses to the charged W bosons and the Z boson. (For a toy example, see Appendix 5.1.) The appearance of Goldstone bosons is stated in the Goldstone theorem, which says:

Be

N = dimension of the algebra of the symmetry group of the complete Lagrangian.
M = dimension of the algebra of the group, under which the vacuum is invariant after spontaneous symmetry breaking.

$\Rightarrow$ There are <u>N-M Goldstone bosons without mass</u> in the theory.

The Goldstone theorem states, that for each spontaneously broken degree of freedom of the symmetry there is one massless Goldstone boson.

In gauge theories, however, the conditions for the Goldstone theorem are not fulfilled: Massless scalar degrees of freedom are absorbed by the gauge bosons to give them mass. The Goldstone phenomenon leads to the Higgs phenomenon.

1.3 Verification of the Higgs mechanism

On the 4th July 2012, the LHC experiments ATLAS and CMS announced the disovery of a new scalar particle with mass $M_H \approx 125$ GeV. The discovery triggered immediately the investigation of the properties of this particle in order to test if it is indeed the Higgs particle, that has been disovered. In order to verify experimentally the Higgs mechanism as the mechanism which allows to generate particle masses without violating gauge principles, we have to perform several steps:

1.) First of all the Higgs particle has to be discovered.

2.) In the next step its couplings to gauge bosons and fermions are measured. If the Higgs mechanism acts in nature these couplings are proportional to the masses (squared) of the respective particles.

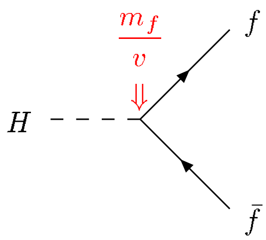
3.) Its spin and parity quantum numbers have to be determined.

4.) And finally, the Higgs trilinear and quartic self-couplings must be measured. This way, the Higgs potential can be reconstructed which, with its typical minimax form, is responsible for the non-vanishing vacuum expectation value, that is essential for the non-zero particle masses.

In the following, we will see how this program can be performed at the hadron collider LHC.

1.4 Higgs boson decays

In order to search for the Higgs boson at existing and future colliders, one has to know what to look for. Hence, one has to study the Higgs decay channels. Since the Higgs boson couples proportional to the mass of the particle its preferred decays will be those into heavy particles, *i.e.* heavy fermions and, when kinematically allowed, into gauge bosons. The branching ratios into fermions are



$$\begin{array}{lcl} BR(H\to b\bar{b}) & \lesssim & 85\% \\ BR(H\to \tau^+\tau^-) & \lesssim & 8\% \\ BR(H\to c\bar{c}) & \lesssim & 4\% \\ BR(H\to t\bar{t}) & \lesssim & 20\% \end{array} . \tag{1.15}$$

They are obtained from the partial width $\Gamma(H\to f\bar{f})$ into fermions and the total width $\Gamma_{\rm tot}$, which is given by the sum of all partial decay widths of the Higgs boson,

$$BR(H\to f\bar{f}) = \frac{\Gamma(H\to f\bar{f})}{\Gamma_{\rm tot}} . \tag{1.16}$$

The tree-level partical decay width into fermions is given by

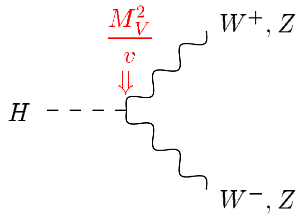
$$\Gamma(H\to f\bar{f}) = \frac{N_{cf} G_F M_H}{4\sqrt{2}\pi} m_f^2 \beta^3 , \tag{1.17}$$

with the velocity

$$\beta = (1-4m_f^2/M_H^2)^{1/2} \tag{1.18}$$

of the fermions, their mass m_f, and the colour factor $N_{cf} = 1(3)$ for leptons (quarks). These decays receive large QCD corrections which have been calculated by various groups and can amount up to -50%. [Braaten, Leveille; Sakai; Inami, Kubota; Drees, Hikasa; Gorishnii, Kataev, Larin, Surguladze; Kataev, Kim; Larin, van Ritbergen, Vermaseren; Chetyrkin, Kwiatkowski; Baikov, Chetyrkin, Kühn]

The branching ratios into gauge bosons reach



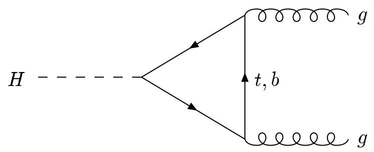
$$\begin{array}{lcl} BR(H \to W^+W^-) & \lesssim & 60-95\% \\ BR(H \to ZZ) & \lesssim & 30\% \end{array} . \tag{1.19}$$

The tree-level decay width into a pair of on-shell massive gauge bosons $V = Z, W$ is given by

$$\Gamma(H \to VV) = \delta_V \frac{G_F M_H^3}{16\sqrt{2}\pi} \beta(1 - 4x + 12x^2) \;, \tag{1.20}$$

with $x = M_V^2/M_H^2$, $\beta = \sqrt{1-4x}$ and $\delta_V = 2(1)$ for $V = W(Z)$. The electroweak corrections to these decays are of the order 5-20%.
[Fleischer, Jegerlehner; Bardin, ...; Kniehl; Ghinculov; Frink, ...] For a Higgs boson of mass $M_H =$ 125 GeV off-shell decays $H \to V^*V^* \to 4l$ are important. The program `PROPHECY4F` includes the complete QCD and EW next-to-leading order (NLO) corrections to $H \to WW/ZZ \to 4f$ [Bredenstein, Denner; Dittmaier, Mück, Weber].

The decay into gluon pairs proceeds via a loop with the dominant contributions from top and bottom quarks:



$$BR(H \to gg) \lesssim 6\% \;. \tag{1.21}$$

At leading order (LO) the decay width can be cast into the form

$$\Gamma_{LO}(H \to gg) = \frac{G_F \alpha_s^2 M_H^3}{36\sqrt{2}\pi^3} \left| \sum_{Q=t,b} A_Q^H(\tau_Q) \right|^2 \;, \tag{1.22}$$

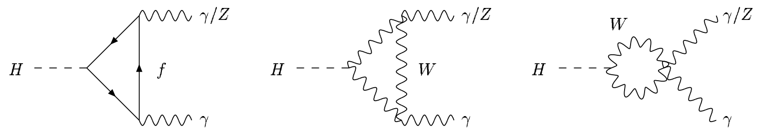
with the form factor

$$A_Q^H = \frac{3}{2}\tau[1 + (1-\tau)f(\tau)] \tag{1.23}$$

$$f(\tau) = \left\{ \begin{array}{ll} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4}\left[\log \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right]^2 & \tau < 1 \end{array} \right. \tag{1.24}$$

The parameter $\tau_Q = 4M_Q^2/M_H^2$ is defined by the pole mass M_Q of the heavy loop quark Q. Note that for large quark masses the form factor approaches unity. The strong coupling constant is denoted by α_s. The QCD corrections have been calculated [Baikov, Chetyrkin; Chetyrkin, Kniehl, Steinhauser; Krämer, Laenen, Spira; Schröder, Steinhauser; Chetyrkin, Kühn, Sturm; Inami eal; Djouadi, Graudenz, Spira, Zerwas; Dawson eal; Harlander, Steinhauser; Harlander, Hofmann]. They are large and increase the branching ratio by about 70% at next-to-leading order (NLO). They are known up to next-to-next-to-next-to leading order(N^3LO).

Further loop-mediated decays are those into 2 photons or a photon and a Z boson. They are mediated by charged fermion and W boson loops, the latter being dominant.



Although they amount only up to

$$BR(H \to \gamma\gamma, Z\gamma) \lesssim 2 \times 10^{-3} \tag{1.25}$$

the $\gamma\gamma$ final state is an important search mode for light Higgs bosons at the LHC. The partial decay width into photons reads

$$\Gamma(H \to \gamma\gamma) = \frac{G_F \alpha^2 M_H^3}{128\sqrt{2}\pi^3} \left| \sum_f N_{cf} e_f^2 A_f^H(\tau_f) + A_W^H(\tau_W) \right|^2 , \tag{1.26}$$

with the form factors

$$\begin{aligned} A_f^H(\tau) &= 2\tau[1 + (1-\tau) f(\tau)] && (1.27)\\ A_W^H(\tau) &= -[2 + 3\tau + 3\tau(2-\tau) f(\tau)] , && (1.28) \end{aligned}$$

with the function $f(\tau)$ defined in Eq. (1.24). The parameters $\tau_i = 4M_i^2/M_H^2$ $(i = f, W)$ are defined by the corresponding masses of the heavy loop particles. N_{cf} denotes again the colour factor of the fermion and e_f the electric charge. For large loop masses the form factors approach constant values,

$$\begin{array}{llll} A_f^H & \to \frac{4}{3} & \text{for } M_H^2 & \ll 4M_Q^2 \\ A_W^H & \to -7 & \text{for } M_H^2 & \ll 4M_W^2 . \end{array} \tag{1.29}$$

The W loop provides the dominant contribution in the intermediate Higgs mass regime, and the fermion loops interfere destructively. The QCD corrections have been calculated and are small in the intermediate Higgs boson mass region. [Zheng, Wu; Djouadi, Graudenz Spira, Zerwas; Melnikov, Spira, Yakovlev; Dawson, Kauffmann; Melnikov, Yakovlev; Inoue, Najima, Okada, Saito] The tree-level decay width into $Z\gamma$ is given

$$\Gamma(H \to Z\gamma) = \frac{G_F^2 M_W^2 \alpha M_H^3}{64\pi^4} \left(1 - \frac{M_Z^2}{M_H^2}\right)^3 \left| \sum_f A_f^H(\tau_f, \lambda_f) + A_W^H(\tau_W, \lambda_W) \right|^2 , \tag{1.30}$$

with the form factors

$$\begin{aligned} A_f^H(\tau, \lambda) &= 2N_{cf} \frac{e_f(I_{3f} - 2e_f \sin^2\theta_W)}{\cos\theta_W} [I_1(\tau,\lambda) - I_2(\tau,\lambda)] \\ A_W^H(\tau, \lambda) &= \cos\theta_W \Big\{ 4(3 - \tan^2\theta_W) I_2(\tau,\lambda) \\ &\quad + \left[\left(1 + \frac{2}{\tau}\right)\tan^2\theta_W - \left(5 + \frac{2}{\tau}\right)\right] I_1(\tau,\lambda) \Big\} . \qquad (1.31) \end{aligned}$$

The functions I_1 and I_2 read

$$I_1(\tau,\lambda) = \frac{\tau\lambda}{2(\tau-\lambda)} + \frac{\tau^2\lambda^2}{2(\tau-\lambda)^2}[f(\tau)-f(\lambda)] + \frac{\tau^2\lambda}{(\tau-\lambda)^2}[g(\tau)-g(\lambda)] \tag{1.32}$$

$$I_2(\tau,\lambda) = -\frac{\tau\lambda}{2(\tau-\lambda)}[f(\tau)-f(\lambda)] \,. \tag{1.33}$$

The function $g(\tau)$ can be cast into the form

$$g(\tau) = \begin{cases} \sqrt{\tau-1}\,\arcsin\frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ \frac{\sqrt{1-\tau}}{2}\left[\log\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right] & \tau < 1 \end{cases} \tag{1.34}$$

The parameters $\tau_i = 4M_i^2/M_H^2$ and $\lambda_i = 4M_i^2/M_Z^2$ $(i = f, W)$ are defined in terms of the corresponding masses of the heavy loop particles. The W loop dominates in the intermediate Higgs mass range, and the heavy fermion loops interfere destructively.

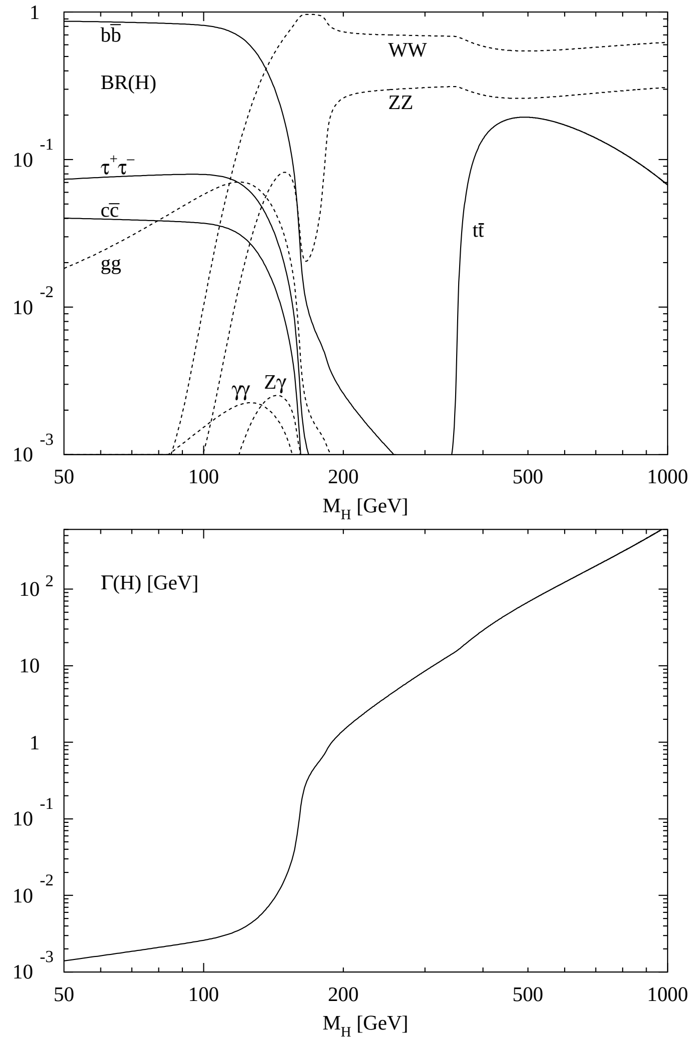


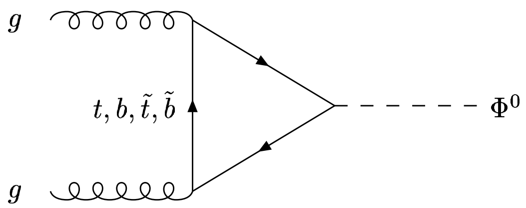
Figure 1.2: The Higgs boson branching ratios (upper) and the total width (lower) as a function of the Higgs boson mass. Made with HDECAY [Djouadi, Kalinowski, Mühlleitner, Spira]

Figs.1.2 show the Higgs boson branching ratios and total width as a function of the Higgs boson mass. One can infer from the figures that the total Higgs boson width is rather small, less than ~ 10 MeV, for masses below about 140 GeV. Once the threshold for gauge boson decays is reached the total widths increases rapidly up to about 600 GeV for $M_H = 1$ TeV. The gauge boson decay widths are proportional to M_H^3. Below the gauge boson threshold the main decay is into $b\bar{b}$, followed by the decay into $\tau^+\tau^-$.

1.5 Higgs boson production at the LHC

There are several Higgs boson production mechanisms at the LHC.
- Gluon fusion: The dominant production mechanism for Standard Model Higgs bosons at the LHC is gluon fusion

[Georgi, et al.;Gamberini, et al.]



$$pp \to gg \to H \ . \tag{1.35}$$

In the Standard Model it is mediated by top and bottom quark loops. The QCD corrections (the next-to leading order calculation involves 2-loop diagrams!) have been calculated and turn out to be large. They are of the order 10-100%. [Spira, Djouadi, Graudenz, Zerwas; Dawson, Kauffmann, Schaffer]; see Fig. 1.3, which shows the NLO K-factor, *i.e.* the ratio of the NLO cross section to the leading order (LO) cross section as a function of the Higgs boson mass for the virtual and real corrections.

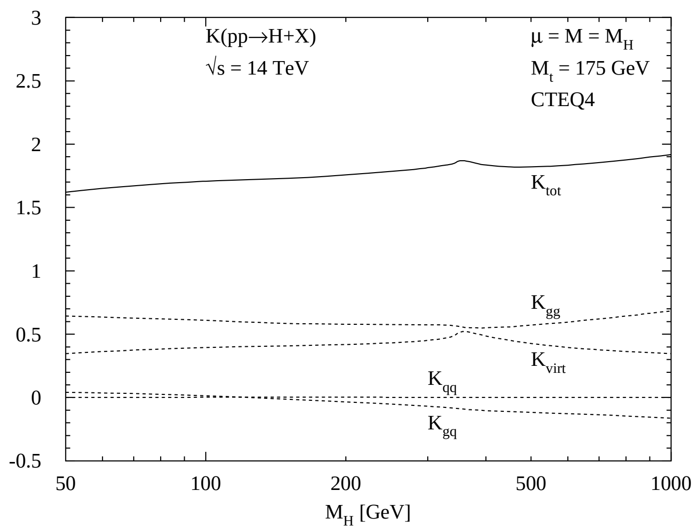


Figure 1.3: The K factor for the gluon fusion process as a function of the Higgs boson mass.

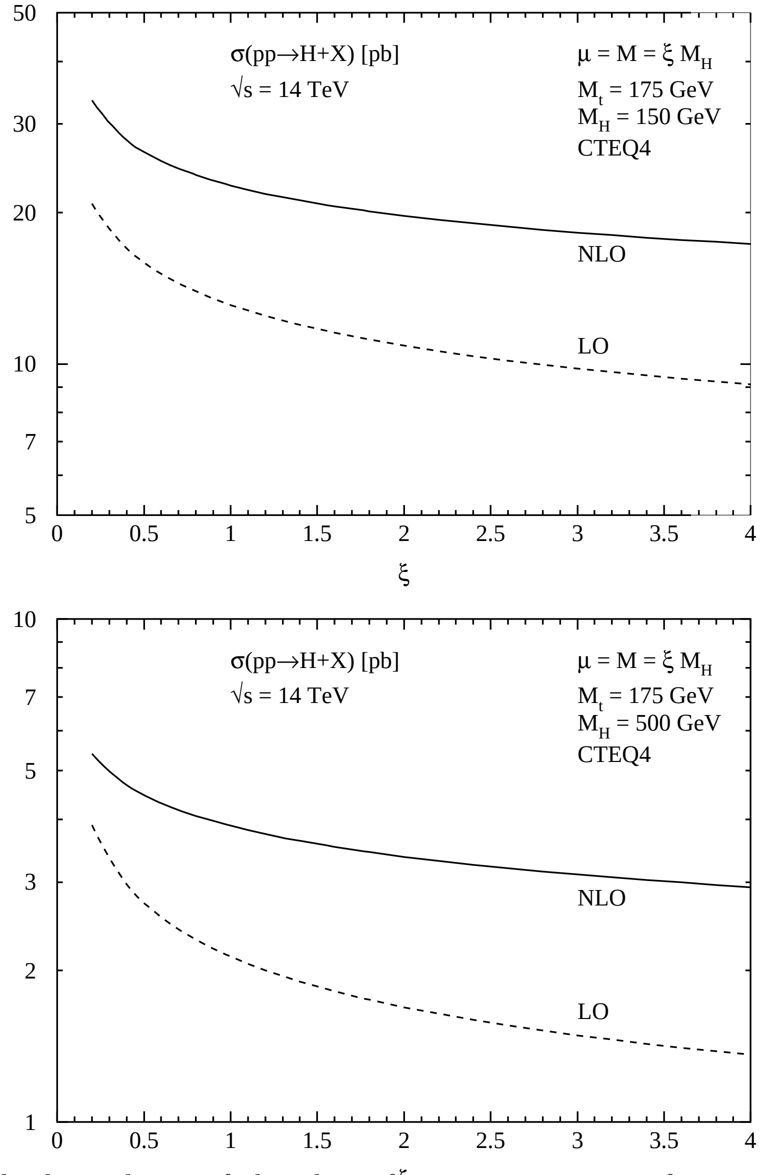


Figure 1.4: The scale dependence of the gluon fusion cross section for two different Higgs masses.

Due to the inclusion of the NLO QCD corrections the scale dependence of the gluon fusion cross section is decreased, *cf.* Fig. 1.4.

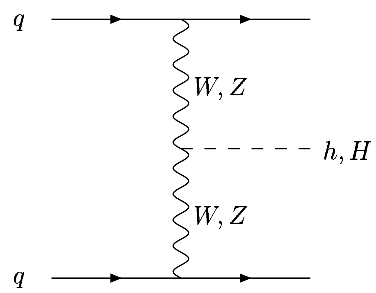
The next-to-next-to leading order (NNLO) corrections have been calculated in the limit of heavy top quark masses ($M_H \ll m_t$) [Harlander,Kilgore;Anastasiou,Melnikov;Ravindran,...]. They lead to a further increase of the cross section by 20-30%. The scale dependence is reduced to $\Delta \lesssim 10-15\%$. Resummation of the soft gluons [Catani, et al.; ...] adds another 10%.

There has been a lot of progress in the computation of the higher order corrections to gluon fusion in the last years.
Status of higher order (HO) corrections:

- ▷ complete NLO: increase σ by $\sim$ 80-100% — Spira,Djouadi,Graudenz,Zerwas; Dawson;Kauffman,Schaffer
- ▷ SM: limit $M_\Phi \ll m_t$ - approximation $\sim$ 20-30% — Krämer,Laenen,Spira

- ▷ NNLO @ $M_\Phi \ll m_t \Rightarrow$ further increase by 20-30%
  Harlander,Kilgore
  Anastasiou,Melnikov
  Ravindran,Smith,van Neerven
- ▷ Estimate of NNNLO effects @ $M_\Phi \ll m_t \rightsquigarrow$ scale stabilisation
  scale dependence: $\Delta \lesssim 10-15\%$
  Moch,Vogt
  Ravindran
- ▷ NNLL resummation: $\sim 6-9\%$
  Catani,de Florian,Grazzini,Nason
  Moch,Vogt; Laenen,Magnea; Idilbi eal
- ▷ leading soft contribution at N^{3}LO in limit $m_t \to \infty$
  Ravindran,Smith,van Nerven; Ahrens eal
- ▷ NNLO mass effects (t loops)
  for $M_H \lesssim 300$ GeV $\Rightarrow \mathcal{O}(0.5\%)$
  Harlander,Ozeren;Pak,Rogal,Steinhauser;
  Marzani et al.
- ▷ NLO electroweak corrections $\sim \mathcal{O}(5\%)$ (SM)
  Aglietti et al.;Degrassi,Maltoni;
  Actis et al
- ▷ mixed QCD and EW corrections
  Anastasiou,Boughezal,Petriello
- ▷ NLO for H+jet $\lesssim 1\%$
  Keung,Petriello; Brein

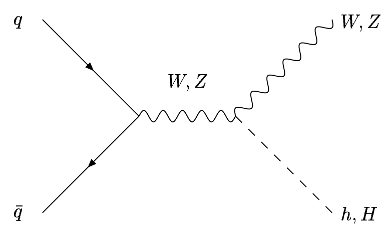
- WW/ZZ fusion: Higgs bosons can be produced in the WW/ZZ fusion processes [Cahn, Dawson; Hikasa; Altarelli, Mele, Pitolli]



$$pp \to W^*W^*/Z^*Z^* \to H \; . \tag{1.36}$$

The QCD corrections have been calculated and amount up to $\sim 10\%$ [Han, Valencia, Willenbrock]. In the meantime more higher order QCD and EW corrections have been calculated. (Not treated here.)

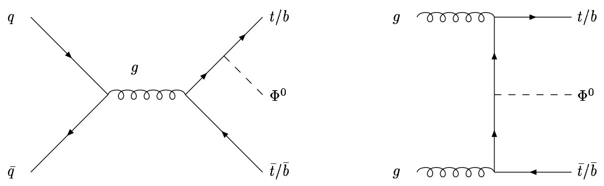
- Higgs-strahlung: Higgs bosons production in Higgs-strahlung [Glashow et al.; Kunszt et al.] proceeds via



$$pp \to W^*/Z^* \to W/Z + H \; . \tag{1.37}$$

The QCD corrections are $\sim 30\%$ [Han,Willenbrock]. The NNLO QCD corrections add another 5-10% [Harlander, Kilgore; Hamberg, Van Neerven, Matsuura; Brein, Djouadi, Harlander]. The theoretical error is reduced to about 5%. The complete electroweak (EW) corrections reduce the cross section by 5-10% [Ciccolini, Dittmaier, Krämer].

- Associated Production: Higgs bosons can also be produced in association with top and bottom quarks [Kunszt; Gunion; Marciano, Paige]



$$pp \to t\bar{t}/b\bar{b} + H \ . \tag{1.38}$$

The NLO QCD corrections to associated top production increase the cross section at the LHC by 20% [Beenakker, et al.;Dawson, et al.].

For all the production and background processes a lot of progress has been made in the last years on the calculation of the higher order (HO) QCD and EW corrections. They are not subject of this lecture, though. For details, see the corresponding literature.

Fig. 1.5 shows the production cross section in pb as a function of the Higgs boson mass.

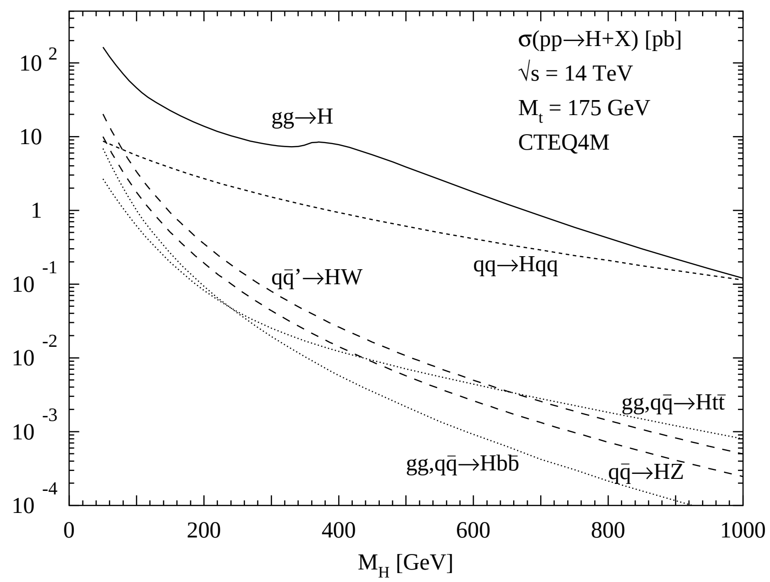
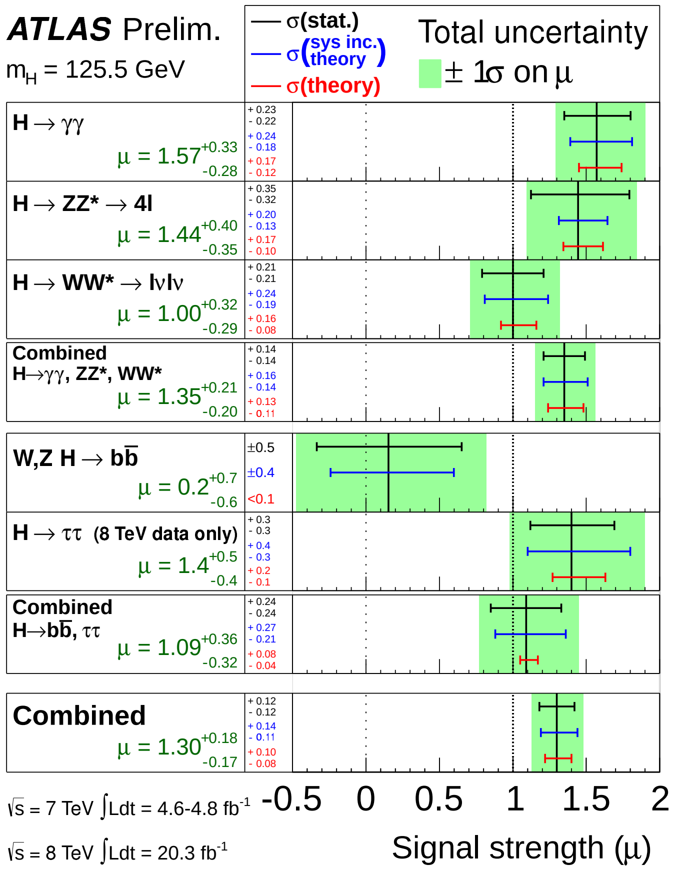


Figure 1.5: The Higgs boson production cross sections at the LHC as a function of the Higgs boson mass.

1.6 Higgs Boson Discovery

The main Higgs discovery channels are the $\gamma\gamma$ and ZZ^* final states. The decay into $\gamma\gamma$ final states has a very small branching ratios, but is very clean. (CMS and ATLAS have an excellent photon-energy resolution. Look for narrow $\gamma\gamma$ invariant mass peak, extrapolate background into the signal region from thresholds.). The ZZ^* final state is the other important search channel. For $M_H = 125$ Gev it is an off-shell decay. It leads to a clean 4 lepton



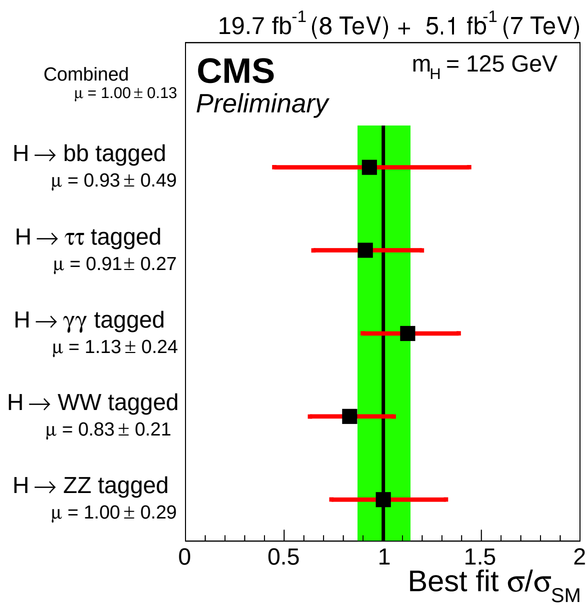


Figure 1.6: Best fits for the μ values reported by ATLAS [Phys. Lett. B726 (2013) 88, ATLAS-CONF-2013-079, ATLAS-CONF-2013-108] (upper) and CMS [CMS-PAS-HIG-14-009] (lower). The green bands/error bars are the 1σ errors.

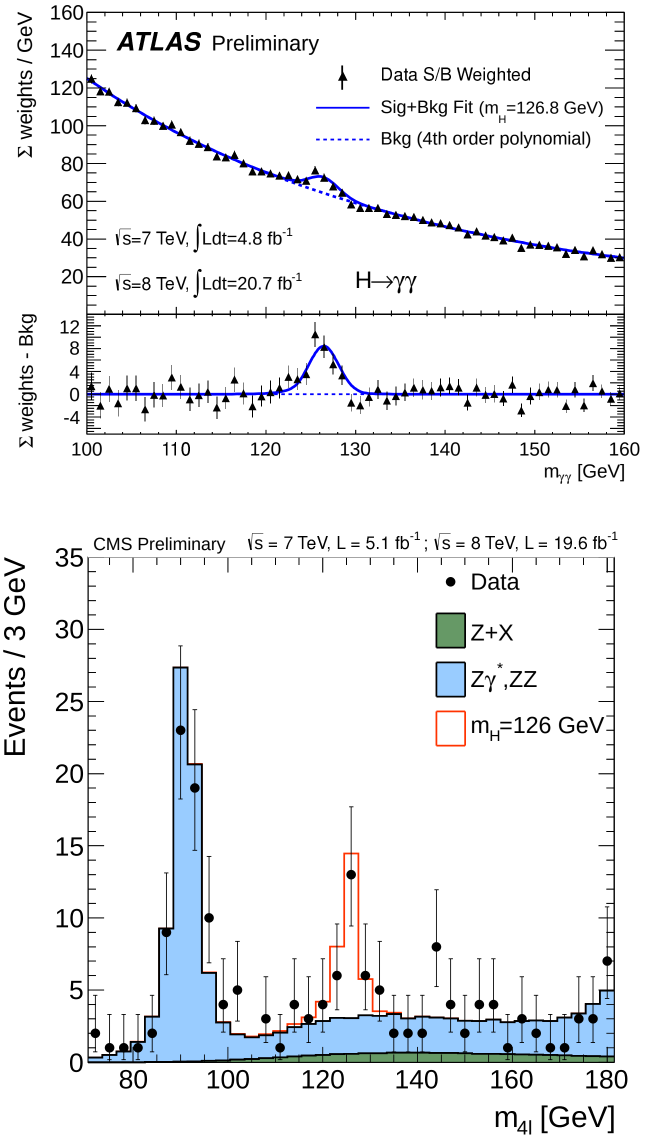


Figure 1.7: The main Higgs discovery channels: Upper: The photon final state, here shown for the ATLAS experiment [ATLAS-CONF-2013-12]. Lower: The ZZ^* final state, here shown for the CMS experiment [CMS-PAS-HIG-13-002].

(4l) final state from the decay of the Z bosons. Also the WW final state is off-shell. The final state signature includes missing energy from the neutrinos of the W boson decays. The $b\bar{b}$ final state is exploited as well. It has the largest branching ratio, but suffers from a large QCD background. Finally, the $\tau\tau$ channel is also used.

The experiments give the best fit values to the reduced μ values in the final state X. These are the production rate times branching ratio into the final state $X = \gamma, Z, W, b, \tau$ normalized to the corresponding value for a SM Higgs boson,

$$\mu = \frac{\sigma_{\rm prod} \times BR(H \to XX)}{(\sigma_{\rm prod} \times BR(H \to XX))_{\rm SM}} \,. \tag{1.39}$$

In case the discovered Higgs boson is a SM Higgs boson they are all equal to 1. Figure 1.6

shows the μ values reported by the LHC experiments. At present the the various final states suffer from uncertainties that leave room for beyond the SM (BSM) physics.

The main discovery channels for the 125 GeV Higgs boson at ATLAS and CMS, *i.e.* the photon and the Z boson final states, are shown in Fig. 1.7.

1.7 Higgs boson couplings at the LHC

In principle the strategy to measure the Higgs boson couplings is to combine various Higgs production and decay channels, from which the couplings can then be extracted. For example, the production of the Higgs boson in W boson fusion with subsequent decay into τ leptons, Fig. 1.8, is proportional to the partial width into WW and the branching ratio into $\tau\tau$. Combination with other production/decay channels and the knowledge of the total width allow then to extract the Higgs couplings. The problem at the LHC, however, is that the total width, which is small for a SM 125 GeV Higgs boson, cannot be measured without model-assumptions, and also not all final states are experimentally accessible. Therefore without applying model-assumptions only ratios of couplings are measureable.

The theoretical approach is to define an effective Lagrangian with modified Higgs couplings. In a first approach the couplings are modified by overall scale factors κ_i and the tensor structure is not changed. With this Lagrangian the signal rates, respectively μ values, are calculated as function of the scaling factors, $\mu(\kappa_i)$. These are then fitted to the experimentally measured μ values. The fits provide then the κ_i values. Such a theoretical Lagrangian for the SM field content with a scalar particle h looks like

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_h - (M_W^2 W_\mu^+ W^{\mu -} + \frac{1}{2} M_Z^2 Z_\mu Z^\mu)[1 + 2\,\kappa_V\,\frac{h}{v} + \mathcal{O}(h^2)] \\ &\quad - m_{\psi_i} \bar{\psi}_i \psi_i [1 + \kappa_F\,\frac{h}{v} + \mathcal{O}(h^2)] + \ldots \end{aligned} \tag{1.40}$$

It is valid below the scale Λ where new physics (NP) becomes important. It implements the electroweak symmetry breaking (EWSB) via $\mathcal{L}_h$ and the custodial symmetry through $\kappa_W = \kappa_Z = \kappa_V$. Furthermore, there are no tree-level flavour changing neutral current (FCNC) couplings as κ_F is chosen to be the same for all fermion generations and does not allow for transitions between fermion generations. The best fit values for κ_f and κ_V are shown in Fig. 1.9.

If the discovered particle is the Higgs boson the coupling strengths are proportional to

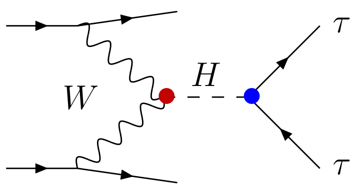


Figure 1.8: Feynman diagram for the production of a Higgs boson in W boson fusion with subsequent decay into $\tau\tau$. It is proportional to the partial width Γ_{WW} and the branching ratio into $\tau\tau$, BR($H \to \tau\tau$).

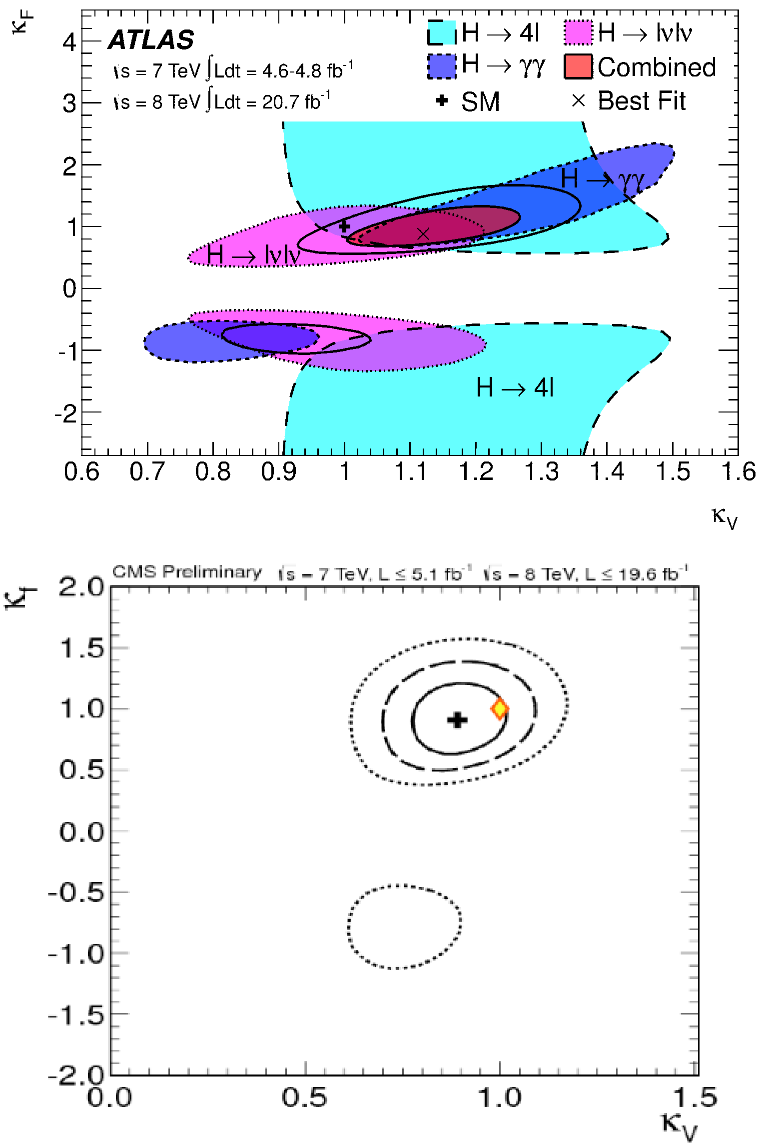


Figure 1.9: The best fit values for κ_f and κ_V by ATLAS [Phys. Lett. B726 (2013) 88] (upper) and CMS [CMS-PAS-HIG-13-005] (lower).

the masses (squared) of the particles to which the Higgs boson couples. This trend can be seen in the plot published by CMS, see Fig. 1.10.

1.8 Higgs Boson Quantum Numbers

The Higgs boson quantum numbers can be extracted by looking at the threshold distributions and the angular distributions of various production and decay processes. The SM Higgs boson has spin 0, positive parity P and is even under charge conjugation C. From the observation of the Higgs boson in the $\gamma\gamma$ final state one can already conclude that it does not have spin 1, due to the Landau-Yang theorem, and that it has $C = +1$, assuming

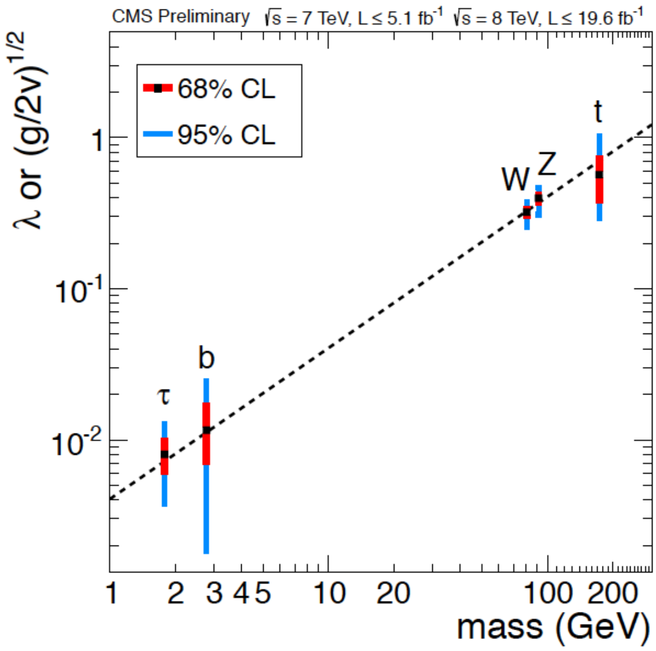


Figure 1.10: Coupling strenghts as function of the mass of the particles coupled to the Higgs boson, CMS [CMS-PAS-HIG-13-005]

charge invariance. However, these are theoretical considerations and have to be proven also experimentally.

The theoretical tools to provide angular distributions for a particle with arbitrary spin and parity are helicity analyses and operator expansion. Let us look as an example at the Higgs decay into ZZ^*, and the Z bosons subsequently decay into 4 leptons,

$$H \to ZZ^{(*)} \to (f_1\bar{f}_1)(f_2\bar{f}_2)\ . \tag{1.41}$$

The decay is illustrated in Fig. 1.11. The angle φ is the azimuthal angle between the decay planes of the Z bosons in the H rest frame. The θ_1 and θ_2 are the polar angles, respectively, of the fermion pairs in, respectively, the rest frame of the decaying Z boson.

For the SM the double polar angle distribution reads

$$\begin{aligned}\frac{1}{\Gamma'}\frac{d\Gamma'}{d\cos\theta_1 d\cos\theta_2} &= \frac{9}{16}\frac{1}{\gamma^4+2}\Big[\gamma^4\sin^2\theta_1\sin^2\theta_2 \\ &\quad +\frac{1}{2}(1+\cos^2\theta_1)(1+\cos^2\theta_2)\Big]\end{aligned} \tag{1.42}$$

and the azimuthal angular distribution is given by

$$\frac{1}{\Gamma'}\frac{d\Gamma'}{d\phi} = \frac{1}{2\pi}\left[1+\frac{1}{2}\frac{1}{\gamma^4+2}\cos 2\phi\right] \tag{1.43}$$

The verification of these distributions is a necessary step for the proof of the 0^+ nature of the Higgs boson.

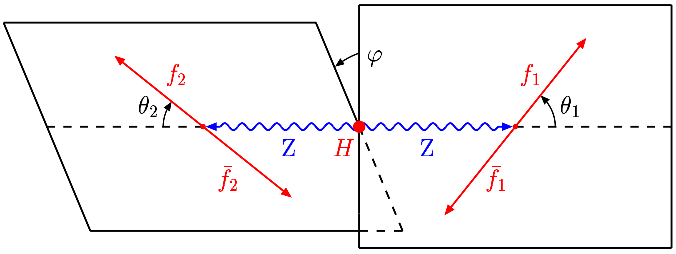


Figure 1.11: The decay $H \to ZZ^{(*)} \to (f_1\bar{f}_1)(f_2\bar{f}_2)$.

The calculation of the azimuthal angular distribution delivers a different behaviour for a scalar and a pseudoscalar boson:

$$\begin{array}{lll} 0^+ & : & d\Gamma/d\phi \sim 1 + 1/(2\gamma^4 + 4)\cos 2\phi \\ 0^- & : & d\Gamma/d\phi \sim 1 - 1/4\cos 2\phi \end{array} \tag{1.44}$$

Here $\gamma^2 = (M_H^2 - M_*^2 - M_Z^2)/(2M_* M_Z)$ and M_* is the mass of the virtual Z boson. Figure 1.12 shows how the azimuthal angular distribution can be exploited to test the parity of the particle. A pseudoscalar with spin-parity 0^- shows the opposite behaviour in this distribution than the scalar, which is due to the minus sign in front of $\cos 2\phi$ in Eq. (1.44). The threshold behaviour on the other hand can be used to determine the spin of the particle. We have for spin 0 a linear rise with the velocity β,

$$\frac{d\Gamma[H \to Z^* Z]}{dM_*^2} \sim \beta = \sqrt{(M_H - M_Z)^2 - M_*^2}/M_H \; . \tag{1.45}$$

A spin 2 particle, *e.g.* shows a flatter rise, $\sim \beta^3$, *cf.* Fig. 1.13.

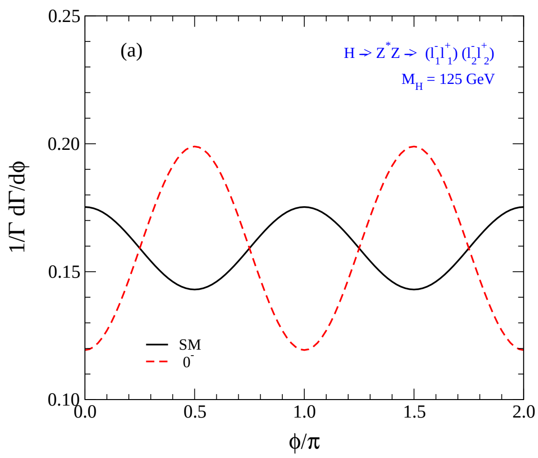


Figure 1.12: The azimuthal distribtuion for the $H \to ZZ^* \to 4l$ decay for the SM scalar Higgs (black) and a pseudoscalar (red). [Choi,Mühlleitner,Zerwas]

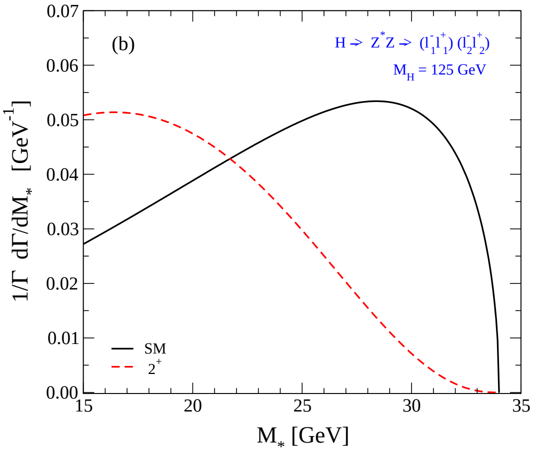


Figure 1.13: The threshold distribution for the $H \to ZZ^* \to 4l$ decay for the SM spin-0 Higgs (black) and a spin-2 particle (red).[Choi,Mühlleitner,Zerwas]

The experiments cannot perform an independent spin-parity measurement. Instead they test various spin-parity hypotheses. Various non-SM spin-parity hypotheses have been ruled out at more than 95% confidence level (C.L.), see *e.g.* Figs. 1.14 and 1.15.

1.9 Determination of the Higgs self-interactions

In order to fully establish the Higgs mechanism as the one responsible for the generation of particle masses without violating gauge principles, the Higgs potential has to be reconstructed. This can be done once the Higgs trilinear and quartic self-interactions have been measured. The trilinear coupling λ_{HHH} is accessible in double Higgs production. The quartic coupling λ_{HHHH} is to be obtained from triple Higgs production.

1.9.1 Determination of the Higgs self-couplings at the LHC

The processes for the extraction of λ_{HHH} [Djouadi,Kilian,Mühlleitner,Zerwas] at the LHC are gluon fusion into a Higgs pair, double Higgs strahlung, double WW/ZZ fusion and radiation of a Higgs pair off top quarks.

$$\begin{array}{llclcl} \text{gluon fusion:} & gg & \to & HH & & \\ \text{double Higgs-strahlung:} & q\bar{q} & \to & W^*/Z^* & \to & W/Z+HH \\ WW/ZZ \text{ double Higgs fusion:} & qq & \to & qq+WW/ZZ & \to & HH \\ \text{associated production:} & pp & \to & t\bar{t}HH & & \end{array} \tag{1.46}$$

The dominant gluon fusion production process proceeds via triangle and box diagrams, see Fig. 1.16.

Due to smallness of the cross sections, *cf.* Fig. 1.17, and the large QCD background the extraction of the Higgs self-coupling at the LHC is extremely difficult. There is an enormous theoretical activity to determine the production processes with high accuracy including HO corrections and to develop strategies and observables for the measurement of the di-Higgs production processes and the trilinear Higgs self-couplings.

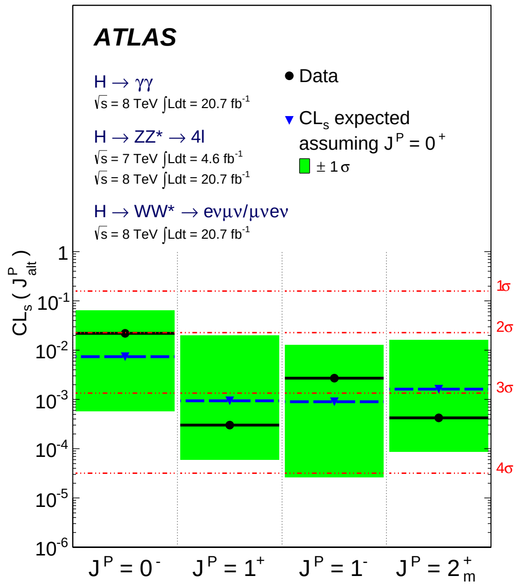


Figure 1.14: Spin-parity hypotheses tests by ATLAS. Details in Phys. Lett. B726 (2013) 120.

1.10 Summary

The measurements of the properties of the discovered particle have identified it as the Higgs boson. CERN therefore officially announced in a press release of March 2013, that the discovered particle is the Higgs boson, *cf.* Fig. 1.18. This lead then to the Nobel Prize for Physics in 2013 to Francois Englert and Peter Higgs.

The SM of particle physics has been very successful so far. At the experiments it has been tested to highest accuracy, including higher order corrections. And with the discovery of the Higgs particle we have found the last missing piece of the SM of particle physics. Still there are many open questions that cannot be answered by the SM. To name a few of them

1. In the SM the Higgs mechanism is introduced ad hoc. There is no dynamical mechanism behind it.

2. In the presence of high energy scales, the Higgs boson mass receives large quantum corrections, inducing the hierarchy problem.

3. We have no explanation for the fermion masses and mixings.

4. The SM does not contain a Dark Matter candidate.

5. In the SM the gauge couplings do not unify.

6. The SM does not incorporate gravity.

7. The CP violation in the SM is not large enough to allow for baryogenesis.

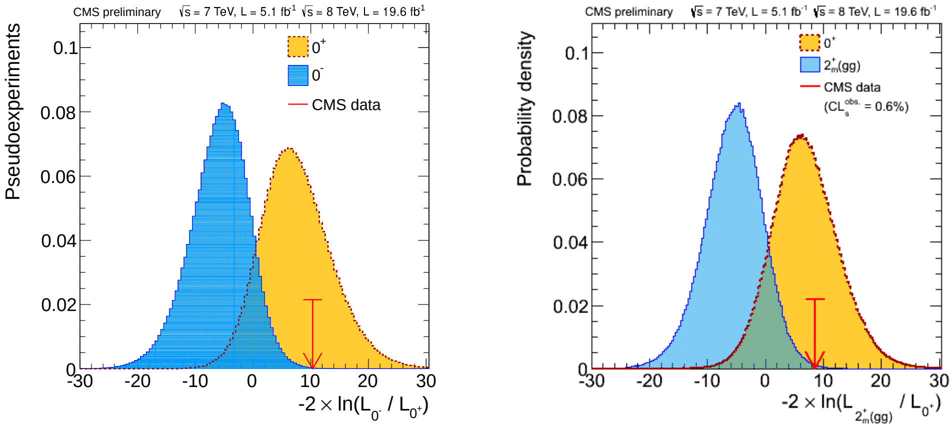


Figure 1.15: Spin-parity hypotheses tests by CMS. Left: 0^- excluded at 95% C.L. [CMS-PAS-HIG-13-002]. Right: $2^+_m(gg)$ excluded at 60% C.L. [CMS-PAS-HIG-13-005].

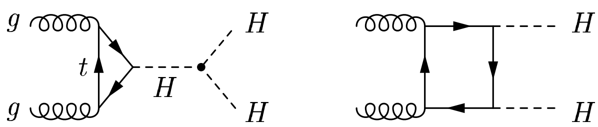


Figure 1.16: The diagrams which contribute to the gluon gluon fusion process $gg \to HH$.

8. ...

We therefore should rather see the SM as an effective low-energy theory which is embedded in some more fundamental theory that becomes apparent at higher scales. The Higgs data so far, although pointing towards a SM Higgs boson, still allow for interpretations within theories beyond the SM. Theses BSM theories can solve some of the problems of the SM. A few of these BSM models shall be presented in this lecture.

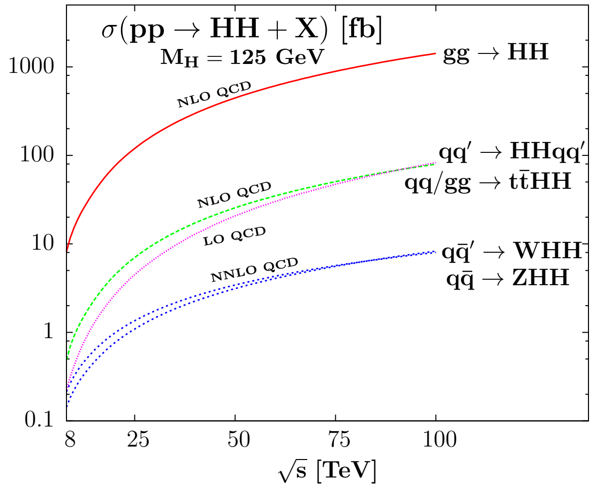
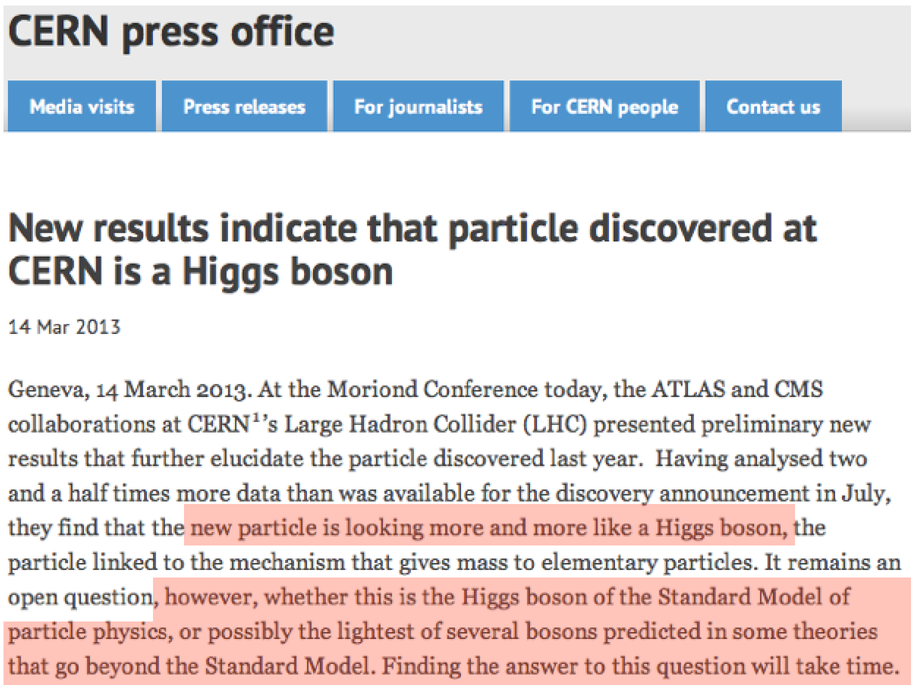


Figure 1.17: Di-Higgs production processes at the LHC with c.m. energy 14 TeV, including HO corrections. [Baglio,Djouadi,Gröber,Mühlleitner,Quévillon,Spira].



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New results indicate that particle discovered at CERN is a Higgs boson

14 Mar 2013

Geneva, 14 March 2013. At the Moriond Conference today, the ATLAS and CMS collaborations at CERN[1]'s Large Hadron Collider (LHC) presented preliminary new results that further elucidate the particle discovered last year. Having analysed two and a half times more data than was available for the discovery announcement in July, they find that the new particle is looking more and more like a Higgs boson, the particle linked to the mechanism that gives mass to elementary particles. It remains an open question, however, whether this is the Higgs boson of the Standard Model of particle physics, or possibly the lightest of several bosons predicted in some theories that go beyond the Standard Model. Finding the answer to this question will take time.

Figure 1.18: CERN press release.

Chapter 2

The 2-Higgs Doublet Model

<ins>*Disclaimer*:</ins> A lot of material for this chapter has been taken from Refs. [3, 4, 5].

So far the experimental Higgs data are compatible with a SM Higgs boson. Still there is room for interpretations of the Higgs data within beyond the SM (BSM) Higgs physics. The SM contains only one complex Higgs doublet. A straightforward minimal extension is given by adding an additional singlet field or another Higgs doublet. When extending our model to BSM physics we have to be careful, however, not to violate experimental and theoretical constraints. Two major constraints are given by the ρ parameter and the severe limits on the existence of flavour-changing neutral currents (FCNC).

<ins>*The ρ parameter constraint:*</ins> The ρ parameter

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \tag{2.1}$$

has been experimentally measured and is very close to one. In the SM the ρ parameter is determined by the Higgs structure of the theory and the tree-level value $\rho = 1$ is automatic. Introducing more generally n scalar multiplets ϕ_i with weak isospin I_i, weak hypercharge Y_i and VEV v_i of the neutral components, we have for the ρ parameter at tree level (demonstrate this)

$$\rho = \frac{\sum_{i=1}^n \left[I_i(I_i+1) - \frac{1}{4}Y_i^2\right] v_i}{\sum_{i=1}^n \frac{1}{2} Y_i^2 v_i} \,. \tag{2.2}$$

Both $SU(2)$ singlets with $Y = 0$ and $SU(2)$ doublets with $Y = \pm 1$ satisfy

$$I(I+1) = \frac{3}{4} Y^2 \tag{2.3}$$

and hence $\rho = 1$. Also models with larger $SU(2)$ multiplets, scalar particles with small or vanishing VEVs and models with triplets and a custodial $SU(2)$ global symmetry satisfy the ρ parameter constraint. But they lead to larger and more complex Higgs sectors.

<ins>*Flavour-changing neutral currents:*</ins> The existence of FCNC is experimentally severly constrained. In the SM tree-level FCNC are autmatically absent, as the mass matrix automatically diagonalizes the Higgs-fermion couplings. This is in general not the case for non-minimal Higgs models. A solution to this problem is given by a theorem by Glashow and

Weinberg [6] and shall be discussed below.

<u>*Unitarity Constraints:*</u> Finally in any model of EWSB it must be ensured, that the amplitude for the scattering of longitudinal gauge bosons V $(V = W, Z)$,

$$V_L V_L \to V_L V_L \tag{2.4}$$

or for fermions f scattering into longitudinal gauge bosons,

$$f_+ \bar{f}_+ \to V_L V_L \ , \tag{2.5}$$

where $+$ denotes the positive helicity of the fermion, do not violate unitarity bounds. This requires non-trivial cancellations of the Feynman diagrams contributing to a process. For example, in $WW \to WW$ scattering, the cancellation happens in the SM due to the existence of a light Higgs boson H with its couplings to the W bosons given by $g_{HWW} = g m_W$. In models with extended Higgs sectors it is not necessary that a single scalar boson ensures the unitary constraints. It is only necessary that sum rules for the scalar boson h_i couplings to VV and $f\bar{f}$ are fulfilled, namely

$$\sum_i g^2_{h_i VV} = g^2_{HVV} \tag{2.6}$$

and

$$\sum_i g_{h_i VV} g_{h_i f\bar{f}} = g_{HVV} g_{Hf\bar{f}} \ . \tag{2.7}$$

Note that these sum rules only hold for models with doublet and singlet Higgs fields. In extensions with triplets or higher Higgs representations there are more complicated sum rules.

The 2-Higgs Doublet Model (2HDM) with 2 complex Higgs doublets is - together with the singlet extension - the simplest possible extension of the SM and shall be discussed in the following. Besides the simple fact that extended Higgs sectors have not been ruled out yet experimentally, one main motivation for considering 2HDMs is supersymmetry (SUSY). Supersymmetry requires the introduction of two Higgs doublets due to the structure of the superpotential and also in order to have an anomaly-free theory. Another motivation is the fact that within the SM it is impossibly to generate a sufficiently large baryon asymmetry of the universe. On the other hand, 2HDMs have more freedom due to their enlarged parameter space and also additional sources for explicit or spontanuous CP violation. The latter is one of the three Sakharov conditions to generate the baryon asymmetry.[1]

2.1 The Higgs Potential

The 2HDM has a very rich vacuum structure due to the large number of parameters. Taking care of respecting the $SU(2)_L \times U(1)_Y$ gauge symmetry and requiring that the theory is renormalizable in $d = 4$ dimensions, there are altogether 14 operator products possible built of the two Higgs doublets Φ_1 and Φ_2 and that have operator dimension ≤ 4. The most general scalar potential can have CP-conserving, CP-violating and charge-violating

[1] The three conditions are baryon number violating processes, C and CP violation and departure from the thermal equilibrium.

minima. When writing up the potential care has to be taken in defining the various bases and in distinguishing between parameters which can be rotated away and those which have physical implications. If we assume, that CP is conserved and not sponaneously broken and if we impose discrete symmetries[2] that eliminate from the potential all quartic terms odd in either of the doublets, while allowing for all real quadratic coefficients, one of which softly breaks these symmetries, then the most general scalar potential for two doublets Φ_1 and Φ_2 with hypercharge +1 is given by [4]

$$\begin{aligned} V &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1\right) + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2 \\ &\quad + \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \left(\Phi_2^\dagger \Phi_1\right)^2\right] , \end{aligned} \tag{2.8}$$

where all parameters are real.[3] In the minimum of the Higgs potential the real components of the Higgs doublet take the VEVs

$$\langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix} \quad \text{and} \quad \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix} . \tag{2.9}$$

The two complex Higgs doublets contain eight real fields,

$$\Phi_a = \begin{pmatrix} \phi_a^+ \\ \frac{v_a + \rho_a + i\eta_a}{\sqrt{2}} \end{pmatrix} , \qquad a = 1, 2 . \tag{2.10}$$

Three out of them provide the longitudinal degrees of freedom for the massive $W^\pm$ and Z bosons. After EWSB we are hence left with five Higgs fields. Assuming CP conservation, we have two neutral scalars, one neutral pseudoscalar and two charged Higgs bosons. Expansion about the minima leads to the mass term for the charged Higgs bosons, given by

$$\mathcal{L}_{\phi^\pm,\text{mass}} = -[m_{12}^2 - (\lambda_4 + \lambda_5)\frac{v_1 v_2}{2}](\phi_1^-, \phi_2^-) \underbrace{\begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix}}_{\mathcal{M}_C'} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix} . \tag{2.11}$$

Here we have already exploited the minimum conditions

$$\left.\frac{\partial V}{\partial \Phi_i^\dagger}\right|_{\langle \Phi_i \rangle = v_i/\sqrt{2}} = 0 , \quad i = 1, 2 , \tag{2.12}$$

which imply

$$m_{11}^2 + \frac{\lambda_1 v_1^2}{2} + \frac{\lambda_3 v_2^2}{2} = m_{12}^2 \frac{v_2}{v_1} - (\lambda_4 + \lambda_5)\frac{v_2^2}{2} , \tag{2.13}$$

$$m_{22}^2 + \frac{\lambda_2 v_2^2}{2} + \frac{\lambda_3 v_1^2}{2} = m_{12}^2 \frac{v_1}{v_2} - (\lambda_4 + \lambda_5)\frac{v_1^2}{2} . \tag{2.14}$$

The mass matrix is diagonalized by the orthogonal transformation matrix

$$\mathcal{U}_C = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} , \tag{2.15}$$

[2]We come back to this point when we discuss the constraints from FCNC couplings.

[3]Note, that in [3], the parameter m_{11}^2 is called m_1^2, m_{22}^2 is m_2^2 and m_{12}^2 is named m_3^2.

where

$$\tan\beta = \frac{v_2}{v_1} \ . \tag{2.16}$$

The parameter $\tan\beta$ is a key parameter of the model. In order to reproduce the W and Z boson masses as in the SM we have

$$v_1^2 + v_2^2 = v^2 \ , \tag{2.17}$$

with

$$v^2 = \frac{1}{\sqrt{2}G_F} \approx 246^2 \ (\mathrm{GeV})^2 \ , \tag{2.18}$$

where G_F denotes the Fermi constant. The mass matrix Eq. (2.11) has one zero eigenvalue, which corresponds to the charged Goldstone boson $G^\pm$. The mass squared of the charged Higgs boson reads

$$m_{H^\pm}^2 = \left(\frac{m_{12}^2}{v_1 v_2} - \frac{\lambda_4 + \lambda_5}{2}\right)(v_1^2 + v_2^2) = M^2 - \frac{1}{2}(\lambda_4 + \lambda_5)v^2 \ , \tag{2.19}$$

where we have introduced

$$M^2 = \frac{m_{12}^2}{\sin\beta\cos\beta} \ . \tag{2.20}$$

Due to CP-invariance, as assumed here, the imaginary and the real parts of the neutral scalar fields decouple. The mass term for the pseudoscalars is given by the imaginary components of the neutral Higgs fields and, again by exploiting the minimum conditions, can be cast into the form

$$\mathcal{L}_{\eta,\mathrm{mass}} = -\frac{1}{2}\frac{m_A^2}{v_1^2 + v_2^2}(\eta_1, \eta_2)\underbrace{\begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix}}_{\mathcal{M}'_P}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} \ . \tag{2.21}$$

The mass matrix is diagonalized by the orthogonal transformation matrix $\mathcal{U}_P$, for which at tree-level

$$\mathcal{U}_P = \mathcal{U}_C \ . \tag{2.22}$$

This leads to one neutral Goldstone boson G^0 and a pseusoscalar, denoted by A, with mass squared

$$m_A^2 = \left(\frac{m_{12}^2}{v_1 v_2} - \lambda_5\right)(v_1^2 + v_2^2) = M^2 - \lambda_5 v^2 \ . \tag{2.23}$$

Note, that when $m_{12} = 0$ and $\lambda_5 = 0$, then the pseudoscalar is massless. The reason behind this is the existence of an additional global $U(1)$ symmetry in that limit, which is spontaneously broken. The mass terms for the scalars, derived by collecting the bilinear terms of the real parts of the neutral Higgs fields and exploiting the minimum conditions, read

$$\mathcal{L}_{\rho,\mathrm{mass}} = -\frac{1}{2}(\rho_1, \rho_2)\underbrace{\begin{pmatrix} m_{12}^2\frac{v_2}{v_1} + \lambda_1 v_1^2 & -m_{12}^2 + \lambda_{345}v_1 v_2 \\ -m_{12}^2 + \lambda_{345}v_1 v_2 & m_{12}^2\frac{v_1}{v_2} + \lambda_2 v_2^2 \end{pmatrix}}_{\mathcal{M}_N}\begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} \ , \tag{2.24}$$

where we have defined

$$\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5 \; . \tag{2.25}$$

The mass matrix is diagonalized by the orthogonal transformation matrix

$$\mathcal{U}_N = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \; . \tag{2.26}$$

The mixing angle α is given in terms of the matrix elements of the mass matrix $\mathcal{M}_N$ as

$$\sin 2\alpha = \frac{2\mathcal{M}_{12}}{\sqrt{(\mathcal{M}_{11}-\mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}} \tag{2.27}$$

$$\cos 2\alpha = \frac{\mathcal{M}_{11}-\mathcal{M}_{22}}{\sqrt{(\mathcal{M}_{11}-\mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}} \tag{2.28}$$

and

$$\tan 2\alpha = \frac{(M^2 - \lambda_{345} v^2)\sin 2\beta}{(M^2-\lambda_1 v^2)\cos^2\beta - (M^2 - \lambda_2 v^2)\sin^2\beta} \; . \tag{2.29}$$

This leads to the CP-even mass eigenstates h and H

$$H = \rho_1 \cos\alpha + \rho_2 \sin\alpha \tag{2.30}$$

$$h = -\rho_1 \sin\alpha + \rho_2 \cos\alpha \; , \tag{2.31}$$

with the mass values

$$m_{H,h}^2 = \frac{1}{2}\left[\mathcal{M}_{11} + \mathcal{M}_{22} \pm \sqrt{(\mathcal{M}_{11}-\mathcal{M}_{22})^2 + 4\mathcal{M}_{12}^2}\right] \; . \tag{2.32}$$

By convention the lighter CP-even state is called h and the heavier one H. Note that the SM Higgs boson would be

$$\begin{aligned} H^{\rm SM} &= \rho_1 \cos\beta + \rho_2 \sin\beta \\ &= H\cos(\alpha-\beta) - h\sin(\alpha-\beta) \; . \end{aligned} \tag{2.33}$$

The SM Higgs boson hence corresponds to h for $\cos\alpha = \sin\beta$ and $\sin\alpha = -\cos\beta$. It corresponds to H for $\cos\alpha = \cos\beta$ and $\sin\alpha = \sin\beta$. That Eq. (2.33) defines the SM Higgs can be seen by multiplying the Higgs doublets with the mixing matrix $\mathcal{U}_C$, respectively $\mathcal{U}_P$,

$$\begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} \cos\beta\Phi_1 + \sin\beta\Phi_2 \\ -\sin\beta\Phi_1 + \cos\beta\Phi_2 \end{pmatrix} \; , \tag{2.34}$$

This leads to two Higgs doublets, one of which

$$\begin{aligned} \Phi_1^{\rm HB} &= \cos\beta\Phi_1 + \sin\beta\Phi_2 = \begin{pmatrix} \cos\beta\phi_1^+ + \sin\beta\phi_2^+ \\ \frac{1}{\sqrt{2}}\left[\cos\beta(v_1+\rho_1+i\eta_1) + \sin\beta(v_2+\rho_2+i\eta_2)\right] \end{pmatrix} \\ &= \begin{pmatrix} G^\pm \\ \frac{1}{\sqrt{2}}\left[iG^0 + (\cos\beta\rho_1 + \sin\beta\rho_2) + v\right] \end{pmatrix} \equiv \begin{pmatrix} G^\pm \\ \frac{1}{\sqrt{2}}\left[iG^0 + S_1 + v\right] \end{pmatrix} \; , \end{aligned} \tag{2.35}$$

contains the massless Goldstone bosons and the VEV v in the neutral component, so that $S_1 \equiv (\cos\beta\rho_1 + \sin\beta\rho_2)$ can be identified with the SM Higgs boson. The superscript HB stands for '*Higgs Basis*'. The other Higgs doublet reads

$$\begin{aligned} \Phi_2^{\mathrm{HB}} &= -\sin\beta\Phi_1 + \cos\beta\Phi_2 = \begin{pmatrix} -\sin\beta\phi_1^+ + \cos\beta\phi_2^+ \\ \frac{1}{\sqrt{2}}\left[-\sin\beta(v_1+\rho_1+i\eta_1) + \cos\beta(v_2+\rho_2+i\eta_2)\right] \end{pmatrix} \\ &= \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(-\sin\beta\rho_1 + \cos\beta\rho_2) + \frac{i}{\sqrt{2}}(-\sin\beta\eta_1 + \cos\beta\eta_2) \end{pmatrix} \equiv \begin{pmatrix} H^+ \\ \frac{S_2+iS_3}{\sqrt{2}} \end{pmatrix} . \end{aligned} \tag{2.36}$$

The advantage of the Higgs Basis is that the three Goldstone fields $G^\pm$ and G^0 get isolated as components of Φ_1. The three neutral scalar mass eigenstates of the physical scalar spectrum, $\varphi_i^0 = (h, H, A)^T$ are related through an orthogonal transformation $\mathcal{R}$ with the S_i fields,

$$\varphi_i^0 = \mathcal{R}_{ij} S_j \; . \tag{2.37}$$

In general the CP-odd component S_3 mixes with the CP-even fields $S_{1,2}$ and the resulting mass eigenstates do not have a definite CP quantum number. If the scalar potential is CP symmetric this admixture disappears. In this case $A \equiv S_3$.

Without loss of generality it can be assumed that β is in the first quadrant, *i.e.* that both v_1 and v_2 are non-negative and real. Furthermore, π can be added to α, which inverts the sign of both the h and H fields, witout affecting any physics. The angle α will be chosen either in the first or the fourth quadrant.

The decoupling and the non-decoupling effect:
The masses of the heavier Higgs bosons ($H, H^\pm$ and A) take the form

$$m_\Phi^2 = M^2 + \lambda_i v^2 (+\mathcal{O}(v^4/M^2)) \; , \tag{2.38}$$

where $\Phi \equiv H, H^\pm, A$ and λ_i is a linear combination of λ_1–λ_5. In case $M^2 \gg \lambda_i v^2$ the mass m_Φ^2 is determined by the soft-breaking scale of the discrete symmetry, M^2. The effective theory below M is then described by one Higgs doublet. And all the tree-level couplings related to the lightest Higgs boson h approach SM values. The loop effects of Φ vanish in the large mass limit due to the decoupling theorem.

In case M^2 is limited to be at the weak scale ($M^2 \lesssim \lambda_i v^2$) a large value of m_ϕ is realized by taking λ_i to be large, so that one is in the strong coupling regime. The squared mass of Φ is then effectively proportional to λ_i, so that the decoupling theorem does not apply, leading to a power-like contribution of m_ϕ in the radiative corrections. This effect is called the non-decoupling effect of Φ. However, theoretical and experimental constraints have to be considered. Thus too large λ_i lead to the breakdown of the validity of perturbation theory. And low-energy precision data impose important constraints on the model parameters.

Parameters of the Higgs Potential:
The parameters of the Higgs potential are $m_{11}^2, m_{22}^2, m_{12}^2$ and λ_1–λ_5. They can be expressed in terms of eight 'physical' parameters, which are the four Higgs mass parameters $m_h, m_H, m_A, m_{H^\pm}$, the two mixing angles α, β, the vacuum expectation value v and the soft-breaking scale of the discrete symmetry, M. The quartic coupling constants can be expressed in terms of these parameters. (Derive the expressions!)

2.2 The problem with flavour conservation

The 2HDM faces the serious problem of possible FCNCs at tree-level. To see this let us look at *e.g.* the Yukawa Lagrangian. The most general Yukawa Lagrangian is given by

$$\begin{aligned}\mathcal{L}_Y &= -\Big\{\bar{Q}'_L(\Gamma_1\Phi_1+\Gamma_2\Phi_2)D'_R - \bar{Q}'_L(\Delta_1\tilde{\Phi}_1+\Delta_2\tilde{\Phi}_2)U'_R \\ &\quad +\bar{L}'(\Pi_1\Phi_1+\Pi_2\Phi_2)E'_R + h.c.\Big\}\,, \end{aligned} \tag{2.39}$$

where Q'_L, L'_L denote the left-handed quark and lepton doublets and $Q \equiv (U,D)^T$, $L \equiv (\nu, E)^T$, with $U \equiv (u,c,t)^T$, $D \equiv (d,s,b)^T$, $\nu \equiv (\nu_e,\nu_\mu,\nu_\tau)^T$ and $E \equiv (e,\mu,\tau)^T$. The indices L, R denote left- and right-handed fermions f given by

$$f_{L,R} = P_{L,R} f \equiv \frac{1}{2}(1 \mp \gamma_5) f\ . \tag{2.40}$$

We have defined $\tilde{\Phi}_a = (\Phi_a^T \epsilon)^\dagger$, with

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\ . \tag{2.41}$$

The couplings Γ_a, Δ_a and Π_a $(a=1,2)$ are 3×3 complex matrices in flavour space. In the Higgs basis the Lagrangian can be cast into the form

$$\begin{aligned}\mathcal{L}_Y &= -\frac{\sqrt{2}}{v}\Big\{\bar{Q}'_L(M'_d\Phi_1^{\rm HB}+Y'_d\Phi_2^{\rm HB})D'_R - \bar{Q}'_L(M'_u\tilde{\Phi}_1^{\rm HB}+Y'_u\tilde{\Phi}_2^{\rm HB})U'_R \\ &\quad +\bar{L}'(M'_l\Phi_1^{\rm HB}+Y'_l\Phi_2^{\rm HB})E'_R + h.c.\Big\}\,, \end{aligned} \tag{2.42}$$

where M'_f $(f = d,u,l)$ are the non-diagonal fermion mass matrices. The matrices Y'_f contain the Yukawa couplings to the scalar doublet with zero vacuum expectation value.

In the basis of the mass eigenstates D, U, E, ν, with diagonal mass matrices M_f $(M_\nu = 0)$, the corresponding matrices Y_f are in general non-diagonal and unrelated to the fermion masses. Therefore the Yukawa Lagrangian leads to FCNC couplings, as there are two different Yukawa matrices coupling to a right-handed fermion field. These can in general not be diagonalized simultaneously. Thus neutral Higgs scalars ϕ can mediate FCNC, as *e.g.* $\bar{d}s\phi$. This would lead to serious phenomenological conflicts. This coupling would lead *e.g.* to K-$\overline{K}$ mixing at tree level. Assuming the coupling to be as large as the b-quark Yukawa coupling, this would require the mass of the exchanged scalar to exceed 10 TeV, in order to achieve a suppression that is in accordance with the experiments.

The problem can be avoided by forcing one of the two matrices to be zero, which can be achieved by imposing that only one scalar doublet couples to a given right-handed fermion field. In other words, if all fermions with the same quantum numbers (so that they can mix) couple to the same Higgs multiplet, then FCNCs are absent. This has been stated in the Paschos-Glashow-Weinberg theorem [7]. It says that a necessary and sufficient condition for the absence of FCNCs at tree level is that all fermions of a given charge and helicity transform according to the same irreducible representation of $SU(2)$, correspond to the same eigenvalue of T_3 and that a basis exists in which they receive their contributions to the mass matrix from a single source. For the SM with left-handed doublets and right-handed singlets, this means that all right-handed quarks of a given charge must couple to a single Higgs multiplet. In the 2HDM, this can only be ensured by introducing discrete or continuous symmetries.

In the 2HDM there are two possibilities to achieve this:

- type I 2HDM: All quarks couple to just one of the Higgs doublets (conventionally chosen to be Φ_2).

- type II 2HDM: The $Q = 2/3$ right-handed (RH) quarks couple to one Higgs doublet (conventionally chosen to be Φ_2) and the $Q = -1/3$ RH quarks couple to the other (Φ_1).

In order to get the type I 2HDM a simple discrete symmetry $\Phi_1 \to -\Phi_1$ is imposed. For the type II 2HDM a $\Phi_1 \to -\Phi_1$, $d_R^i \to -d_R^i$ discrete symmetry is enforced. Note, that SUSY models lead to the same Yukawa couplings as the type II model. They use, however, continuous symmetries.

In the type I and type II 2HDMs it is conventionally assumed, that the right-handed leptons satisfy the same discrete symmetry as the d_R^i, so that the leptons couple to the same Higgs boson as the down-typ quarks. The Glashow-Weinberg theorem, however, does not require this. There are therefore two more possibilities:

- Lepton-specific model: The RH quarks all couple to Φ_2 and the RH leptons couple to Φ_1.

- Flipped model: The RH up-type quarks couple to Φ_2, the RH down-type quarks couple to Φ_1, as in type II, but now the RH leptons couple to Φ_2.

There circulate a lot of different names for these models in the literature. Thus Model III and Model IV were used for the flipped and lepton-specific models, respectively, in one of the earliest works on them. In other papers lepton-specific and flipped were named, respectively, Model I and Model II. Also the terms IIA and IIB were used. Recently, lepton-specific was called X-type and flipped Y-type models.

The explicit implementation of the discrete symmetry is scalar-basis dependent. If it is imposed in the Higgs Basis, all fermions are forced to couple to the field Φ_1^{HB} in order to get non-vanishing masses. This *inert* doublet model provides a natural frame for dark matter. Note, however, that although Φ_2^{HB} does not couple to fermions, it has nevertheless electroweak interactions.

Tree-level FCNC interactions can be avoided in a softer and more general way by requiring the alignment in flavour space of the Yukawa couplings of the two scalar doublets. A convenient way to implement this condition is given by the form

$$\Gamma_2 = \xi_d e^{-i\theta}\Gamma_1 \,, \quad \Delta_2 = \xi_u^* e^{i\theta}\Delta_1 \,, \quad \Pi_2 = \xi_l e^{-i\theta}\Pi_1 \,. \tag{2.43}$$

The proportionality parameters ξ_f are arbitrary complex numbers. The explicit phases $e^{\mp i\theta}$ can be introduced to cancel the relative global phases between the two scalar doublets. They will be omitted in the following. Through the Yukawa alignment the Y_f' and M_f' matrices are guaranteed to be proportional to each other, so that they can be diagonalized simultaneously, leading to

$$Y_{d,l} = \zeta_{d,l} M_{d,l} \,, \quad Y_u = \zeta_u^* M_u \,, \quad \zeta_f \equiv \frac{\xi_f - \tan\beta}{1 + \xi_f \tan\beta} \,. \tag{2.44}$$

The Yukawa interactions in terms of the mass eigenstate fields then take the form

$$\begin{aligned}\mathcal{L}_Y &= -\frac{\sqrt{2}}{v}H^+\bar{U}[\zeta_d V M_d P_R - \zeta_u M_u V P_L]D - \frac{\sqrt{2}}{v}H^+\zeta_l\bar{\nu}M_l P_R E \\ &\quad -\frac{1}{v}\sum_{\varphi_i^0,f}\varphi_i^0 y_f^{\varphi_i^0}\bar{f}M_f P_R f + h.c.\ ,\end{aligned}\tag{2.45}$$

where V is the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The flavour alignment of the Yukawa couplings leads to a very specific structure of the scalar couplings to the fermions:

i) All fermion couplings of the physical scalar fields are proportional to the corresponding fermion mass matrices.

ii) The neutral Yukawa couplings are diagonal in flavour space. The couplings of the physical scalar fields h, H and A are proportional to the corresponding elements of the orthogonal matrix $\mathcal{R}$, namely

$$y_{d,l}^{\varphi_i^0} = \mathcal{R}_{i1} + (\mathcal{R}_{i2} + i\mathcal{R}_{i3})\zeta_{d,l} \tag{2.46}$$

$$y_u^{\varphi_i^0} = \mathcal{R}_{i1} + (\mathcal{R}_{i2} - i\mathcal{R}_{i3})\zeta_u^*\ . \tag{2.47}$$

iii) The only source of flavour-changing couplings is given by the CKM matrix V. It regulates the quark couplings of the $W^\pm$ bosons and the charged scalars $H^\pm$.

iv) All leptonic couplings are diagonal in flavour space. This is because we assume the neutrinos to be massless in our low-energy Lagrangian. (Although we know in the meantime that the neutrinos have mass.) Since we assume the neutrinos to be massless the leptonic mixing matrix can be reabsorbed through a redefinition of the neutrino fields.

v) The only new couplings introduced by the Yukawa Lagrangian are the three parameters ζ_f, which encode all possible freedom allowed by the alignment conditions. The couplings satisfy universality among the different generations, as all fermions of a given electric charge have the same universal coupling ζ_f. Furthermore, the parameters ζ_f are invariant under global $SU(2)$ transformations of the scalar fields [9], *i.e.* $\Phi_a \to \Phi'_a = U_{ab}\Phi_b$. This means that they are independent of the basis choice adopted in the scalar space.

vi) The models where a single scalar doublet couples to each type of right-handed fermions are recovered by taking the appropriate limits $\xi_f \to 0$ or $\xi_f \to \infty$, *i.e.* $\zeta_f \to -\tan\beta$ of $\zeta_f \to \cot\beta$. Thus the type-I model corresponds to $(\xi_d, \xi_u, \xi_l) = (\infty, \infty, \infty)$, type II to $(0, \infty, 0)$, the lepton-specific to $(\infty, \infty, 0)$ and the flipped model to $(0, \infty, \infty)$. (Compare with Table 2.1.) The *inert* model corresponds to $\zeta_f = 0$ ($\xi_f = \tan\beta$).

vii) The ζ_f can be arbitrary complex numbers, so that one can have new sources of CP violation without tree-level FCNCs.

We will now determine the Yukawa couplings. In the type II model, *e.g.* the Yukawa Lagrangian is given by[4]

$$\begin{aligned}\mathcal{L}_Y &= -\left\{ \bar{E}_R \begin{pmatrix} h_e & 0 & 0 \\ 0 & h_\mu & 0 \\ 0 & 0 & h_\tau \end{pmatrix} \Phi_1^\dagger E_L + \bar{D}'_R V \begin{pmatrix} h_d & 0 & 0 \\ 0 & h_s & 0 \\ 0 & 0 & h_b \end{pmatrix} V^\dagger \Phi_1^\dagger \begin{pmatrix} U \\ D' \end{pmatrix}_L \right. \\ & \left. -\bar{U}_R \begin{pmatrix} h_u & 0 & 0 \\ 0 & h_c & 0 \\ 0 & 0 & h_t \end{pmatrix} \Phi_2^T \epsilon \begin{pmatrix} U \\ D' \end{pmatrix}_L + h.c. \right\} . \end{aligned} \tag{2.48}$$

Here $U \equiv (u,c,t)^T$, $D \equiv (d,s,b)^T$ and $E \equiv (e,\mu,\tau)^T$. The h_f denote the various Yukawa couplings. Coupling the Higgs doublets for the various models as described above and rotating to the mass eigenstates, one gets, in the notation of Ref. [8], the Yukawa Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}}^{\text{2HDM}} &= -\sum_{f=u,d,l} \frac{m_f}{v} \left(\xi_h^f \bar{f} f h + \xi_H^f \bar{f} f H - i \xi_A \bar{f} \gamma_5 f A \right) \\ & - \left\{ \frac{\sqrt{2} V_{ud}}{v} \bar{u} (m_u \xi_A^u P_L + m_d \xi_A^d P_R) d H^+ + \frac{\sqrt{2} m_l \xi_A^l}{v} \bar{\nu}_L l_R H^+ + h.c. \right\} \end{aligned} \tag{2.49}$$

Here we have replaced the Yukawa coupling h_f of the fermions f to the Higgs boson by $\sqrt{2} m_f / v_i$. In the Lagrangian the u, d, l, ν stand for all three generations. The Lagrangian defines the parameters $\xi_h^f, \xi_H^f, \xi_A^f$. They are defined in Table 2.1.

	Type I	Type II	Lepton-specific	Flipped
ξ_h^u	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$
ξ_h^d	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$
ξ_h^l	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$-\sin\alpha/\cos\beta$	$\cos\alpha/\sin\beta$
ξ_H^u	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$
ξ_H^d	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$
ξ_H^l	$\sin\alpha/\sin\beta$	$\cos\alpha/\cos\beta$	$\cos\alpha/\cos\beta$	$\sin\alpha/\sin\beta$
ξ_A^u	$\cot\beta$	$\cot\beta$	$\cot\beta$	$\cot\beta$
ξ_A^d	$-\cot\beta$	$\tan\beta$	$-\cot\beta$	$\tan\beta$
ξ_A^l	$-\cot\beta$	$\tan\beta$	$\tan\beta$	$-\cot\beta$

Table 2.1: The u, d, l (they stand for all three generations) Yukawas couplings to the neutral Higgs bosons h, H, A in the four different models.

2.3 Branching Ratios

For the determination of the Higgs decays widths, we also need the couplings to the gauge bosons. The couplings of the Higgs bosons to the gauge bosons are derived from

$$\sum_{i=1}^{2} (D_\mu \Phi_i)^\dagger (D^\mu \Phi_i) , \tag{2.50}$$

[4]Compare with "Theoretische Teilchenphysik" winter semester 2013/14, section 2.7.2.

with

$$D_\mu = \partial_\mu + i\frac{g}{2}\vec{\tau}\vec{W}_\mu + i\frac{g'}{2}B\mu \;, \tag{2.51}$$

where $\vec{\tau} = (\tau_1, \tau_2, \tau_3)^T$ in terms of the Pauli matrices. Using $\Phi_i = (\phi_i^+, 1/\sqrt{2}(v_i + \rho_i + i\eta_i))^T$ and

$$\rho_1 \;=\; Hc_\alpha - hs_\alpha \;, \qquad \rho_2 = Hs_\alpha + hc_\alpha \;, \tag{2.52}$$

$$\eta_1 \;=\; G^0 c_\beta - As_\beta \;, \qquad \eta_2 = G^0 s_\beta + Ac_\beta \;, \tag{2.53}$$

one finds for all four 2HDM models for the Higgs couplings to the gauge bosons normalized to the corresponding SM coupling $g_{H^{\rm SM}VV}$ $(V = W, Z)$

$$g_{hWW} \;=\; \sin(\beta-\alpha) g_{H^{\rm SM}WW} \;, \qquad g_{hZZ} = \sin(\beta-\alpha) g_{H^{\rm SM}ZZ} \;, \tag{2.54}$$

$$g_{HWW} \;=\; \cos(\beta-\alpha) g_{H^{\rm SM}WW} \;, \qquad g_{HZZ} = \cos(\beta-\alpha) g_{H^{\rm SM}ZZ} \;, \tag{2.55}$$

$$g_{AWW} \;=\; g_{AZZ} = 0 \;. \tag{2.56}$$

Note that in the 2HDM the Higgs couplings to the gauge bosons are always suppressed compared to the SM, and in the case of the pseudoscalar they vanish.

In the 2HDM we can have additional decays of the neutral Higgs bosons, such as Higgs-to-Higgs decays

$$h, H \to AA \;, \qquad H \to hh \;, \qquad h, H \to H^+H^- \;, \tag{2.57}$$

and Higgs decays into a Higgs boson and a gauge boson,

$$h, H \to ZA \;, \qquad h, H, A \to W^\pm H^\mp \;, \qquad A \to Zh, ZH \;. \tag{2.58}$$

The Higgs-to-Higgs decays require the derivation of the trilinear Higgs self-couplings (excercise!). They can be expressed in terms of the Higgs masses, M, α and β and can be found in Ref. [3] in Eqs. (E1)-(E6), (E11) and (E12). The Eqs. (E9), (E10), (E15) and (E16) contain the Higgs-Higgs-gauge boson couplings.

With these couplings at hand the decay widths and branching ratios can be calculated. There are several public programs which have implemented the calculation of the branching ratios of the 2HDM including the state-of-the-art higher order corrections, such as:

- `HDECAY`
  Ref.: A. Djouadi, J. Kalinowski and M. Spira, Comput. Phys. Commun. **108** (1998) 56 [hep-ph/9704448]; A. Djouadi, M. M. Mühlleitner and M. Spira, Acta Phys. Polon. B **38** (2007) 635 [hep-ph/0609292].
  webpage: `http://tiger.web.psi.ch/hdecay/`

- `2HDMC`
  Ref.: D. Eriksson, J. Rathsman and O. Stål, Comput. Phys. Commun. **181** (2010) 189 [arXiv:0902.0851 [hep-ph]]; D. Eriksson, J. Rathsman and O. Stål, Comput. Phys. Commun. **181** (2010) 833.
  webpage: `http://2hdmc.hepforge.org/`

Exercise: Determine the branching ratios. Produce plots for the branching ratios of respectively, $H, A, H^{\pm}$ as a function of their mass. Choose $m_h = 125$ GeV, $\sin(\beta - \alpha) = 0.9$ (to be close to the SM) and two values of $\tan\beta$, $\tan\beta = 2, 10$. For the time being, we are not interested in Higgs-to-Higgs decays, so that m_{12}^2 can be chosen arbitrarily. For simplicity choose H and $H^{\pm}$ to be close in mass (*e.g.* 150 and 600 GeV, respectively). As for the H mass, choose it such that in one case the $H \to H^+H^-$ and $H \to ZA$ decays are possible, in the other case not.

2.4 Higgs Production

For the neutral Higgs bosons of the 2HDM the same production mechanisms apply as in the SM. The dominant production process at the LHC is given by gluon fusion. The cross section can readily be taken over from the SM by making the appropriate replacements of the Higgs couplings to the top and bottom quarks. So we have for $\phi = h, H, A$

$$\sigma(gg \to \phi) = m_\phi^2 \delta(\hat{s} - m_\phi^2)\hat{\sigma} \ , \tag{2.59}$$

where $\hat{s}$ denotes the partonic c.m. energy and

$$\hat{\sigma} = \frac{G_F \alpha_s^2}{512\sqrt{2}\pi} \left| \sum_{q=t,b} g_{\phi qq} A_{1/2}^{\phi}(\tau_q) \right|^2 \ . \tag{2.60}$$

Here we have defined $\tau_q = 4m_q^2/m_\phi^2$ and the Yukawa coupling modification factors for the four 2HDM models are summarised in Tab. 2.1. Furthermore, we have the form factors

$$A_{1/2}^{h/H} = 2\tau[1 + (1-\tau)f(\tau)] \tag{2.61}$$

$$A_{1/2}^{A} = 2\tau f(\tau) \ , \tag{2.62}$$

with $f(\tau)$ defined in Eq. (1.24). For large quark masses, *i.e.* $\tau_q \ll 1$, they approach

$$A_{1/2}^{h/H} \to \frac{4}{3} \qquad \text{and} \qquad A_{1/2}^{A} \to 2 \ . \tag{2.63}$$

Note in particular, that while b-quark loops in the SM do not play a role in the type II and the flipped 2HDMs they can become crucial for large values of $\tan\beta$ as the Higgs couplings to down-type quarks are proportional to $\tan\beta$.

The production cross sections for h and H in gauge boson fusion and Higgs radiation can be obtained from the corresponding SM cross sections by multiplying them with the coupling modification factors $\sin^2(\beta - \alpha)$ for h and $\cos^2(\beta - \alpha)$ for H. The pseudocscalar does not couple to the gauge bosons and cannot be produced through these processes. The $t\bar{t}\phi$ production cross section is obtained from the SM formula by multiplying it with $(\cos\alpha/\sin\beta)^2$ for h, $(\sin\alpha/\sin\beta)^2$ for H and $\cot^2\beta$ for A in all four 2HDM models. In the type II and flipped 2HDMs also $b\bar{b}\phi$ production can become important due to the $\tan\beta$ enhanced Higgs couplings to b-quarks for large values of $\tan\beta$.

In the 2HDM there are further production mechanisms. Thus a resonantly produced heavy scalar can decay into a Higgs pair. Higgs bosons can also be produced in di-Higgs production through non-resonant channels and from gauge bosons produced in the Drell-Yan process, that subsequently decay into a Higgs pair. As the 2HDM has a large parameter space and the trilinear couplings are not given by the gauge couplings (as in supersymmetric theories) the double Higgs production cross sections can in general be larger than in the Minimal Supersymmetric extension of the SM (MSSM). Charged Higgs bosons finally can be produced in H^+H^- production or, if they are light enough, from top decays.

2.5 Type II 2HDM and the MSSM

As stated earlier the Higgs coupling structure to the fermions of the type II 2HDM is the same as in the MSSM. However, there are some crucial differences between these models:

- The type II 2HDM does not have a strict upper bound on the mass of the lightest Higgs boson. This is the case in the MSSM, as the Higgs potential, due to supersymmetry, is given in terms of the gauge couplings.
- For the same reason in the 2HDM the scalar self-couplings are now arbitrary.
- Also the mixing angle α, which in the MSSM is given in terms of $\tan\beta$ and the scalar and pseudocalar masses, is now arbitrary.
- In the MSSM the charged scalar and pseudoscalar masses are so close that the decay of the charged Higgs boson into a pseudoscalar and a real W is kinematically forbidden, while it is generally allowed in the type II 2HDM.

2.6 The Scalar Sector of the 2HDM

In its most general form the Higgs potential has 14 independent parameters. The Higgs doublets Φ_1 and Φ_2 are not physical observables, only the scalar mass eigenstates are physical particles. One therefore has the freedom to redefine the doublets, provided the form of their kinetic terms is preserved. Through such basis changes some of the parameters in the potential can be absorbed. They are essential to understand the number of physical parameters really present in the potential.

It is common to impose a variety of global symmetries on the 2HDM, *e.g.* in order to avoid tree-level FCNC couplings. Thereby the number of free parameters is reduced. It has been proven that there are only *six* such symmetries which have distinct effects on the scalar potential. The resulting six models have different physical implications:

> different spectra of scalars, different interactions with gauge bosons, in some cases predictions of massless axions or potential dark matter candidates.

The scalar potential determines the vacuum of the 2HDM. Contrary to the SM this vacuum is not unique. With two Higgs doublets it is possible that the model spontaneously breaks the CP symmetry. For certain parameter values of the potential is is also possible to have vacua that violate the electromagnetic symmetry and thus give mass to the photon. These have to be avoided of course. Even if only vacua are considered that preserve both CP and the usual gauge symmetries of the SM, the 2HDM has a rich vacuum structure. Thus some potentials can have two different electromagnetism-preserving minima, with different predictions for the masses of the gauge bosons for example. The 2HDM, however, has a feature which distinguishes it from other multi-Higgs models, such as SUSY or the 3HDM:

Its vacua are stable and no tunneling from a neutral, CP-conserving vacum to a deeper, CP- or charge-breaking vacuum is possible. Vice-versa, any CP- or charge-breaking minimum that one finds is guaranteed to be the global minimum of the model.

Not all values of the parameters of the 2HDM potential, however, ensure a stable minimum, unless the potential can be ensured to be bounded from below. This basic requirement imposes constraints on the quartic scalar couplings and translates in possibly severe bounds on the masses of the physical scalar particles through renormalization-group improvement.

2.6.1 Notations of the Scalar Potential

<u>*Notation 1:*</u> The most general renormalizable scalar potential can be written as [10]

$$\begin{aligned} V_H &= m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - m_{12}^2\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) + \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 \\ &\quad +\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &\quad + \left[\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + h.c.\right] . \end{aligned} \tag{2.64}$$

The parameters m_{11}^2, m_{22}^2 and $\lambda_{1,2,3,4}$ are real, whereas m_{12}^2 and $\lambda_{5,6,7}$ are complex. This leads to 14 parameters for the Higgs potential of Eq. (2.64). However, the freedom to redefine the basis means that in reality only eleven degrees of freedom are physical.

<u>*Notation 2:*</u> An alternative notation has been given in [11] and reads

$$V_H = \sum_{a,b=1}^{2} \mu_{ab}\Phi_a^\dagger\Phi_b + \frac{1}{2}\sum_{a,b,c,d=1}^{2} \lambda_{ab,cd}(\Phi_a^\dagger\Phi_b)(\Phi_c^\dagger\Phi_d) , \tag{2.65}$$

where by definition

$$\lambda_{ab,cd} = \lambda_{cd,ab} . \tag{2.66}$$

Hermiticity in Eq. (2.65) implies

$$\mu_{ab} = \mu_{ba}^* \qquad \text{and} \qquad \lambda_{ab,cd} = \lambda_{ba,dc}^* . \tag{2.67}$$

The notation of Eq. (2.65) is useful for the study of invariants, basis transformations and symmetries. The correspondance between notation 1 and 2 is given by

$$\begin{array}{lcllcl} \mu_{11} &=& m_{11}^2 , & \mu_{22} &=& m_{22}^2 , \\ \mu_{12} &=& -m_{12}^2 , & \mu_{21} &=& -m_{12}^{2*} , \\ \lambda_{11,11} &=& \lambda_1 , & \lambda_{22,22} &=& \lambda_2 , \\ \lambda_{11,22} = \lambda_{22,11} &=& \lambda_3 , & \lambda_{12,21} &=& \lambda_{21,12} = \lambda_4 , \\ \lambda_{12,12} &=& \lambda_5 , & \lambda_{21,21} &=& \lambda_5^* , \\ \lambda_{11,12} = \lambda_{12,11} &=& \lambda_6 , & \lambda_{11,21} &=& \lambda_{21,11} = \lambda_6^* , \\ \lambda_{22,12} = \lambda_{12,22} &=& \lambda_7 , & \lambda_{22,21} &=& \lambda_{21,22} = \lambda_7^* . \end{array}$$

<u>*Notation 3:*</u> While the previous notations consider the scalar doublets Φ_a $(a = 1, 2)$ individually, the third notation presented here emphasises the presence of field bilinears $\Phi_a^\dagger\Phi_b$ in the scalar potential. It can be written as [12]

$$V_H = \sum_{\mu=0}^{3} M_\mu r_\mu + \sum_{\mu,\nu=0}^{3} \Lambda_{\mu\nu} r_\mu r_\nu , \tag{2.68}$$

where

$$\Lambda_{\mu\nu} = \Lambda_{\nu\mu} \tag{2.69}$$

and

$$\begin{aligned} r_0 &= \frac{1}{2}(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2) \ , \\ r_1 &= \frac{1}{2}(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) = \Re(\Phi_1^\dagger\Phi_2) \\ r_2 &= -\frac{i}{2}(\Phi_1^\dagger\Phi_2 - \Phi_2^\dagger\Phi_1) = \Im(\Phi_1^\dagger\Phi_2) \\ r_3 &= \frac{1}{2}(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2) \ . \end{aligned} \tag{2.70}$$

This notation is convenient for studies of features such as the existence and number of minima of the scalar potential. Since the Yukawa couplings involve the Higgs doublets individually rather than bilinears, notation 3 cannot be applied for studies of the full theory with both scalars and fermions. The correspondence between notations 1 and 3 is given by

$$M_\mu = (m_{11}^2 + m_{22}^2, -2\Re(m_{12}^2), 2\Im(m_{12}^2), m_{11}^2 - m_{22}^2) \ , \tag{2.71}$$

$$\Lambda_{\mu\nu} = \begin{pmatrix} (\lambda_1+\lambda_2)/2+\lambda_3 & \Re(\lambda_6+\lambda_7) & -\Im(\lambda_6+\lambda_7) & (\lambda_1-\lambda_2)/2 \\ \Re(\lambda_6+\lambda_7) & \lambda_4+\Re(\lambda_5) & -\Im(\lambda_5) & \Re(\lambda_6-\lambda_7) \\ -\Im(\lambda_6+\lambda_7) & -\Im(\lambda_5) & \lambda_4-\Re(\lambda_5) & -\Im(\lambda_6-\lambda_7) \\ (\lambda_1-\lambda_2)/2 & \Re(\lambda_6-\lambda_7) & -\Im(\lambda_6-\lambda_7) & (\lambda_1+\lambda_2)/2-\lambda_3 \end{pmatrix} . \tag{2.72}$$

In the following we will discuss constraints on the 2HDM Higgs potential and the implications on its parameter values.

2.6.2 Stability of the 2HDM Potential

In order to ensure the stability of the 2HDM potential, we have to make sure that it is bounded from below, *i.e.* that there is no direction in field space along which the potential tends to minus infinity. The existence of a stable minimum, around which perturbative calculations can be performed, is a basic requirement for any physical theory. The scalar potential of the SM satisfies this requirement through the trivial condition $\lambda > 0$, where λ is the quartic coupling of the SM scalar potential. The 2HDM scalar potential of Eq. (2.64) is much more complicated than the one of the SM. All possible directions along which the fields Φ_1 and Φ_2, respectively their eight component fields, tend to arbitrarily large values, have to be studied. In order to have a non-trivial minimum, *i.e.* the fields Φ_i acquire non-zero VEVs, two conditions have to be fulfilled: The quartic part of the scalar potential, V_4, is positive for arbitrarily large values of the component fields, but the quadratic part of the scalar potential, V_2, can take negative values for at least some values of the fields. In this respect, demanding $V_4 > 0$ for all $\Phi_i \to \infty$ may be a too strong requirement, since several interesting models are excluded by it. Thus in tree-level SUSY potentials there is a direction, $\langle\Phi_1\rangle = \langle\Phi_2\rangle$ for which $V_4 = 0$. A simple way to obtain *necessary* conditions on the quartic parameters of the potential is to study its behaviour along specific field directions. Considering for example the direction $|\Phi_1| \to \infty$ and $|\Phi_2| = 0$, the expression Eq. (2.64) for the potential obviously leads to the conclusion that one can have positive values for V_4 if and only if $\lambda_1 \geq 0$. Likewise, the direction $|\Phi_1| = 0$ and $|\Phi_2| \to \infty$ gives the condition $\lambda_2 \geq 0$. By studying several such directions it is possible to reach other conditions on the couplings, arriving at

$$\begin{aligned} &\lambda_1 \geq 0\,, \qquad \lambda_2 \geq 0 \\ &\lambda_3 \geq -\sqrt{\lambda_1\lambda_2}\,, \qquad \lambda_3+\lambda_4-|\lambda_5| \geq -\sqrt{\lambda_1\lambda_2}\,, \end{aligned} \tag{2.73}$$

where λ_5 has been taken real. In potentials, where $\lambda_6 = \lambda_7 = 0$ these are actually necessary and sufficient conditions to ensure the positivity of the quartic potential along all directions.

The conditions Eq. (2.73) have been obtained through a tree-level analysis. The inclusion of higher order corrections is done by considering only the tree-level expressions Eq. (2.73), but taking the values of the couplings which appear in these expressions at different renormalization scales. One hence takes the bounds of Eq. (2.73) and runs the couplings therein, using the β-functions of the model along a range of scales μ, *i.e.* from the weak scale M_Z to an upper scale Λ. At all scales in the interval chosen the bounds must hold. Note, that combinations of parameters which at one scale might be acceptable would violate the bounds at another scale.

Such an analysis has been performed for the SM. The Higgs potential quartic coupling λ at the scale Q is given in terms of the β-function by

$$\frac{d\lambda}{d\ln Q} = \beta(g_i)\,, \tag{2.74}$$

where g_i generically denotes the couplings of the model. The β-function is derived by considering the quantum corrections to the Higgs potential, and reads

$$\begin{aligned} 16\pi^2\beta &= 24\lambda^2 - (3g'^2+9g^2-12y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2g^2 + \frac{9}{8}g^4 - 6y_t^4 \\ &\quad + \text{higher order terms}\,. \end{aligned} \tag{2.75}$$

Here g and g' denote the SM electroweak gauge couplings and y_t the top Yukawa coupling. The β-function has a sizeable negative contribution from the top quark Yukawa coupling. As the top is so heavy, this term tends to decrease the value of λ at higher renormalization scales. If the starting value of λ at the weak scale is too small, the coupling can become negative at some higher scale and the potential would be unbounded from below. This allows us to put a lower bound on λ and thus on the Higgs mass. Let us have a closer look at this. For small masses (hence small λ – g and g' are anyway small) the renormalization group equation (RGE) Eq. (2.75) is dominated by y_t, hence

$$16\pi^2\frac{d\lambda}{d\ln Q} = -6y_t^4\,. \tag{2.76}$$

Integration leads to

$$\lambda(Q) = \lambda_0 - \frac{\frac{3}{8\pi^2}y_0^4\ln\frac{Q}{Q_0}}{1-\frac{9}{16\pi^2}y_0^2\ln\frac{Q}{Q_0}}\,. \tag{2.77}$$

Therefore λ decreases with Q. In order to have vacuum stability we have to require

$$\Lambda \leq v e^{4\pi^2 M_H^2/(3y_t^4v^2)}\,. \tag{2.78}$$

New Physics must appear before this point to ensure vacuum stability. For a fixed value of Λ this leads to a lower bound on M_H.

If the starting value of λ is too large, the RG evolution of the coupling will increase its value immensely and eventually the theory becomes non-perturbative. We will come back to this point later, when we discuss unitarity bounds. The RG analysis thus allows to impose higher and lower bounds on the masses of the Higgs particles.

In the 2HDM the same type of phenomena can occur. If for example the Φ_1 only couples to the up-type quarks, the β-function for the λ_1 quartic coupling will have a large negative top Yukawa contribution, and a similar analysis to the SM case will hold. However, in the 2HDM many other quartic couplings are present and more bounds need to be obeyed. Nevertheless the main conclusions hold: Smaller values for some of the λ_i at the weak scale are disfavoured as they lead to unbounded from below potentials at higher scales. Large values of these couplings lead to Landau poles at high scales and thus the breakdown of perturbation theory. These translate into bounds on the several Higgs masses.

2.6.3 Vacuum Stability

In the SM, apart from the trivial minimum, there is only *one* possible type of minimum. In the 2HDM, however, there exist three types of vacua:[5]

- "Normal" (N) vacua, with VEVs which do not have any complex relative phase and can thus trivially be rendered real:

$$\langle\Phi_1\rangle_N = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix}, \quad \langle\Phi_2\rangle_N = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix}, \tag{2.79}$$

  where $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV and $\tan\beta = v_2/v_1$.

- CP breaking vacua, where the VEVs have a relative complex phase,

$$\langle\Phi_1\rangle_{CP} = \begin{pmatrix} 0 \\ \frac{\bar{v}_1}{\sqrt{2}} e^{i\theta} \end{pmatrix}, \quad \langle\Phi_2\rangle_{CP} = \begin{pmatrix} 0 \\ \frac{\bar{v}_2}{\sqrt{2}} \end{pmatrix}, \tag{2.80}$$

  where $\bar{v}_1$ and $\bar{v}_2$ are real.

- Charge breaking (CB) vacua, in which one of the VEVs carries electric charge,

$$\langle\Phi_1\rangle_{CB} = \begin{pmatrix} \frac{\alpha}{\sqrt{2}} \\ \frac{v_1'}{\sqrt{2}} \end{pmatrix}, \quad \langle\Phi_2\rangle_{CB} = \begin{pmatrix} 0 \\ \frac{v_2'}{\sqrt{2}} \end{pmatrix}, \tag{2.81}$$

  with real numbers v_1', v_2', α. Because of the presence of a non-zero VEV in an upper component (charged) of the fields, this vacuum breaks electrical charge conservation, so that the photon acquires a mass. Such a vacuum therefore has to be avoided.

[5]Any stationary point of the potential, regardless of whether it is a minimum or not, is considered a vacuum.

The minima of the potential are defined by solving the minimization conditions. With the potential written in terms of $\tilde{v}_i$ for any of the three sets Eqs. (2.79)-(2.81) a stationary point of the potential is found if the set of equations $\partial V/\partial \tilde{v}_i = 0$ has solutions. The different CP and CB stationary points are determined by a set of three equations and a normal one by only two. Since the 2HDM potential depends on eight real component fields, in fact, any stationary point should be the solution of a set of eight equations on eight unknowns. As one can always choose the simplified forms of the VEVs given in Eqs. (2.79)-(2.81), most of those equations are trivially satisfied. It has been shown in [13] that the charge breaking VEVs can always be obtained analytically and are given by

$$\begin{pmatrix} v_1'^2 + \alpha^2 \\ v_2'^2 \\ v_1' v_2' \end{pmatrix} = 2 \begin{pmatrix} \lambda_1 & \lambda_3 & 2\Re(\lambda_6) \\ \lambda_3 & \lambda_2 & 2\Re(\lambda_7) \\ 2\Re(\lambda_6) & 2\Re(\lambda_7) & 2(\lambda_4 + \Re(\lambda_5)) \end{pmatrix}^{-1} \begin{pmatrix} m_{11}^2 \\ m_{22}^2 \\ -2\Re(m_{12}^2) \end{pmatrix} . \tag{2.82}$$

This implies, that if Eq. (2.82) has a solution, then this is unique up to trivial sign changes ($\alpha \to -\alpha$, $v_1' \to -v_1'$, $v_2' \to -v_2'$) with no physical impact. Charge breaking is hence impossible in several symmetry-constrained 2HDM.

The CP breaking VEVs can be obtained analytically in terms of the parameters of the potential. Assuming potentials where the CP symmetry is defined one obtains

$$\begin{pmatrix} \bar{v}_1^2 \\ \bar{v}_2^2 \\ \bar{v}_1 \bar{v}_2 \cos\theta \end{pmatrix} = 2 \begin{pmatrix} \lambda_1 & \lambda_3 + \lambda_4 - \Re(\lambda_5) & 2\Re(\lambda_6) \\ \lambda_3 + \lambda_4 - \Re(\lambda_5) & \lambda_2 & 2\Re(\lambda_7) \\ 2\Re(\lambda_6) & 2\Re(\lambda_7) & 4\Re(\lambda_5) \end{pmatrix}^{-1} \begin{pmatrix} m_{11}^2 \\ m_{22}^2 \\ -2\Re(m_{12}^2) \end{pmatrix} \tag{2.83}$$

Up to physically irrelevant sign changes the CP vacuum is unique.

The most difficult vacuum to be solved is the normal one. For many potentials the minimization conditions cannot be solved analytically. The equations $\partial V/\partial v_1 = 0$ and $\partial V/\partial v_2 = 0$ result for the most general 2HDM potential in

$$\begin{aligned} m_{11}^2 v_1 - \Re(m_{12}^2) v_2 + \frac{\lambda_1}{2} v_1^3 + \frac{\lambda_{345}}{2} v_1 v_2^2 + \frac{1}{2}[3\Re(\lambda_6) v_1^2 v_2 + \Re(\lambda_7) v_2^3] &= 0 & (2.84) \\ m_{22}^2 v_2 - \Re(m_{12}^2) v_1 + \frac{\lambda_2}{2} v_2^3 + \frac{\lambda_{345}}{2} v_2 v_1^2 + \frac{1}{2}[\Re(\lambda_6) v_1^3 + 3\Re(\lambda_7) v_2 v_1^2] &= 0\,, & (2.85) \end{aligned}$$

with $\lambda_{345} = \lambda_3 + \lambda_4 + \Re(\lambda_5)$.

With the possibility of minima of different natures in theories with more than one scalar, the theory may allow for tunneling from one minimum to another. In the 2HDM therefore the question arises: Can the vacua of different natures coexist with one another? Could one tunnel from a normal minimum to a deeper charge-breaking one? In other words, given a minimum in the 2HDM, is it stable? In Refs. [13, 14, 15] it has been shown:

- Suppose we have a potential where a normal stationary point and a charge breaking one exist, and with the VEVs given by Eqs. (2.82) and (2.84), (2.85), then the difference in the values of the scalar potential at both those vacua is given by

$$V_{CB} - V_N = \left(\frac{M_{H^\pm}^2}{4v^2}\right)_N \underbrace{[(v_1' v_2 - v_2' v_1)^2 + \alpha^2 v_2^2]}_{>0} . \tag{2.86}$$

Note that $(M_{H^\pm}^2/v^2)_N$ is the ratio of the squared mass of the charged scalar to the sum of the square of the VEVs, $v^2 = v_1^2 + v_2^2$, as computed in the normal stationary point. This implies: If the normal stationary point is a minimum, which implies $M_{H^\pm}^2 > 0$, then one will necessarily have $V_{CB} - V_N > 0$. Hence, if there is a normal minimum, any CB stationary point will lie above it. The normal minimum is stable against charge breaking. It was also proven in [13] that in such a case the CB stationary point is necessarily a saddle point. Therefore normal and CB minima cannot co-exist in the 2HDM. In case the set of parameters is chosen such that the global minimum of the potential breaks charge, there are no normal minima.

- In case we have a potential where a normal stationary point and a CP breaking one exist with the VEVs given by Eqs. (2.83) and (2.84), (2.85), the difference in the values of the scalar potential at both those vacua is given by

$$V_{CP} - V_N = \left(\frac{M_A^2}{4v^2}\right)_N \underbrace{\left[(\bar{v}_1 v_2 \cos\theta - \bar{v}_2 v_1)^2 + \bar{v}_1^2 v_2^2 \sin^2\theta\right]}_{>0} . \tag{2.87}$$

Note that $(M_A^2/4v^2)_N$ is the ratio of the squared mass of the pseudoscalar to the sum of the square of the VEVs, $v^2 = v_1^2 + v_2^2$, as computed in the normal stationary point. Therefore, if the normal stationary point is a minimum, which implies that $M_A^2 > 0$ then we necessarily have $V_{CP} - V_N > 0$. If there is a normal minimum, any CP stationary point will hence be above it. The normal minmum ist stable against CP breaking. In [16] it was proven that in that case the CP stationary point is necessarily a saddle point, and therefore normal and CP minima cannot co-exist in the 2HDM. If the set of parameters of the potential is chosen such that the global minimum of the potential breaks CP, then there are no normal minima.

- Also no CB and CP minima can co-exist. This is because for the CP vacuum the square of the charged Higgs mass is given by

$$(M_{H^\pm}^2)_{CP} = -\frac{1}{2}[\lambda_4 - \Re(\lambda_5)](\bar{v}_1^2 + \bar{v}_2{}^2) , \tag{2.88}$$

whereas in a CB vacuum one of the squared mass matrix eigenvalues is

$$M_{CB}^2 = \frac{1}{2}[\lambda_4 - \Re(\lambda_5)](v_1'^2 + v_2'^2 + \alpha^2) . \tag{2.89}$$

The sign of $\lambda_4 - \Re(\lambda_5)$ determines that both these vacua cannot be simultaneously minima. Therefore if a CP minimum exists the (unique) CB stationary point, if it exists, cannot be a minimum as well, and vice-versa.

- The normal minimization conditions, however, allow for multiple solutions, so that one can have an N_1 vacuum with VEVs $\{v_{1,1}, v_{2,1}\}$ and an N_2 vacuum with different VEVs $\{v_{1,2}, v_{2,2}\}$. The difference in the values of the potential in these two vacua is given by

$$V_{N_2} - V_{N_1} = \frac{1}{4}\left[\left(\frac{M_{H^\pm}^2}{v^2}\right)_{N_1} - \left(\frac{M_{H^\pm}^2}{v^2}\right)_{N_2}\right](v_{1,1}v_{2,2} - v_{2,1}v_{2,2})^2 , \tag{2.90}$$

where $(M_{H^\pm}^2/v^2)_{N_1}$ is the ratio of the charged mass squared to the sum of the square VEVs, $(v^2)_{N_1} = v_{1,1}^2 + v_{2,1}^2$, as computed in the N_1 stationary point, and analogously for $(M_{H^\pm}^2/v^2)_{N_2}$. This equation shows that nothing favours N_1 over N_2. The deepest stationary point is simply determined by the values of the parameters. This is to be expected as the two vacua have the same symmetries. It was proven in [16] that it is possible to have two co-existing normal minima. On the other hand it is very easy to find a set of parameters where N_1 would be the global minimum, with N_2 above it or not even existing.

In summary, for the 2HDM vacua the following holds:

- Minima of different natures cannot coexist in the 2HDM.
- Whenever a normal minimum exists in the 2HDM, the global minimum of the potential is normal. No tunneling to a deeper CB or CP minimum is possible.
- If a CP (CB) violating minimum exists, it is the global minimum of the theory and thouroughly stable. No tunnelling to a deeper normal or CB (CP) minimum can occur.

2.6.4 Unitarity Constraints

We have already seen that the condition that the potential must have a minimum and is not unbounded from below leads to constraints on the parameter values of the Higgs potential. Another theoretical constraint arises from the requirement, that all the (tree-level) scalar-scalar scattering amplitudes must respect unitarity. In the SM this requirement is equivalent to ensuring that the quartic coupling in the scalar potential is not too large. This leads then to an upper bound on the Higgs boson mass. We can see this by looking again at the RGE in Eq. (2.75). For large masses (and hence large λ), the RGE is dominated by the λ term, hence

$$16\pi^2 \frac{d\lambda}{d\ln Q} = 24\lambda^2 \; . \tag{2.91}$$

This is solved by

$$\lambda(Q) = \frac{M_H^2}{2v^2 - \frac{3}{2\pi^2} M_H^2 \ln \frac{Q}{v}} \; . \tag{2.92}$$

The coupling λ hence increases with Q. It diverges at the Landau pole. We therefore have to require that new physics appears before this point, in order to restore stability, hence

$$\Lambda \leq v e^{4\pi^2 v^2/(3M_H^2)} \; . \tag{2.93}$$

For fixed Λ this translates into an upper bound on M_H. Extending this bound to the 2HDM is complicated. Due to the richer scalar spectrum many scattering amplitudes need to be taken into account. Furthermore the existence of many quartic couplings makes things more complicated. This leads to an analysis of the eigenvalues of the S matrix for scalar-scalar scattering amplitudes. The relevant ones are given by

$$a_\pm = \frac{3}{2}(\lambda_1+\lambda_2) \pm \sqrt{\frac{9}{4}(\lambda_1-\lambda_2)^2 + (2\lambda_3+\lambda_4)^2}\ , \tag{2.94}$$

$$b_\pm = \frac{1}{2}(\lambda_1+\lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1-\lambda_2)^2 + 4\lambda_4^2}\ , \tag{2.95}$$

$$c_\pm = \frac{1}{2}(\lambda_1+\lambda_2) \pm \frac{1}{2}\sqrt{(\lambda_1-\lambda_2)^2 + 4\lambda_5^2}\ , \tag{2.96}$$

$$e_1 = \lambda_3 + 2\lambda_4 - 3\lambda_5\ , \tag{2.97}$$

$$e_2 = \lambda_3 - \lambda_5\ , \tag{2.98}$$

$$f_+ = \lambda_3 + 2\lambda_4 + 3\lambda_5\ , \tag{2.99}$$

$$f_- = \lambda_3 + \lambda_5\ , \tag{2.100}$$

$$f_1 = \lambda_3 + \lambda_4\ , \tag{2.101}$$

$$p_1 = \lambda_3 - \lambda_4\ . \tag{2.102}$$

The requirement of tree-level perturbative unitarity leads to

$$|a_\pm|, |b_\pm|, |c_\pm|, |f_\pm|, |e_{1,2}|, |f_1|, |p_1| < 8\pi\ . \tag{2.103}$$

2.6.5 Further Constraints

The Higgs bosons of the 2HDM also contribute to the electroweak precision observables. New physics contributions to these observables can conveniently be parametrized in terms of the oblique parameters. With the vacuum polarization tensors written as

$$\Pi^{\mu\nu}_{VV'}(q) = g^{\mu\nu} A_{VV'}(q^2) + q^\mu q^\nu B_{VV'}(q^2)\ , \tag{2.104}$$

where VV' is either $\gamma\gamma$, γZ, ZZ or W^+W^- and $q = (q^\alpha)$ is the four-momentum of the gauge bosons, and defining

$$\bar{A}_{VV'}(q^2) = A_{VV'}(q^2)|_{\text{2HDM}} - A_{VV'}(q^2)|_{\text{SM}}\ , \tag{2.105}$$

the oblique parameters S, T, U, V, W, X of the 2HDM can be expressed in terms of the $\bar{A}_{VV'}$. Electroweak precision constraints lead to

$$m_A = m_{H^\pm} \tag{2.106}$$

$$\sin(\beta-\alpha) = 1 \quad \Rightarrow \quad m_{H^\pm} = m_H \tag{2.107}$$

$$\sin(\beta-\alpha) = 0 \quad \Rightarrow \quad m_{H^\pm} = m_h\ . \tag{2.108}$$

Other constraints arise from the measurement of muon anomalous magnetic moment and from B-physics. In particular the charged Higgs boson can have a significant effect on B-physics observables. For all four models without FCNC the Yukawa couplings of the charged Higgs boson can be written as

$$\mathcal{L}_{H^\pm} = -H^+\left(\frac{\sqrt{2}V_{ud}}{v}\bar{u}(m_u X P_L + m_d Y P_R)d + \frac{\sqrt{2}m_l}{\nu} Z\bar{\nu}_L l_R\right) + \text{h.c.}\ . \tag{2.109}$$

The values of X, Y and Z are given in Table 2.2 for the various models. In Type I the couplings to all fermions are suppressed if $\tan\beta \gg 1$, implying a fermiophobic charged

	Type I	Type II	Lepton-specific	Flipped
X	$\cot\beta$	$\cot\beta$	$\cot\beta$	$\cot\beta$
Y	$\cot\beta$	$-\tan\beta$	$\cot\beta$	$-\tan\beta$
Z	$\cot\beta$	$-\tan\beta$	$-\tan\beta$	$\cot\beta$

Table 2.2: The parameters X, Y and Z for the four models without FCNC.

Higgs. In the same limit one has in the lepton-specific model a quark-phobic but lepto-philic charged Higgs, which could lead to a huge branching ratio for $H^\pm \to \tau^\pm \nu_\tau$. In both models the quark-phobic nature of the charged Higgs eliminates constraints from rare B decays. In the type II and flipped model large contributions to rare B decays are possible. The data on $B \to X_s\gamma$ lead for these models then to a constraint on the charged Higgs mass given by

$$m_{H^\pm} \gtrsim 360 \text{ GeV} . \tag{2.110}$$

All models are constraint by the data on $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixing. The measured

$$R_b = \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to q\bar{q})} \tag{2.111}$$

constrains

$$\tan\beta \gtrsim 1 . \tag{2.112}$$

Last but not least there are constraints from the Higgs data from LEP, Tevatron and LHC. It has to be made sure that the 2HDM Higgs sector is not in conflict with the reported exclusion limits and the Higgs data of the discovered 126 GeV scalar. Thus at LEP it was looked for the production of charged Higgs bosons in

$$e^+e^- \to H^+H^- , \tag{2.113}$$

with the charged Higgs decaying into $\tau^+\nu_\tau$. For any model the non-observaton of the charged Higgs leads then to a constraint of $m_{H^\pm} \gtrsim 80$ GeV. And for the lepton-specific one, the limit is $m_{H^\pm} \gtrsim 94$ GeV. At ATLAS and CMS a charged Higgs boson is looked for in

$$pp \to \bar{t}t \to \bar{b}bW^+H^- . \tag{2.114}$$

The non-observation translates into exclusion limits in the $\tan\beta$-$m_{H^\pm}$ plane.

There are dedicated public programs available that allow to check for the Higgs data constraints, namely `HiggsBounds` [17, 18, 19] and `HiggsSignals` [20]. The program `HiggsBounds` requires as inputs the effective couplings of the Higgs bosons of the investigated model, normalized to the corresponding SM values, as well as the masses, the widths and the branching ratios of the Higgs bosons. This allows then to check for the compatibility with the non-observation of the 2HDM Higgs bosons, in particular whether or not the Higgs spectrum is excluded at the 95% confidence level (CL) in view of the LEP, Tevatron and LHC measurements. The package `HiggsSignals` uses the same input and validates the compatibility of the SM-like Higgs boson with the Higgs observation data. A p-value is given, which when demanded to be at least 0.05 corresponds to a non-exclusion at 95% CL.

A tool for performing scans of the parameter space of scalar sectors is given by `ScannerS` [24]. It automatises scans for tree-level renormalizable scalar potentials. It is interfaced with

- `SuShi` [21] for the Higgs production at NNLO in gluon fusion and associated production with $b\bar{b}$.
- `HDECAY` [22] for the computation of the Higgs decays.
- `Superiso` [23] for the check of some flavour physics observables.
- `HiggsBounds` for the limits from the Higgs searches at LEP, Tevatron and the LHC.
- `HiggsSignals` for the signal rates at the Tevatron and LHC.

Furthermore `ScannerS` checks for the global minimum and has implemented checks of the constraints from vacuum stability (potential bounded from below), unitarity, electroweak precision observables and some alternative sources for B-physics constraints. The webpage of the program is given by:
`http://www.hepforge.org/archive/scanners/`

In order to check for the 2HDM allowed parameter space with the available LHC data as given in September 2014, a random scan has been performed, setting $m_h = 125.9$ GeV, over the parameter values

$$\begin{aligned} 50\text{ GeV} \le m_{H^\pm} &\le 1\text{ TeV} \\ m_h + 5\text{ GeV} \le m_A, m_H &\le 1\text{ TeV} \\ -900^2\text{ GeV}^2 \le m_{12}^2 &\le 900^2\text{ GeV}^2 \\ 0.5 \le \tan\beta &\le 50 \\ -\frac{\pi}{2} \le \alpha &\le \frac{\pi}{2} \, . \end{aligned} \tag{2.115}$$

The theoretical and pre-LHC experimental constraints have been imposed. The branching ratios and production rates at the LHC have been calculated and the collider constraints have been checked with `HiggsBounds` and `HiggsSignals`. The result for the type II model is shown in Fig. 2.1. As can be inferred from the plot there are two regions that are favoured. One is given by the SM-like limit. Here $\sin(\beta - \alpha) = 1$, leading to $\kappa_F = 1$ and $\kappa_V = 1$, where κ_x denotes the 2HDM coupling of the SM-like h with mass around 126 GeV to the SM particles x normalized to the corresponding coupling of the SM Higgs boson with same mass. Hence all tree-level coupling to fermions and massive gauge bosons are as in the SM. The other favoured region is the so-called 'wrong-sign' limit [25], as here

$$\kappa_D \kappa_V < 0 \qquad \text{or} \qquad \kappa_U \kappa_V < 0 \, . \tag{2.116}$$

This means that the Yukawa couplings and couplings to massive gauge bosons have a relative minus sign. This can be easily checked by re-writing the coupling factor κ_D to down-type fermions, which in the type II model is given by

$$\kappa_D = -\frac{\sin\alpha}{\cos\beta} = -\sin(\beta+\alpha) + \cos(\beta+\alpha)\tan\beta \tag{2.117}$$

and analogously

$$\kappa_U = \frac{\cos\alpha}{\sin\beta} = \sin(\beta+\alpha) + \cos(\beta+\alpha)\cot\beta \, . \tag{2.118}$$

For $\sin(\beta+\alpha) = 1$ this leads to $\kappa_D = -1$ ($\kappa_U = 1$) and with

$$\kappa_V = \sin(\beta - \alpha) = \frac{\tan^2\beta - 1}{\tan^2\beta + 1} \tag{2.119}$$

we have $\kappa_V \ge 0$ if $\tan\beta \ge 1$.

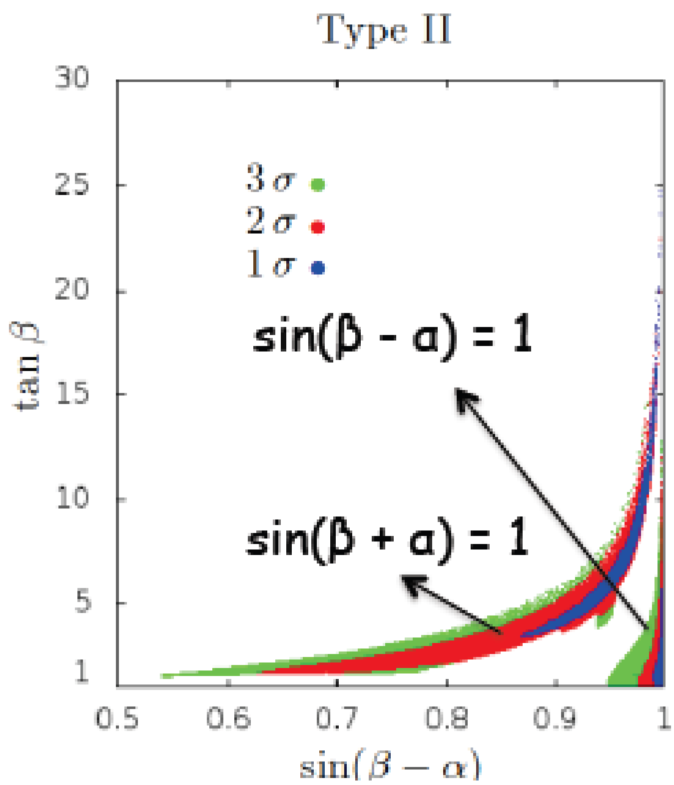

Figure 2.1: Allowed parameters space of the type II 2HDM model as in September 2014. Taken from the talk given by R. Santos at HiggsDays 2014 in Santander.

Chapter 3

Supersymmetry

3.1 Literature

- H. Kalka und G. Soff, *Supersymmetrie*, Teubner Studienbücher 1997. (For notation, SUSY algebra etc.)
- Wess & Bagger, *Supersymmetry and Supergravity*, Princeton University Press.
- H.P. Nilles, *Sypersymmetry, Supergravity and Particle Physics*, Phys. Rep. **110** (1984) 1.
- H.E. Haber and G.L. Kane, *The Search for Supersymmetry: Probing Physics beyon the Standard Model*, Phys. Rep. **117** (1985) 75.
- S. Martin, *A Sypersymmetry Primer*, hep-ph/9709356.
- S. Dawson, *The MSSM and why it works*, hep-ph/9712464.
- S. Dawson, J.F. Gunion, H.E. Haber and G. Kane, *The Higgs Hunter's Guide*, Frontiers in Physics.
- M. Drees, R.M. Godbole and P. Roy, *Theory and Phenomenology of Sparticles*, World Scientific.
- H. Baer and X. Tata, *Weak Scale Supersymmetry*, Cambridge University Press.
- A. Djouadi, *The Anatomy of electro-weak symmetry breaking. II. The Higgs bosons in the minimal supersymmetric model*, Phys. Rept. **459** (2008) 1 [hep-ph/0503173].

3.2 The Standard Model and its Flaws

The SM of particle physics describes the today known fundamental structures of matter and forces, with the exception of gravity. It is a consistent renormalizable quantum field theory and describes a large amount of experimental data over a large range of energies. It has been tested at the quantum level in electroweak precision experiments. Still, the SM is incomplete. A few of the flaws and open questions are listened here (and have also partly been discussed in Section 1.10):

- Experimental arguments:
  - Neutrinos are not massless.
  - Astrophysical and cosmological data point towards the existence of Dark Matter (DM).
  - Gravity cannot be described in the framework of the SM.
- Theory arguments:
  - Is there a scale where the strong, weak and electromagnetic forces unify? – Not in the SM.
  - Is there a dynamical explanation for the mass and mixing patterns?
  - Is there a dynamical explanation for the Higgs mechanism and EW symmetry breaking? In the SM the Higgs potential is added by hand.
  - How can the value of the Higgs mass be explained in a natural way, without fine-tuning, if the SM is valid up to high energies? (Hierarchy problem)

A few of these open questions and their solution within supersymmetry (SUSY) are discussed in the following.

3.3 The Hierarchy Problem

Let us first have a look at the radiative corrections in quantum electrodynamics (QED), which is described by the Lagrangian

$$\mathcal{L} = i\bar{\psi}\gamma_\mu D^\mu \psi - m\bar{\psi}\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \ . \tag{3.1}$$

Here ψ denotes the 4-component spinor of a Dirac fermion, γ_μ the Dirac matrices andD_μ the covariant derivative, given by

$$D_\mu = \partial_\mu + iq_\psi A_\mu(x) \ , \tag{3.2}$$

with the vector potential $A_\mu(x)$ and the coupling constant q_ψ, which is identified with the charge of the Dirac field. The field strength tensor $F_{\mu\nu}$, expressed through the vector potential A_μ, reads

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \ . \tag{3.3}$$

The theory decsribes the interaction between fermions and a photon. These interactions conserve chirality. This means, that a left-handed fermion remains left-handed, when it emits (or absorbs) a photon, and a right-handed one remains right-handed. The kinetic term also conserves chirality. Since the emission or the absorption of a photon cannot change the chirality of a fermion, this immediately entails that any radiative correction to the fermion mass (which is an operator that connects ψ_L and ψ_R) has to vanish in all orders of perturbation theory, if the fermion mass is equal to zero. This means that

$$\delta m \sim m \ . \tag{3.4}$$

The loop integrals which appear in the computation of the radiative corrections are divergent. If we regularize the divergence through a cut-off Λ, then we find by applying dimensional analysis, that the dependence on the cut-off parameter must be given by

$$\delta m \sim m \ln \frac{\Lambda}{m} \; . \tag{3.5}$$

Naive dimensional analysis would have lead to $\delta m \sim \Lambda$. Bechause of the chiral symmetry the actual divergence is milder. The chiral symmetry protects the fermion masses against large radiative corrections. Analogously the gauge invariance of the photon protects against acquiring a mass. In fact, in QED the leading divergence in all quantities is logarithmic.

The structure of the divergence in field theories with elementary scalars, however, is very different. Let us look at the radiative corrections to the scalar mass in a toy model, which is described by the following Lagrangian

$$\mathcal{L}_1 = \bar{\psi}(i\partial\!\!\!/ - m_F)\psi + \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}m_S^2 S^2 - \frac{\lambda_F}{2}\bar{\psi}\psi S \; , \tag{3.6}$$

where the Dirac spinor ψ describes a fermion of mass m_F, S a real scalar field of mass m_S, and the coupling between the scalar field S and two fermions is given by the coupling constant λ_F. The radiative corrections to the fermion and scalar masses are given by the diagrams displayed in Fig. 3.1. Depending on the cut-off parameter Λ we obtain for the correction δm_F to the fermion mass and δm_S^2 to the scalar mass

$$\begin{aligned} \delta m_F &= -\frac{3\lambda_F^2 m_F}{64\pi^2} \ln \frac{\Lambda^2}{m_F^2} + ... &(3.7)\\ \delta m_S^2 &= -\frac{\lambda_F^2}{8\pi^2}\left[\Lambda^2 - m_F^2 \ln \frac{\Lambda^2}{m_F^2}\right] + ... \; . &(3.8) \end{aligned}$$

While the radiative correction to the fermion mass has the above discussed logarithmic divergence, the correction to the scalar mass is *quadratically* divergent. In the framework of

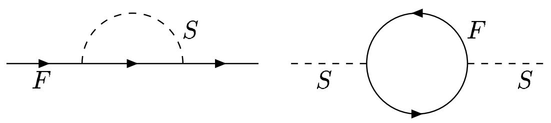

Figure 3.1: Radiative corrections to the fermion mass (left) and to the boson mass (right).

the SM there are further quadratically divergent contributions to the scalar mass from the gauge boson loops and other fermion loops.

The quadratic divergences to the scalar mass, within the SM the ones to the Higgs boson mass, are in principle no problem. Since the SM is renormalizable radiative corrections can be treated to any accuracy. What is hence the problem? Let us have a look at the radiatively corrected Higgs boson mass at 1-loop level,

$$m_{H_{SM}}^2(\text{phys}) \approx m_{H_{SM}}^2 + \frac{c}{16\pi^2}\Lambda^2 \; , \tag{3.9}$$

where $m^2_{H_{SM}}$ is the quadratic Higgs mass of the Lagrangian and the second term is the quadratically divergent correction to the Higgs boson mass. The terms logarithmic in Λ have been neglected for simplicity. The coefficient c depends on the various couplings constants of the SM. In Eq. (3.9) we have only integrated over the energy-momentum range, in which the SM is expected to be valid. The scale Λ can be of the order of only a few TeV, but for sure not higher than the Planck scale $M_P \approx 1.2 \times 10^{19}$ GeV, where quantum gravitational effects are expected to become important.

How can we know which values of Λ are reasonable? We know now that the Higgs mass is about 126 GeV. If we now demand that Eq. (3.9) is fulfilled without excessive fine-tuning between the terms on the right-hand side, we would deduce $\Lambda \leq \mathcal{O}(\text{TeV})$. If we assume, however, that the SM is valid up to the scale of grand unification (GUT) and hence choose $\Lambda = M_{GUT} \sim 10^{16}$ GeV, then the mass parameter $m^2_{H_{SM}}$ in the Lagrangian has to be adjusted up to 1 part in 10^{26}, in order to ensure the cancellation necessary to achieve a Higgs mass of 126 GeV. The logarithmic term on the other hand, contributes a correction which is $\sim m^2_{H_{SM}}$, even for $\Lambda \sim M_P$. The $\ln \Lambda$ correction, which also appear in the fermion masses, are hence not large. In other words, the large Λ^2 corrections imply: If we use the high-energy theory, from which the SM results as effective low-energy theory, to make predictions at TeV energies, then these predictions are extremely sensitive to the parameters of the high-energy theory, if $\Lambda \gg 1$ TeV. This is the so-called *fine-tuning problem* of the SM.

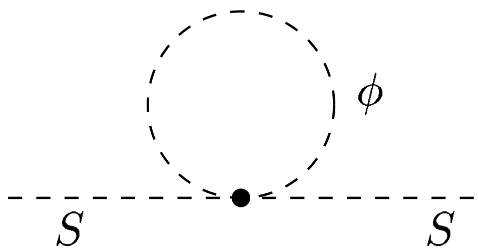

Figure 3.2: Radiative corrections to the mass of the boson S due to the fields ϕ_1 and ϕ_2.

How can supersymmetry solve this problem? In order to understand this we add to our toy Lagrangian Eq. (3.6) the Lagrangian $\mathcal{L}_2$,

$$\mathcal{L}_2 = |\partial_\mu \phi_1|^2 + |\partial_\mu \phi_2|^2 + \frac{\lambda_S}{2} S^2 (|\phi_1|^2 + |\phi_2|^2) - m_\phi^2 (|\phi_1|^2 + |\phi_2|^2) \ . \tag{3.10}$$

The fields ϕ_1, ϕ_2 describe complex scalar particles of mass m_ϕ, which interact with the scalar field with the coupling strength λ_S. These new particles also contribute to the radiative corrections of the scalar mass of S. The corresponding diagram is displayed in Fig. 3.2. The calculation of the radiative correction $\delta m'^2_S$ to the mass of S, which stems from the loops with ϕ_1, ϕ_2, leads to

$$\delta m'^2_S = + \frac{\lambda_S^2}{8\pi^2} [\Lambda^2 - m_\phi^2 \ln \frac{\Lambda^2}{m_\phi^2}] + \ldots \ . \tag{3.11}$$

If $\lambda_S = \lambda_F$, then this contribution in the term which is quadratically divergend in Λ is exactly equal to the contribution that is stemming from the fermionic mass correction, but with different sign. This contribution hence cancels exactly the quadratically divergent term in Λ so that the hierarchy problem or the problem of finetunig of the parameter does not exist any more. The opposite sign in the contributions of the fermion and boson loops is a consequence of the Pauli principle ($\rightarrow$ "closed fermion loops obtain a factor -1.") In order for this cancellation to take place, the following is necessary:

- The number of degrees of freedom must be the same. Since there are four fermionic degrees of freedom, we need to introduce two complex scalar fields.
- The couplings of the new particles ϕ_1, ϕ_2 to the scalar field must be equal to the coupling of the fermion to the scalar field,

$$\lambda_F = \lambda_S \equiv \lambda \, . \tag{3.12}$$

The remaining logarithmic divergence

$$\delta m_S^2 \sim \frac{\lambda^2}{8\pi^2}(m_F^2 - m_\phi^2) \ln \Lambda^2 \tag{3.13}$$

is no problem for fine-tuning if m_ϕ is not too large.

Supersymmetry ensure this by relating fermions and bosons. The fermionic particles of the SM acquire in SUSY bosonic partners and the bosonic particles acquire fermionic partners. In this way the bosonic masses can be kept small.

> Supersymmetry: Bosonic masses can be kept small in a natural way if bosons and fermions are related to each other.

In order to avoid another fine-tuning through the introduction of new fields and their masses, we demand additionally that the masses of these new SUSY partners of the fermion are maximally of the order of TeV, $m_\phi \lesssim \mathcal{O}(1 \text{ TeV})$ (see Eq. 3.13). In summary, the properties of low-scale SUSY are:

> - Doubling of the particle spectrum (in the minimal version of the SUSY extension of the SM).
> - Equality of the fermionic and bosonic coupling constants.
> - $m_{SM} \sim \mathcal{O}(100 \text{ GeV}) \Rightarrow m_\phi \equiv \tilde{m} \lesssim \mathcal{O}(\text{few TeV})$.

3.4 Some Basics about Supersymmetry

3.4.1 The Coleman-Mandula Theorem

The Coleman-Mandula theorem is a so-called no-go theorem. It says, that the maximal set for symmetry transformations, that concern the space-time coordinates, is given by shifts, rotations and the Lorentz transformation. In other words, the most general Lie algebra of the symmetries of the S matrix contains the energy-momentum vector P^μ, the generators of the Lorentz rotation $M^{\mu\nu}$ and a finite number of opertors B^ρ that are Lorentz scalars. The latter must be part of the Lie algebra of a compact Lie group.

The theorem is based on the following assumptions

(1) The S matrix is based on a local relativistic quantum field theory in four space-time dimensions.

(2) There is only a finite number of different particles that are associated with a one-particle state of given mass.

(3) There is an energy gap between the vacuum and 1-particle states.

Supersymmetric theories evade the constraints of the Coleman-Mandula theorem by relaxing one condition. They generalize the Lie algebra in such a way, that algebraic systems are included, which contain as defining relations both commutators and anti-commutators. These new algebras are called superalgebras or graded Lie algebras.

The theorem by Haag, Sohnius and Lopuszanski then says the following: The Supersymmetry algebra is the only graded Lie algebra of symmetries of the S matrix, that is consistent with relativistic quantum field theory.

3.4.2 Graded algebras

The simplest form of a *graded algebra*, the so-called $\mathbf{Z}_2$ graded algebra, consists of a vector space $\mathbf{L}$, which is the direct sum of two subspaces,

$$\mathbf{L} = \mathbf{L}_0 \oplus \mathbf{L}_1 \ , \tag{3.14}$$

and of a product $\circ$ with the following properties,

$$\begin{array}{lll} u_1 \circ u_2 & \in \ \mathbf{L}_0 & \forall u_1, u_2 \in \mathbf{L}_0 \ , \\ u \circ v & \in \ \mathbf{L}_1 & \forall u \in \mathbf{L}_0, v \in \mathbf{L}_1 \ , \\ v_1 \circ v_2 & \in \ \mathbf{L}_0 & \forall v_1, v_2 \in \mathbf{L}_1 \ . \end{array} \tag{3.15}$$

A $\mathbf{Z}_n$ graded algebra is the direct sum of n subspaces $\mathbf{L}_i$,

$$\mathbf{L} = \mathbf{L}_0 \oplus \mathbf{L}_1 \oplus ... \oplus \mathbf{L}_{n-1} \tag{3.16}$$

and has a product with the properties

$$u_j \circ u_k \in \mathbf{L}_{j+k \bmod n} \ , \tag{3.17}$$

where $u_i \in \mathbf{L}_i$. A product $\circ$ with this property is called *graded*.

3.4.3 Graded Lie algebras

A *graded Lie algebra* is constructed from a $\mathbf{Z}_2$ graded algebra, by imposing on the product $\circ$ the following properties:

$$\begin{array}{ll} \text{Graduation:} & x_i \circ x_j \in \mathbf{L}_{i+j \bmod 2} \\ \text{Supersymmetry:} & x_i \circ x_j = -(-1)^{i \cdot j} x_j \circ x_i \\ \text{Jacobi identity:} & x_k \circ (x_l \circ x_m)(-1)^{k \cdot m} + x_l \circ (x_m \circ x_k)(-1)^{l \cdot k} + \\ & x_m \circ (x_k \circ x_l)(-1)^{m \cdot l} = 0 \ , \end{array} \tag{3.18}$$

where $x_i \in \mathbf{L}_i$ and $i = 0, 1$. According to supersymmetry the product can be both symmetric and anti-symmetric. Only the subspace $\mathbf{L}_0$ defines a Lie algebra, as the product is anti-symmetric. The product in the subspace $\mathbf{L}_1$ is symmetric. This one even is not an algebra, as because of $x_1 \circ y_1 \in \mathbf{L}_0$ the product is outside $\mathbf{L}_1$.

3.4.4 The Poincaré Superalgebra

The $\mathbf{Z}_2$ graduation of the Poincaré algebra leads to the Poincaré superalgebra.

The subspace $\mathbf{L}_0$ is built from the 10 generators P^μ and $M^{\mu\nu}$ of the Poincaré algebra. It is enlarged by N SUSY generators Q_a, where here we choose $N = 4$. Now the products *in* and *between* the two subspaces are to be defined. In the subspace $\mathbf{L}_0$ we have

$$\mathbf{L}_0 \times \mathbf{L}_0 \to \mathbf{L}_0 : \qquad \text{Poincaré algebra} . \tag{3.19}$$

For the product $\mathbf{L}_0 \times \mathbf{L}_1$ commutations of Q_a both with P^μ and with $M^{\mu\nu}$ have to be defined. The matrix of the structure constants must be an $N \times N$ representation matrix of the generators of $\mathbf{L}_0$. In case of the generators of the Lorentz transformation $M^{\mu\nu}$ it is the 4×4 matrix

$$\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu] . \tag{3.20}$$

where γ^μ, γ^ν are the 4×4 Dirac γ matrices. For translations the trivial transformation is chosen. We hence have

$$\mathbf{L}_0 \times \mathbf{L}_1 \to \mathbf{L}_1 : \qquad [P^\mu, Q_a] = 0 \tag{3.21}$$
$$[M^{\mu\nu}, Q_a] = -\Sigma^{\mu\nu}_{ab} Q_b . \tag{3.22}$$

And therefore the Q_a behaves like a spinor under rotations. With the third product

$$\mathbf{L}_1 \times \mathbf{L}_1 \to \mathbf{L}_0 \tag{3.23}$$

we then find the SUSY algebra, that is given below. It consists of 14 generators given by P^μ, $M^{\mu\nu}$, and Q_a These fullfil the following relations:

Poincaré-Algebra

$$\begin{aligned} [P^\mu, P^\nu] &= 0 \\ [P^\mu, M^{\rho\sigma}] &= i(g^{\mu\rho}P^\sigma - g^{\mu\sigma}P^\rho) \\ [M^{\mu\nu}, M^{\rho\sigma}] &= -i(g^{\mu\rho}M^{\nu\sigma} - g^{\mu\sigma}M^{\nu\rho} - g^{\nu\rho}M^{\mu\sigma} + g^{\nu\sigma}M^{\mu\rho}) . \end{aligned} \tag{3.24}$$

und

$$[P^\mu, Q_a] = 0 , \tag{3.25}$$
$$[M^{\mu\nu}, Q_a] = -\Sigma^{\mu\nu}_{ab} Q_b , \tag{3.26}$$
$$\{Q_a, \bar{Q}_b\} = 2\gamma^\mu_{ab} P_\mu . \tag{3.27}$$

From the last relation one can see that two SUSY transformations performed one after the other lead to a translation. As local translation invariance (Poincaré invariance) is the symmetry that leads to general relativity, one can expect a connection between supersymmetry and gravitation.

3.4.5 Varia

It can be shown that the Hamilton operator of the SUSY generators has non-negative mass eigenvalues.

The energy spectrum is non-negative: $E \geq 0$.

States with zero energy are supersymmetric ground states. *Ground states* as the expectation value of H has its minimum at zero. *Supersymmetric* as it can be shown that

$$< 0|H|0 >= 0 \Rightarrow Q|0 >= \bar{Q}|0 >= 0 \; . \tag{3.28}$$

This means that ground states with positive energy spontaneously break superymmetry. Small reminder:

Spontaneous symmetry breaking: Be the Hamiltonian H invariant under a transformation $[H, G] = 0$, where G denotes the generator of this transformation. In case of spontaneous symmetry breaking the ground state does *not* respect this symmetry, $G|0 >\neq 0$. This means

$$[H, G] = 0 \quad \text{and} \quad \left\{ \begin{array}{cl} G|0 >= 0 & \text{(symmetry exact)} \\ G|0 >\neq 0 & \text{(spontaneously broken)} \end{array} \right. \tag{3.29}$$

In supersymmetric theories, where Q relates fermions to bosons and vice versa, we have

number of bosonic degrees of freedom = number of fermionic degrees of freedom

Furthermore it follows from $[Q, P^\mu] = 0$ and from the fact, that the operator Q transforms fermions into bosons and vice versa, ($H = P^0$), that

$$\begin{aligned} H|B> &= m_B|B> \\ QH|B> &= HQ|B >= H|F >= m_F|F> \qquad (3.30) \\ &= m_B|F> \\ \Rightarrow \quad & m_B|F >= m_F|F >\Rightarrow m_B = m_F \; . \qquad (3.31) \end{aligned}$$

This means

The masses of the fermionic and bosonic states that are connected through SUSY transformations, are equal: $m_B = m_F$.

3.4.6 The breaking of supersymmetry

Supersymmetry says that the particles and their SUSY partners have the same masses. As we have not found SUSY particles yet, SUSY must be broken. Supersymmetry breaking cannot take place in our world though. The reason is the Ferrara-Girardello-Palumbo mass sum rule. It says:

Ferrara-Girardello-Palumbo sum rule: $\sum(-1)^{2J}(2J+1)m_J^2 = 0$ (3.32)

This would mean for the electron and its two complex valued superpartners, $\tilde{e}_L$ and $\tilde{e}_R$, that

$$m_{\tilde{e}_L}^2 + m_{\tilde{e}_R}^2 - 2m_e^2 = 0 \; . \tag{3.33}$$

This is not compatible with the observation, as either both selectrons would have to have the same mass as the electron or one of the selectrons would have to be lighter than the electron. A possible solution would be that the right-hand side of the equation is non-zero because of *e.g.* a supergravity sector $\sim m_{3/2}^2$. One would hence have to add matter terms. These matter terms should not directly interact with our world, however, in order not to destroy the Standard Model. This means they would exist in a hidden sector. The breaking in the hidden sector is then communicated to our world via messenger particles. Depending on the realisation it is called gravity mediated, gauge mediated, anomaly mediated SUSY breaking etc. Phenomenologically this is realized by adding to the SUSY Lagrangian a soft SUSY breaking Lagrangian. It contains SUSY breaking operators which are such that they do not introduce new quadratic divergences. This is why it is called soft SUSY breaking. For the soft SUSY breaking Lagrangian which we require to be renormalizable, we make the following ansatz

$$\boxed{\begin{array}{rcll} \mathcal{L}_{soft} & = & -\frac{1}{2}M_i\bar{\lambda}_i\lambda_i & \text{for the gauginos} \\ & - & m_{\tilde{f}}^2|\tilde{f}|^2 + ... & \text{for sfermions, Higgs} \\ & - & U_2(\varphi) - U_3(\varphi) + h.c. & \text{super potential} \end{array}}$$

This Lagrangian parametrizes our ignorance about the mechanism that leads to SUSY breaking. The Lagrangian contains scalar mass terms, gaugino mass terms for each gauge group and bilinear terms as well as cubic scalar couplings.

3.5 The MSSM

In the Minimal Supersymmetric extension of the SM (MSSM) each degree of freedom of the SM acquires a supersymmetric partner degree of freedom. The MSSM is the phenomenologically most intensely studied SUSY extension of the SM. Also the Next-to-Minimal SUSY exension of the SM (NMSSM) has been studied in quite detail. Compared to the MSSM the Higgs potential is enlarged by a singlet. Here we start by studying the MSSM.

The superpotential, part of the SUSY Lagrangian, contains a left-chiral superfield

$$\hat{H}_2 = \begin{pmatrix} \hat{h}_2^+ \\ \hat{h}_2^0 \end{pmatrix} \; , \tag{3.34}$$

which has hypercharge $Y = +1$. The vacuum expectation value of the scalar component of the superfield $\hat{h}_2^0$ gives masses to the up-type quarks. In order to give masses to the down-type quarks we need a superfield with hypercharge $Y = -1$. However, the right-chiral superfield $\hat{H}_2^\dagger$, which has $Y = -1$ is not allowed in the holomorphic superpotential. We therefore have to add a second left-chiral superfield with $Y = -1$,

$$\hat{H}_1 = \begin{pmatrix} \hat{h}_1^{0*} \\ -\hat{h}_1^- \end{pmatrix} \; . \tag{3.35}$$

This also solves another problem. The extension of the scalar Higgs doublet to the superfield Higgs doublet $\hat{H}_2$ introduces further fermions (called higgsinos). Their presence would destroy the cancellation of the triangle anomalies, which works in the SM. The higgsinos of the $Y = -1$ doublet, however, have the correct quantum numbers to restore the cancellation of the anomaly. The matter- and Higgs-superfield-content of the MSSM for one generation is summarised in Table 3.5 together with their behaviour under gauge transformations and with the weak hypercharges. The vector fields are summarized in Table 3.5.

Superfeld	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Teilcheninhalt
$\hat{L} = \begin{pmatrix} \hat{\nu}_{eL} \\ \hat{e}_L \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	-1	$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}$
$\hat{E}^c$	$\mathbf{1}$	$\mathbf{1}$	2	$\bar{e}_R, \tilde{e}_R^*$
$\hat{Q} = \begin{pmatrix} \hat{u}_L \\ \hat{d}_L \end{pmatrix}$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{3}$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}$
$\hat{U}^c$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$-\frac{4}{3}$	$\bar{u}_R, \tilde{u}_R^*$
$\hat{D}^c$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\frac{2}{3}$	$\bar{d}_R, \tilde{d}_R^*$
$\hat{H}_2 = \begin{pmatrix} \hat{h}_2^+ \\ \hat{h}_2^0 \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2}$	1	$\begin{pmatrix} H_2 \\ \tilde{h}_2 \end{pmatrix}$
$\hat{H}_1 = \begin{pmatrix} \hat{h}_1^{0*} \\ -\hat{h}_1^- \end{pmatrix}$	$\mathbf{1}$	$\mathbf{2^*}$	-1	$\begin{pmatrix} H_1 \\ \tilde{h}_1 \end{pmatrix}$

Table 3.1: The matter and Higgs-superfield and particle content of the MSSM for one generation together with gauge quantum numbers and weak hypercharge.

Superfeld	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	Teilcheninhalt
$\hat{G}^a$	$\mathbf{8}$	$\mathbf{1}$	0	$G^\mu, \tilde{g}$
$\hat{W}^i$	$\mathbf{1}$	$\mathbf{3}$	0	$W_i^\mu, \tilde{w}_i$
$\hat{B}$	$\mathbf{1}$	$\mathbf{1}$	0	$B^\mu, \tilde{b}$

Table 3.2: Gauge-superfield and particle content of the MSSM with gauge quantum numbers and weak hypercharge.

We introduce an additional discrete symmetry in order avoid rapid proton decay. We require the conservation of R-parity. It involves a multiplicative quantum number, and we have

$$\boxed{\begin{array}{lll} R\text{-Parität:} & = +1 & \text{for SM particles} \\ & = -1 & \text{for SUSY particles} \end{array}}$$

Phenomenologically the consequences are

◇ SUSY particles are produced in pairs.

◇ There is a lightest SUSY particle, the LSP. This particle is stable.

3.5.1 The scalar potential of the MSSM and EWSB

In the following we assume CP conservation and the absence of flavour-changing neutral currents. In order to investigate EWSB we have to investigate the minima of the scalar potential of the MSSM. The Higgs potential of the MSSM is given by

$$\boxed{\begin{aligned} V_{Higgs} &= (m_{H_1}^2 + |\mu|^2)|H_1|^2 + (m_{H_2}^2 + |\mu|^2)|H_2|^2 - B\mu\epsilon_{ij}(H_1^i H_2^j + h.c.) \\ &= +\frac{g^2+g'^2}{8}[|H_1|^2 - |H_2|^2]^2 + \frac{g^2}{2}|H_1^\dagger H_2|^2 \qquad (3.36) \end{aligned}}$$

Here H_1 and H_2 are the two complex Higgs doublets, which we have to introduce to give masses to the up- and down-type quarks and to ensure an anomaly-free theory,

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} h_1^{0*} \\ -h_1^- \end{pmatrix} \qquad H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} h_2^+ \\ h_2^0 \end{pmatrix} . \tag{3.37}$$

The g and g' are the $SU(2)$ and $U(1)$ gauge couplings and μ is the so-called higgsino mass parameter. The term proportional to $B\mu$ arises from the soft SUSY breaking Lagrangian. We want that the minimum of the potential breaks the electroweak symmetry down to the electromagnetic symmetry, *i.e.* $SU(2)_L \times U(1)_Y \to U(1)_{em}$. The $SU(2)_L$ invariance allows to rotate away a possible VEV for one of the weak isospin components of one of the scalar fields. Without restriction of generality we assume $h_1^- = 0$ in the minimum of the potential. It can then be shown, that a minimum of the potential which fulfills $\partial V/\partial h_1^- = 0$ also has to have $h_2^+ = 0$ in the minimum. This means that in the minimum of the potential electromagnetism is unbroken, as the charged components of the Higgs scalars do not acquire VEVs. After setting $h_1^- = h_2^+ = 0$ only the potential for the neutral fields has to be minimised. It reads

$$\begin{aligned} V_{scalar} &= (m_{H_1}^2 + \mu^2)|h_1^0|^2 + (m_{H_2}^2 + \mu^2)|h_2^0|^2 - B\mu(h_1^{0*}h_2^0 + h.c.) \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|h_2^0|^2 - |h_1^0|^2)^2 . \end{aligned} \tag{3.38}$$

Without restriction of the generality we can set $< h_1^0 >$ and $< h_2^0 >$ real and positive. In order to ensure that the minimum of the potential is not acquired for $< h_1^0 >=< h_2^0 >= 0$, so that we have EWSB, we require that there is a local maximum at the origin. This leads to the condition

$$(B\mu)^2 > (m_{H_1}^2 + \mu^2)(m_{H_2}^2 + \mu^2) . \tag{3.39}$$

Furthermore, the potential shall have a stable mimimum and shall not be unbounded from below. For most of the field values this is no problem, as the positive definite quartic term of the scalar potential dominates at large field values. However, the quartic term vanishes in the direction of the field space where $< h_1^0 >=< h_2^0 >$. We have to require that the scalar potential is positive along this direction. This leads to the condition

$$m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 > 2B\mu . \tag{3.40}$$

If these conditions are fulfilled, the potential has a well-defined local minimum in which the electroweak symmetry is spontaneously broken. We now require that the breaking is compatible with the phenomenology of the EWSB $SU(2)_L \times U(1)_Y \to U(1)_{em}$. We set

$$< H_1^0 >\equiv \frac{v_1}{\sqrt{2}} \qquad \text{and} \qquad < H_2^0 >\equiv \frac{v_2}{\sqrt{2}} . \tag{3.41}$$

These VEVs are connected with the known mass of the Z boson and the electroweak couplings,

$$v_1^2 + v_2^2 = v^2 = 4\frac{m_Z^2}{g^2 + g'^2} \approx 246 \text{ GeV} . \tag{3.42}$$

The ratio of the VEVs is written as

$$\tan\beta = \frac{v_2}{v_1} . \tag{3.43}$$

The mixing angle β plays an important role in phenomenological studies of the MSSM. Since we have chosen $v_1 = v\cos\beta$ and $v_2 = v\sin\beta$ positive and real, we have $0 < \beta < \pi/2$. We can now write up the conditions $\partial V/\partial h_1^0 = \partial V/\partial h_2^0 = 0$, for which the potential (3.38) takes a minimum and which fulfills the conditions (3.41) and (3.42):

$$\begin{aligned} (m_{H_1}^2 + \mu^2)h_1^0 - B\mu h_2^0 - \frac{1}{4}(g^2 + g'^2)h_1^0(|h_2^0|^2 - |h_1^0|^2) &= \\ \Rightarrow \qquad (m_{H_1}^2 + |\mu|^2) - B\mu\tan\beta + \frac{m_Z^2}{2}\cos(2\beta) &= 0 \qquad (3.44) \\ (m_{H_2}^2 + \mu^2)h_2^0 - B\mu h_1^0 + \frac{1}{4}(g^2 + g'^2)h_2^0(|h_2^0|^2 - |h_1^0|^2) &= \\ \Rightarrow \qquad (m_{H_2}^2 + |\mu|^2) - B\mu\cot\beta - \frac{m_Z^2}{2}\cos 2\beta &= 0 . \qquad (3.45) \end{aligned}$$

These conditions allow us to eliminate $B\mu$ and $|\mu|$ and to replace them by $\tan\beta$ and m_Z. However, we cannot determine the phase of μ.

Remarks:

The "μ problem": If we choose $|\mu|^2, B\mu, m_{H_1}^2, m_{H_2}^2$ as input parameters we get

$$\begin{aligned} \sin(2\beta) &= \frac{2B\mu}{m_{H_1}^2 + m_{H_2}^2 + 2|\mu|^2} \qquad (3.46) \\ m_Z^2 &= \frac{|m_{H_1}^2 - m_{H_2}^2|}{\sqrt{1 - \sin^2(2\beta)}} - (m_{H_1}^2 + m_{H_2}^2) - 2|\mu|^2 . \qquad (3.47) \end{aligned}$$

We can read off Eq. (3.47), that all input parameters should be within one or two orders of magnitude of m_Z. However, in the MSSM μ is a SUSY conserving parameter, that appears in the superpotential, while $B\mu$, $m_{H_1}^2$, $m_{H_2}^2$ are SUSY breaking parameters. There it is assumed that the MSSM has to be enlarged at high energies, to include a mechanism that somehow relates the effective value of μ with the SUSY breaking mechanism.

The Higgs boson mass: So far we have looked at the tree-level potential of the EWSB sector of the MSSM. A characteristic property of this potential is that the quartic self-interactions of the Higgs fields are only given by the $SU(2)_L \times U(1)_Y$ gauge couplings. This implies that the Higgs sector of the MSSM automatically fulfills the unitarity constraints, in contrast to the SM where the Higgs self coupling value is an independent parameter. We will see, that the structure of the self-couplings in the Higgs sector of the MSSM implies an *upper* bound of m_Z for the mass of the lightest Higs boson! This, however, is a tree-level result, and higher order corrections significantly change this value.

3.5.2 The Higgs bosons

In the SM we are left after the EWSB out of the four degrees of the complex Higgs doublet with one physical Higgs boson. The other three degrees of freedom, the would-be Goldstone bosons, are "eaten" in order to provide the longitudinal components of the massive gauge bosons $W^{\pm}$ and Z. The symmetry breaking pattern in the MSSM is the same as in the SM. We therefore expect the same set of would-be Goldstone bosons. Since we have, however, two complex Higgs doublets, there will remain 2 charged and 3 neutral spin-0 bosons in the physical spectrum of the MSSM. In order to identify these states and to calculate their masses, we have to investigate the following Higgs potential,

$$\begin{aligned} V_{Higgs} &= (m^2_{H_1}+|\mu|^2)(|h_1^0|^2+|h_1^+|^2)+(m^2_{H_2}+|\mu|^2)(|h_2^0|^2+|h_2^+|^2)-B\mu(h_2^+h_1^-+h_2^0h_1^{0*}+h.c.) \\ &\quad +\frac{g^2}{8}\{(|h_2^+|^2-|h_2^0|^2+|h_1^0|^2-|h_1^-|^2)^2+4|h_2^+|^2|h_2^0|^2+4|h_1^0|^2|h_1^-|^2 \\ &\quad -4(h_2^{+*}h_1^{-*}h_2^{0*}h_1^0+h_2^0h_1^{0*}h_2^+h_1^-)\} \\ &\quad +\frac{g'^2}{8}[|h_2^+|^2+|h_2^0|^2-|h_1^0|^2-|h_1^-|^2]^2 \; . \end{aligned} \tag{3.48}$$

The neutral fields can be split into real and imaginary components, *i.e.*

$$h_2^0 = h_{2R}^0 + ih_{2I}^0 \tag{3.49}$$
$$h_1^0 = h_{1R}^0 + ih_{1I}^0 \; . \tag{3.50}$$

The scalar potential can be viewed as a function of 8 independent fields, $V(h^0_{2R},h^0_{2I},h^0_{1R},h^0_{1I},h_2^+,h_2^{+*},h_1^-,h_1^{-*})$. Since we are interested in excitations of the vacuum, we expand the Higgs potential about its minimum as

$$\begin{aligned} V^{Higgs} &= V_{min}+\sum_{h_i}\frac{\partial V}{\partial h_i}\Big|_{h_i=<h_i>}(h_i-<h_i>) \\ &\quad +\frac{1}{2}\sum_{h_i,h_j}\frac{\partial^2 V}{\partial h_i \partial h_j}\Big|_{h_{i,j}=<h_{i,j}>}(h_i-<h_i>)(h_j-<h_j>)+\ldots \; . \end{aligned} \tag{3.51}$$

Here the h_i denote the eight fields of the potential.

- The only non-vanishing VEVs are $<h_{2R}>=v_2$ and $<h_{1R}>=v_1$.
- The coefficients of the linear terms all have to vanish, as the derivatives are evaluated at the minimum of the potential.
- The quadratic terms wil then lead to the Higgs boson mass terms. Since in general there is mixing, this will be mass matrices.
- The conservation of the electric charge means that there is no mixing between charged an neutral Higgs fields, so that there is a mass matrix for the charged sector and one for the neutral sector.
- Since we have assumed CP conservation, the real and imaginary components of the neutral Higgs bosons do not mix either, so that the 4×4 mass matrix in the neutral sector decomposes into two 2×2 blocks.

Let us first look at the mass matrices that contain the would-be Goldstone bosons. These are in the charged and in the CP-odd sector (*i.e.* the imaginary components) of the neutral fields. The states orthogonal to the Goldstone bosons are then automatically the physical states of this sector. Let us start with the charged fields. The corresponding Lagrangian has the form

$$(h_2^{+*} h_1^-)\mathcal{M}^2_{H^\pm} \begin{pmatrix} h_2^+ \\ h_1^{-*} \end{pmatrix} , \tag{3.52}$$

where

$$\mathcal{M}^2_{h\pm} = \begin{pmatrix} \frac{\partial^2 V}{\partial h_2^+ \partial h_2^{+*}}\Big|_{h_i \to v_i} & \frac{\partial^2 V}{\partial h_2^{+*} \partial h_1^{-*}}\Big|_{h_i \to v_i} \\ \frac{\partial^2 V}{\partial h_2^+ \partial h_1^-}\Big|_{h_i \to v_i} & \frac{\partial^2 V}{\partial h_1^- \partial h_1^{-*}}\Big|_{h_i \to v_i} \end{pmatrix} . \tag{3.53}$$

The derivatives are easily obtained as they are evaluated for the VEVs of the Higgs fields. This means, that after performing the dervatives we can let drop the terms which are proportional to $h_2^+, h_1^-, h_{2I}^0, h_{1I}^0$, as these fields vanish in the vacuum. For example, we find

$$\begin{aligned} \frac{\partial^2 V}{\partial h_2^+ \partial h_2^{+*}}\Big|_{h_i \to v_i} &= (m_{H_2}^2 + \mu^2) + \frac{g^2}{8}(v_1^2 + v_2^2) + \frac{g'^2}{8}(v_2^2 - v_1^2) \\ &= B\mu \cot\beta + \frac{g^2}{4} v_1^2 , \end{aligned} \tag{3.54}$$

where in the last step the minimisation condition (3.45) has been used to replace $m_{H_2}^2 + \mu^2$ by $B\mu$. The quadratic mass matrix in the charged sector finally reads

$$\mathcal{M}^2_{H^\pm} = \begin{pmatrix} B\mu \cot\beta + \frac{g^2}{4} v_1^2 & -B\mu - \frac{g^2}{4} v_1 v_2 \\ -B\mu - \frac{g^2}{4} v_1 v_2 & B\mu \tan\beta + \frac{g^2}{4} v_2^2 \end{pmatrix} . \tag{3.55}$$

Here Eq. (3.44) was used, in order to eliminate in the right lower entry $m_{H_1}^2 + \mu^2$. The eigenvalues of this matrix are given by

$$m_{G^\pm} = 0 \qquad \text{and} \qquad m_{H^\pm} = B\mu(\cot\beta + \tan\beta) + m_W^2 . \tag{3.56}$$

In the unitary gauge the Goldstone bosons $G^\pm$ do not appear. They are absorbed to give masses to the $W^\pm$ bosons. The other states $H^\pm$ remain in the spectrum. The mixing matrix has the following form

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} h_1^{-*} \\ h_2^+ \end{pmatrix} . \tag{3.57}$$

In the neutral sector we get for the mass terms of the imaginary components of the neutral fields

$$\frac{1}{2}(h_{2I}^0 h_{1I}^0)\mathcal{M}_{h_{iI}^0} \begin{pmatrix} h_{2I}^0 \\ h_{1I}^0 \end{pmatrix} , \tag{3.58}$$

with

$$\mathcal{M}_{h_{iI}^0} = \begin{pmatrix} \frac{\partial^2 V}{\partial h_{2I}^{02}}\Big|_{h_i \to v_i} & \frac{\partial^2 V}{\partial h_{2I}^0 \partial h_{1I}^0}\Big|_{h_i \to v_i} \\ \frac{\partial^2 V}{\partial h_{2I}^0 \partial h_{1I}^0}\Big|_{h_i \to v_i} & \frac{\partial^2 V}{\partial h_{1I}^{02}}\Big|_{h_i \to v_i} \end{pmatrix} . \tag{3.59}$$

The calculation results in

$$\mathcal{M}^2_{h^0_{iI}} = \begin{pmatrix} B\mu\cot\beta & B\mu \\ B\mu & B\mu\tan\beta \end{pmatrix} . \tag{3.60}$$

The eigenvalues are

$$m_{G^0} = 0 \qquad \text{and} \qquad m_A^2 = B\mu(\cot\beta + \tan\beta) . \tag{3.61}$$

Comparison of the eigenvalues for $H^\pm$ and A leads to

$$m^2_{H^\pm} = m_A^2 + m_W^2 , \tag{3.62}$$

so that at tree level $m_{H^\pm} \geq m_W$ and $m_{H^\pm} \geq m_A$. Again the Goldstone boson G^0 disappears from the Lagrangian to provide the longitudinal degree of freedom of the Z boson. The massive A boson remains as *pseudoscalar*[1] Higgs boson. The mixing matrix for G^0 and A reads

$$\begin{pmatrix} G^0 \\ A \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} h^0_{2I} \\ h^0_{1I} \end{pmatrix} . \tag{3.63}$$

Finally we look at the mass matrix for the remaining neutral scalars, h^0_{2R} and h^0_{1R}. We have for the mass matrix of the real components of the neutral Higgs scalars

$$\frac{1}{2}(h^0_{2R} h^0_{1R})\mathcal{M}^2_{h^0_{iR}} \begin{pmatrix} h^0_{2R} \\ h^0_{1R} \end{pmatrix} , \tag{3.64}$$

with

$$\begin{aligned} \mathcal{M}_{h^0_{iR}} &= \begin{pmatrix} \left.\frac{\partial^2 V}{\partial h^{02}_{2R}}\right|_{h_i\to v_i} & \left.\frac{\partial^2 V}{\partial h^0_{2R}\partial h^0_{1R}}\right|_{h_i\to v_i} \\ \left.\frac{\partial^2 V}{\partial h^0_{2R}\partial h^0_{1R}}\right|_{h_i\to v_i} & \left.\frac{\partial^2 V}{\partial h^{02}_{1R}}\right|_{h_i\to v_i} \end{pmatrix} \\ &= \begin{pmatrix} m_A^2\cos^2\beta + m_Z^2\sin^2\beta & -(m_A^2+m_Z^2)\sin\beta\cos\beta \\ -(m_A^2+m_Z^2)\sin\beta\cos\beta & m_A^2\sin^2\beta + m_Z^2\cos\beta \end{pmatrix} . \end{aligned} \tag{3.65}$$

The eigenvalues of the mass matrix read

$$m^2_{h,H} = \frac{1}{2}[(m_A^2+m_Z^2) \mp \sqrt{(m_A^2+m_Z^2)^2 - 4m_A^2 m_Z^2\cos^2 2\beta}] , \tag{3.66}$$

where h and H denote, respectively, the lighter and the heavier one of the neutral scalar mass eigenstates. The physical Higgs scalars as function of h^0_{2R} and h^0_{1R} read

$$\begin{pmatrix} h \\ H \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h^0_{2R} \\ h^0_{1R} \end{pmatrix} . \tag{3.67}$$

The mixing angle α is given by

$$\tan\alpha = \frac{(m_A^2 - m_Z^2)\cos 2\beta + \sqrt{(m_A^2+m_Z^2)^2 - 4m_A^2 m_Z^2\cos^2 2\beta}}{(m_A^2+m_Z^2)\sin 2\beta} \tag{3.68}$$

[1]That it is a pseudoscalar boson can be seen from the couplings to massive fermions. The A is also at loop-level a pseudoscalar eigenstate as CP is conserved and as the CP-odd A cannot mix with the CP-even scalar Higgs bosons.

From Eq. (3.66) follows

$$m_h \leq m_A|\cos 2\beta| \leq m_H \tag{3.69}$$
$$m_h \leq m_Z|\cos 2\beta| \leq m_H \; . \tag{3.70}$$

We have the following inequalities

$$\begin{aligned} M_h &< m_Z, M_A \\ M_H &> m_Z, M_A \\ M_{H^\pm} &> M_A, m_W \; . \end{aligned} \tag{3.71}$$

In particular the lightest Higgs mass is smaller than the Z boson mass. We will see, however, that the radiative corrections will make m_h considerably larger than m_Z.

Chapter 4

Composite Higgs

A very good introduction in Higgs models is given in

Roberto Contino
The Higgs as a Composite Nambu-Golstone Boson
arXiv:1005.4269 [hep-ph].

4.1 Electroweak Symmetry Breaking

The data that have been collected so far in high-energy experiments can be explained by the Lagrangian

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_0 + \mathcal{L}_{\text{mass}} \\
\mathcal{L}_0 &= -\frac{1}{4}W^a_{\mu\nu}W^{a\,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \sum_{j=1}^{3}\bar{\Psi}^{(j)} i\not{D}\Psi^{(j)} \\
\mathcal{L}_{\text{mass}} &= M_W^2 W^+_\mu W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu \\
&\quad - \sum_{ij}\left(\bar{u}^{(i)}_L M^u_{ij} u^{(j)}_R + \bar{d}^{(i)}_L M^d_{ij} d^{(j)}_R + \bar{e}^{(i)}_L M^e_{ij} e^{(j)}_R + \bar{\nu}^{(i)}_L M^\nu_{ij} \bar{\nu}^{(j)}_R\right) + h.c.\ ,
\end{aligned} \tag{4.1}$$

where $\Psi = \{q^i_L, u^i_R, d^i_R, l^i_L, e^i_R, \nu^i_R\}$ stands for the SM fermions and i, j denote generation indices. All fundamental interactions described by the Lagrangian are invariant under local $SU(2)_L \times U(1)_Y$ transformations, but the mass spectrum is not. This means that the electroweak symmetry is broken by the vacuum. The Lagrangian describes the data very well and can be applied at sufficiently low energies, but it leads to inconsistencies if extrapolated to arbitrarily high energies. It predicts scatterig amplitudes that grow with the energy and violate the unitarity bound. The latter requires that the elastic scattering amplitude a_l of each l-th partial wave muss satisfy

$$\Im(a_l) = |a_l|^2 + |a_l^{in}|^2\ , \tag{4.2}$$

where a_l^{in} denotes the inelastic scattering amplitude. At energies below the inelastic threshold a_l is constrained to lie on the unitarity circle $\Re^2(a_l) + (\Im(a_l) - 1/2)^2 = 1/4$ and at higher energies is is bounded to lie inside it. At tree level the amplitude is real and an imaginary part only arises at the 1-loop level. Perturbativity is therefore lost when the imaginary and the real part are of the same order, *i.e.* when the scattering phase is large $\delta \approx \pi$. As we have seen before perturbativity is violated in processes that involve longitudinally polarized vector bosons as external states. At tree level, the amplitude for the elastic scattering of two longitudinally polarized W's grows as E^2 at energies $E \gg m_W$,

$$\mathcal{A}(W_L^+ W_L^- \to W_L^+ W_L^-) \sim \frac{g^2}{4m_W^2}(s+t) \ , \tag{4.3}$$

where s and t are the kinematic Mandelstam variables. Term subleading in m_W/E have been dropped. By projecting on the partial wave amplitudes,

$$a_l = \frac{1}{32\pi}\int_{-1}^{+1} d\cos\theta \mathcal{A}(s,\theta) P_l(\cos\theta) \ , \tag{4.4}$$

with the Legendre polynomials $P_l(x)$ ($P_0(x) = 1, P_1(x) = x, P_2(x) = 3x^2 - 1/2$, etc.), one has for the s-wave amplitude ($l = 0$)

$$a_0(W_L^+ W_L^- \to W_L^+ W_L^-) \sim \frac{1}{32\pi}\frac{s}{v^2} \ . \tag{4.5}$$

Perturbative unitarity in the s-wave scattering is lost for

$$\pi \approx \delta \approx 2\Re(a_0) \tag{4.6}$$

and hence for

$$\sqrt{s} \approx \Lambda = 4\pi v \approx 3 \text{ TeV} \ . \tag{4.7}$$

The role of the longitudinally polarized vector bosons suggests that the inconsistency of the Lagrangian (4.1) is in the sector that breaks spontaneously the electroweak symmetry and gives mass to the vector bosons. The Nambu-Goldstone bosons, as we know, correspond to the longitudinal polarizations of the W and Z bosons. At high energies the Goldstone boson equivalence theorem says that the longitudinal gauge bosons can be described by the Nambu-Goldstone bosons, which are denoted by χ^a in the following. This means at high energies the longitudinal gauge boson scattering is described by the scattering of four Goldstone bosons

$$\mathcal{A}(\chi^+\chi^- \to \chi^+\chi^-) = \frac{1}{v^2}(s+t) \ . \tag{4.8}$$

There are two possibilities to solve this problem. *i*) Either new particles associated to new dynamics come in to restore unitarity before perturbativity is lost. *ii*) Or the $\chi\chi$ scattering grows strong until the interaction among four χ's becomes non-perturbative. These are two paradigms for electroweak symmetry breaking (EWSB) are well expemplified by two theories: the Higgs model and models based on strong dynamics, like composite Higgs models. We have already discussed the Higgs model. In this chapter the composite Higgs model will be discussed as a model where EWSB is strongly realized.

4.2 The Higgs Boson as a composite Nambu-Goldstone boson

In composite Higgs models a light Higgs boson emerges as the bound state of a strongly interacting sector and is not an elementary field. These models interpolate between the Higgs model and technicolor theories.[1] A composite Higgs boson solves the hierarchy problem of the SM in the sense, that its mass is not sensitive to virtual effects above the compositeness scale. In contrast the simple technicolor constructions, the Higgs is light and allows to satisfy the electroweak precision tests and to comply with the Higgs boson mass determined by the LHC experiments. In the eighties, Georgi and Kaplan pointed out that the composite Higgs boson can be naturally lighter than the other resonances if it emerges as the pseudo Nambu-Goldstone boson of an enlarge global symmetry of the strong dynamics. Let us consider the general case in which the strongly interacting sector has a global symmetry $\mathcal{G}$ that is dynamically broken to $\mathcal{H}_1$ at the scale f, and the subgroup $\mathcal{H}_0 \subset \mathcal{G}$ is gauged by external vector bosons. The global symmetry breaking $\mathcal{G} \to \mathcal{H}_1$ implies $n = \dim(\mathcal{G}) - \dim(\mathcal{H}_1)$ Goldstone bosons. Out of these $n_0 = \dim(\mathcal{H}_0) - \dim(\mathcal{H})$ are eaten to give mass to as many vector bosons, so that $\mathcal{H} = \mathcal{H}_1 \cap \mathcal{H}_0$ is the unbroken gauge group. The remaining $n - n_0$ are pseudo Nambu-Goldstone bosons. In this picture the SM fields are assumed to be external to the strong sector. They are therefore referred to as 'elementary', in contrast to the composite nature of the resonances of the strong dynamics. The SM gauge fields, in particular, belong to the vector bosons associated with the gauge group $\mathcal{H}_0$. In the following, for simplicity, $\mathcal{H}_0$ wil be identified with the SM electroweak group, $\mathcal{H}_0 = G_{SM} \equiv SU(2)_L \times U(1)_Y$, so that the SM vector bosons are the only elementary gauge fields coupled to the strong sector. Two conditions have to be fulfilled in order to have a composite pseudo Nambu-Goldstone boson (pNG):

1. The SM electroweak group G_{SM} must be embeddable in the unbroken group $\mathcal{H}_1$,

$$\mathcal{G} \to \mathcal{H}_1 \supset G_{SM} \ . \tag{4.9}$$

2. The coset $\mathcal{G}/\mathcal{H}_1$ contains at least one $SU(2)_L$ doublet, which is to be identified with the Higgs doublet.

If these two conditions are satisfied, at tree level G_{SM} is unbroken and the Higgs doublet is one of the pNG bosons living on the coset $\mathcal{G}/\mathcal{H}_1$. As a consequence of the non-linear Goldstone symmetry acting on it, its potential vanishes at tree level. The global symmetry $\mathcal{G}$ on the other hand is explicitly broken by the couplings of the SM fields to the strong sector, as they will be invariant under G_{SM} but not in general under $\mathcal{G}$. The Higgs potential is generated by loops of SM fermions and gauge bosons. The Higgs potential can break the electroweak symmetry.[2] In this context the electroweak scale v is dynamically determined and can be smaller than the scale f. This is in contrast to technicolor theories, wher no separation of scales exists. The ratio

$$\xi = \frac{v^2}{f^2} \tag{4.10}$$

[1]In technicolor theories, the Higgs boson is no fundamental particle but a bound state of newly introduced fermions. In technicolor models the the electroweak symmetry is directly broken by the strong dynamics. In composite Higgs models the composite Higgs gets a VEV which in turn breaks the symmetry.

[2]The EWSB is triggered by the fermion loops.

is determined by the orientation of G_{SM} with respect to $\mathcal{H}$ in the true vacuum (degree of misalignment). It sets the size of the parametric suppression in all corrections to the precision observables. Naive dimensional analysis shows, that the mass scale of the resonances of the strong sector is $m_\rho \sim g_\rho f$, with $1 \lesssim g_\rho \lesssim 4\pi$. The Higgs boson gets a much ligher mass at one-loop, $m_h \sim g_{SM} v$, where $g_{SM} \lesssim 1$ is a generic SM coupling. The limit $f \to \infty$ $(f \to 0)$ with fixed v is a decoupling limit where the Higgs stays light and all the other resonances become infinitely heavy.

4.3 Minimal Composite Higgs Models

The physics of a Strongly Interacting Light Higgs boson (SILH) can be described by an effective Lagrangian involving higher dimensional operators for the low-energy degrees of freedom. There are two classes of higher dimensional operators: (i) those that are genuinely sensitive to the new strong force and will affect qualitatively the physics of the Higgs boson and (ii) those that are sensitive to the spectrum of the resonances only and will simply act as form factors. The effective Lagrangian generically takes the form

$$\begin{aligned}\mathcal{L}_{\rm SILH} &= \frac{c_H}{2f^2}\left(\partial_\mu |H|^2\right)^2 + \frac{c_T}{2f^2}\left(H^\dagger \overleftrightarrow{D}_\mu H\right)^2 - \frac{c_6\lambda}{f^2}|H|^6 + \left(\frac{c_y y_f}{f^2}|H|^2 \bar{f}_L H f_R + \text{h.c.}\right) \\ &\quad + \frac{i c_W g}{2m_\rho^2}\left(H^\dagger \sigma^i \overleftrightarrow{D^\mu} H\right)(D^\nu W_{\mu\nu})^i + \frac{i c_B g'}{2m_\rho^2}\left(H^\dagger \overleftrightarrow{D^\mu} H\right)(\partial^\nu B_{\mu\nu}) + \dots \qquad (4.11)\end{aligned}$$

where g, g' are the SM EW gauge couplings, λ is the SM Higgs quartic coupling and y_f is the SM Yukawa coupling to the fermions $f_{L,R}$. All the coefficients, $c_H, c_T \dots$, appearing in Eq. (4.11) are expected to be of order one unless protected by some symmetry. For instance, in every model in which the strong sector preserves custodial symmetry, the coefficient c_T vanishes and only three coefficients, c_H, c_y and c_6, give sizable contributions to the Higgs (self-)couplings. The operator c_H gives a correction to the Higgs kinetic term which can be brought back to its canonical form at the price of a proper rescaling of the Higgs field, inducing a universal shift of the Higgs couplings by a factor $1 - c_H\,\xi/2$. For the fermions, this universal shift adds up to the modification of the Yukawa interactions

$$g^{\xi}_{Hf\bar{f}} = g^{\rm SM}_{Hf\bar{f}} \times [1 - (c_y + c_H/2)\xi], \qquad (4.12)$$

$$g^{\xi}_{HVV} = g^{\rm SM}_{HVV} \times (1 - c_H\,\xi/2), \quad g^{\xi}_{HHVV} = g^{\rm SM}_{HHVV} \times (1 - 2c_H\,\xi) \qquad (4.13)$$

where $V = W, Z$, $g^{\rm SM}_{Hf\bar{f}} = m_f/v$ (m_f denotes the fermion mass), $g^{\rm SM}_{HW^+W^-} = gM_W$, $g^{\rm SM}_{HZZ} = \sqrt{g^2 + g'^2}M_Z$, $g^{\rm SM}_{HHW^+W^-} = g^2/2$ and $g^{\rm SM}_{HHZZ} = (g^2 + g'^2)/2$.

The effective SILH Lagrangian should be seen as an expansion in $\xi = (v/f)^2$ where $v = 1/\sqrt{\sqrt{2}G_F} \approx 246$ GeV and f is the typical scale of the Goldstone bosons of the strong sector. Therefore, it can be used to describe composite Higgs models in the vicinity of the SM limit, $\xi \to 0$. To reach the technicolor limit, $\xi \to 1$, a resummation of the full series in ξ is needed. Explicit models, built in five-dimensional (5D) warped space, provide concrete examples of such a resummation. In the following we will discuss two 5D models that exhibit different behaviors of the Higgs couplings. In these explicit models, the two extra parameters that generically control the couplings[3] of a composite Higgs boson are related

[3] These couplings will be called *anomalous* couplings since they differ from the SM ones.

to each other and the deviations from the SM Higgs couplings are only controlled by the parameter $\xi = (v/f)^2$ which varies from 0 to 1.

The Holographic Higgs models of Refs. [26, 27, 28] are based on a five-dimensional theory in Anti de-Sitter (AdS) space-time. The bulk gauge symmetry $SO(5) \times U(1)_X \times SU(3)$ is broken down to the SM gauge group on the UV boundary and to $SO(4) \times U(1)_X \times SU(3)$ on the IR. Since the symmetry-breaking pattern of the bulk and IR boundary is given by $SO(5) \to SO(4)$, we expect four Goldstone bosons parametrized by the $SO(5)/SO(4)$ coset [27]:

$$\Sigma = \langle\Sigma\rangle e^{\Pi/f} \ , \qquad \langle\Sigma\rangle = (0,0,0,0,1) \ , \qquad \Pi = \begin{pmatrix} 0_4 & \mathcal{H} \\ -\mathcal{H}^T & 0 \end{pmatrix} , \tag{4.14}$$

where $\mathcal{H}$ is a real 4-component vector, which transforms as a doublet under the weak $SU(2)$ group and can be associated with the Higgs. The couplings between the Higgs boson and the gauge fields are obtained from the pion kinetic term

$$\mathcal{L}_{\rm kin} = \frac{f^2}{2}(D_\mu\Sigma)(D^\mu\Sigma)^T \ . \tag{4.15}$$

In the unitary gauge where $\Sigma = (\sin H/f, 0, 0, 0, \cos H/f)$, Eq. (4.15) gives

$$\mathcal{L}_{\rm Kin} = \frac{1}{2}\partial_\mu H \partial^\mu H + m_W^2(H)\left[W_\mu W^\mu + \frac{1}{2\cos^2\theta_W} Z_\mu Z^\mu\right] \ \text{with} \ \ m_W(H) = \frac{gf}{2}\sin\frac{H}{f} \ . \tag{4.16}$$

Expanding Eq. (4.16) in powers of the Higgs field, we obtain the Higgs couplings to the gauge fields

$$g_{HVV} = g_{HVV}^{\rm SM}\ \sqrt{1-\xi} \ , \qquad g_{HHVV} = g_{HHVV}^{\rm SM}\ (1-2\xi) \ , \tag{4.17}$$

with the compositeness parameter ξ defined as

$$\xi = \left(\frac{v}{f}\right)^2 = \sin^2\frac{\langle H\rangle}{f} \ . \tag{4.18}$$

The couplings of the Higgs boson to the fermions can be obtained in the same way, but they will depend on the way the SM fermions are embedded into representations of the bulk symmetry. In the MCHM4 model [27] with SM fermions transforming as spinorial representations of $SO(5)$, the interactions of the Higgs to the fermions take the form

$$\mathcal{L}_{\rm Yuk} = -m_f(H)\bar{f}f \ \ \text{with} \ \ m_f(H) = M\sin\frac{H}{f} \ . \tag{4.19}$$

We then obtain

$$\text{MCHM4:} \qquad g_{Hff} = g_{Hff}^{\rm SM}\ \sqrt{1-\xi} \ . \tag{4.20}$$

In the MCHM5 model [28] with SM fermions transforming as fundamental representations of $SO(5)$, the interactions of the Higgs to the fermions take the following form (M is a constant of mass-dimension one)

$$\mathcal{L}_{\rm Yuk} = -m_f(H)\bar{f}f \ \ \text{with} \ \ m_f(H) = M\sin\frac{2H}{f} \ . \tag{4.21}$$

We then obtain

$$\text{MCHM5:} \qquad g_{Hff} = g_{Hff}^{\rm SM} \, \frac{1-2\xi}{\sqrt{1-\xi}} \; . \tag{4.22}$$

In both models, the Higgs couplings to gauge boson are always reduced compared to the SM ones. On the contrary, the two models exhibit different characteristic behaviors in the Higgs couplings to fermions: in the vicinity of the SM, i.e., for low values of ξ, the couplings are reduced, and the reduction is more important for MCHM5 than for MCHM4, but, for larger values of ξ, the couplings in MCHM5 are raising back and can even get much larger than the SM ones. This latter effect is at the origin of an enhancement of the Higgs production cross-section by gluon fusion, enhancement that will significantly affect the Higgs searches.

4.3.1 Branching ratios and total widths

The partial widths in the composite Higgs models can be easily obtained from the SM partial widths by rescaling the couplings involved in the Higgs decays. Since in MCHM4 all Higgs couplings are modified by the same universal factor $\sqrt{1-\xi}$, the branching ratios are the same as in the SM model. The total width will be different though by an overall factor $1-\xi$.

In MCHM5, all partial widths for decays into fermions are obtained from the SM widths by multiplication with the modification factor of the Higgs Yukawa coupling squared,

$$\Gamma(H \to f\bar{f}) = \frac{(1-2\xi)^2}{(1-\xi)} \, \Gamma^{SM}(H \to f\bar{f}) \; . \tag{4.23}$$

The Higgs decay into gluons is mediated by heavy quark loops, so that the multiplication factor is the same as for the fermion decays:

$$\Gamma(H \to gg) = \frac{(1-2\xi)^2}{(1-\xi)} \, \Gamma^{SM}(H \to gg) \; . \tag{4.24}$$

For the Higgs decays to massive gauge bosons V we obtain

$$\Gamma(H \to VV) = (1-\xi) \, \Gamma^{SM}(H \to VV) \; . \tag{4.25}$$

The Higgs decay into photons proceeds dominantly via W-boson and top and bottom loops. Since the couplings to gauge bosons and fermions scale differently in MCHM5, the various loop contributions have to be multiplied with the corresponding Higgs coupling modification factor. The leading order width is given by

$$\Gamma(H \to \gamma\gamma) = \frac{\Gamma^{SM}(H \to \gamma\gamma)}{[I_\gamma(M_H) + J_\gamma(M_H)]^2} \left[\frac{1-2\xi}{\sqrt{1-\xi}} I_\gamma(M_H) + \sqrt{1-\xi} J_\gamma(M_H) \right]^2 \; , \tag{4.26}$$

where

$$\begin{gathered} I_\gamma(M_H) = \tfrac{4}{3} F_{1/2}(4M_t^2/M_H^2), \quad J_\gamma(M_H) = F_1(4M_W^2/M_H^2), \\ F_{1/2}(x) \equiv -2x\left[1 + (1-x)f(x)\right], \quad F_1(x) \equiv 2 + 3x\left[1 + (2-x)f(x)\right], \\ f(x) \equiv \arcsin[1/\sqrt{x}]^2 \text{ for } x \geq 1 \text{ and } f(x) \equiv -\tfrac{1}{4}\left[\log \tfrac{1+\sqrt{1-x}}{1-\sqrt{1-x}} - i\pi\right]^2 \text{ for } x < 1. \end{gathered} \tag{4.27}$$

Both decays into gluons and photons are loop-induced and might in principle be affected by possible new particles running in the loops. The set-ups we are considering, however, assume that the only chiral degrees of freedom the Higgs couples to are the SM ones. This will certainly be modified if the top quark, for instance, is a composite particle since additional top-partners would then also be expected to have a significant coupling to the Higgs. Under our original assumption, the corrections to the $H\gamma\gamma$ and Hgg vertices originate from the modified Yukawa interactions only and the loop-decays can be safely computed in the framework of our effective theory. The higher order corrections to the decays are unaffected as long as QCD corrections are concerned, since they do not involve the Higgs couplings.

Figure 4.1 shows the branching ratios (BRs) as a function of ξ for $M_H = 125$. The Higgs branching have been calculated with the programe`eHDECAY`[4] The BRs into fermions are governed by the $(1-2\xi)^2/(1-\xi)$ prefactor of the corresponding partial widths: as ξ increases from 0, there is first a decrease of the fermionic BRs, until they vanish at $\xi = 0.5$ and then grow again with larger ξ. The same behaviour is observed in the decay into gluons, which is loop-mediated by quarks. The decays into gauge bosons show a complementary behaviour: for small ξ, due to the decreasing decay widths into fermions, the importance of the vector boson decays becomes more and more pronounced until a maximum value at $\xi = 0.5$ is reached. Above this value the branching ratios into gauge boson decrease with increasing Higgs decay widths into fermion final states: the Higgs boson becomes gaugephobic in the technicolor limit ($\xi \to 1$).

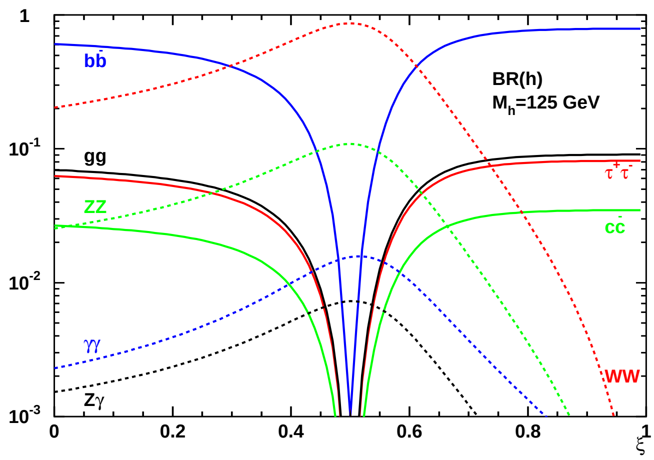

Figure 4.1: The branching ratios of MCHM5 as a function of ξ for $M_H = 125$ GeV.

4.4 Partial Compositeness

The question of the generation of fermion masses in composite Higgs models is solved by the hypothesis of partial compositeness. It assumes that the SM fermions, which are elementary, couple linearly to heavy states of the strong sector with the same quantum numbers, implying in particular the top quark to be largely composite. These couplings explicitly break the global symmetry of the strong sector. The Higgs potential is generated from loops of SM

[4]The program implements among others the MCHCM4/5 parametrisations and can be downloaded at the url: http://www.itp.kit.edu/~maggie/eHDECAY/.

particles with EWSB triggered by the top loops which provide the dominant contribution. The Higgs self-couplings therefore also depend on the representation of the fermions, and the Higgs boson mass is related to the fermion sector. It has been shown that a low-mass Higgs boson of ~ 125 GeV can naturally be accommodated only if the heavy quark partners are rather light, *i.e.* for masses below about 1 TeV. A Lagrangian with elementary fermions ψ that couple linearly to the heavy states χ of the strong sector, and which have the same quantum numbers, would look like

$$\mathcal{L}_{pc} = \bar{\psi} i \partial\!\!\!/ \psi + \bar{\chi}(i\partial\!\!\!/ - m_\star)\chi - \Delta_L \bar{\psi}_L \chi_R - \Delta_R \bar{\chi}_L \psi_R \ . \tag{4.28}$$

The fermions acquire their masses through mixing with the new vector-loke strong sector fermions. Due to the large Yukawa couplings the top quarks are largely composite. The linear couplings violate $\mathcal{G}$ explicitly and a Higgs potential is induced through the fermion loops.

4.5 Phenomenological Implications

Because of the modified couplings of the composite Higgs boson to the SM gauge bosons and fermions unitarity cannot be restored any more in longitudinal gauge boson scattering. Processes like $V_L V_L \to V_L V_L$ or Higgs pair production from gauge boson fusion will grow with the energy squared and hence be a smoking gun signature for composite Higgs models. The anomalous couplings also influence the compatibility with EW precision data. And of course also the Higgs production and decay rates are changed.

Constraints from electroweak precision tests (EPWT): The PeskinTakeuchi S, T, and U parameters parameterize potential new physics contributions to electroweak radiative corrections. The oblique corrections, to which the Peskin-Takeuchi parameters are sensitive, can be parameterized in terms of four vacuum polarization functions: the self-energies of the photon, Z boson, and W boson, and the mixing between the photon and the Z boson induced by loop diagrams. Due to the modified Higgs couplings to gauge bosons there is no cancellation of the UV divergencies and the S, equivalently ϵ_3, and T, equivalently ϵ_1 diverge logarithmically. The divergence is cut-off, hence regularized, by the mass m_ρ of the first resonance,

$$\Delta\epsilon_3^{IR} = \frac{\alpha(m_Z^2)}{48\pi \sin^2\theta_W} \xi \log\left(\frac{m_\rho^2}{m_Z^2}\right) \tag{4.29}$$

$$\Delta\epsilon_1^{IR} = -\frac{3\alpha(m_Z^2)}{16\pi \sin^2\theta_W} \xi \log\left(\frac{m_\rho^2}{m_Z^2}\right) \ . \tag{4.30}$$

In addition there are contributions from new fermions in loop. They can relax the EWPT constraints, *cf.* Fig. 4.2.

Further contributios to the S parameter or equivalently ϵ_3 arise from the mixing of the elementary gauge fields with new vector (ρ) and axialvector (a) resonances,

$$\Delta\epsilon_3^{\mathrm{UV}} = \frac{m_W^2}{m_\rho^2}\left(1 + \frac{m_\rho^2}{m_a^2}\right) \ . \tag{4.31}$$

By choosing m_ρ large enough, these contributions can be suppressed to a level that is compatible with the EW precision data. There are new contributions to the precisely measured

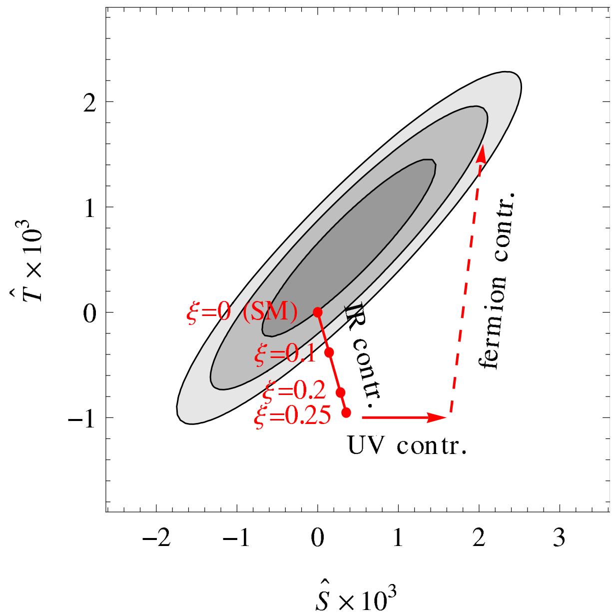

Figure 4.2: Constraints on the oblique EW parameters $\hat{S}$ and $\hat{T}$. The gray ellipses correspond to the 68%, 95% and 99% confidence level contours for mh = 126 GeV and mt = 173 GeV. The red lines show the contributions that arise in composite Higgs models as explained in the main text. From C. Grojean, O. Matsedonskyi and G. Panico, JHEP **1310** (2013) 160.

Zb_Lb_L coupling due to new fermions in the loop corrected coupling.

The new heavy fermions from the hypothesis of partial compositeness could be produced at the LHC. Their non-observation places constraints on the lower bound of their masses. Figure 4.3 shows the mass of the lightest composite fermion as a function of ξ. The points in the plot are the ones which pass the EWPT at 99% C.L. and fulfill $|V_{tb}| > 0.92$. The light blue points are excluded by direct searches at 95% C.L., the dark blue points are not excluded. The line in the plot marks the exclusion limit from CMS of 770 GeV on charge-5/3 fermions. As can be inferred from the plot this exclusion limit eliminates quite some parameter space for $m_{\text{lightest}} > 770$ GeV. No points are excluded above masses of the lightest partner of 770 GeV.

As already mentioned before, the modified Higgs couplings change the production and decay rates and hence the Higgs boson signal rates. The requirement of compatibility with the Higgs data therefore put further constaints on composite Higgs models.

Flavour physics can lead to further constraints on composite Higgs models. They depend, however, on the exact flavour structure of the model and shall not be discussed here further.

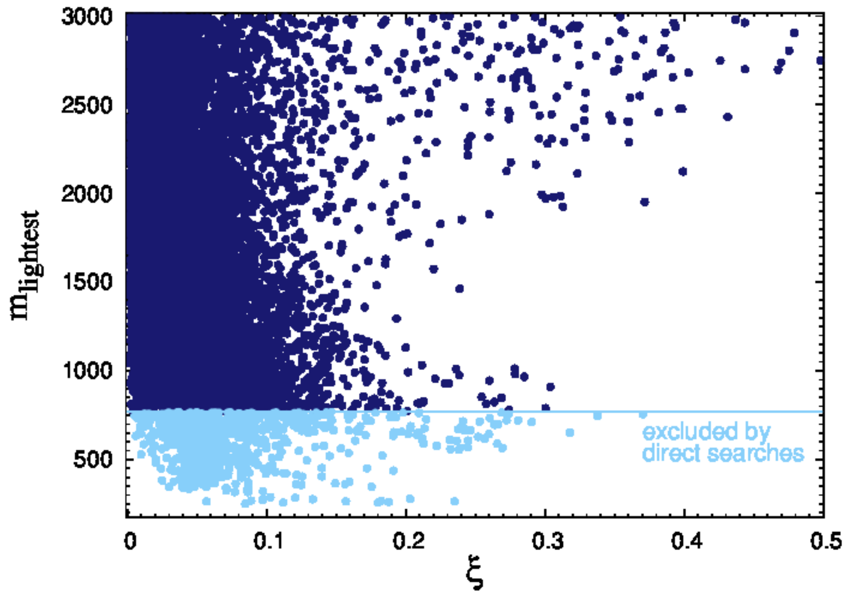

Figure 4.3: Spectrum of the lightest composite fermion as function of ξ. The points in the plot are obtained from a scan over the parameters of a model with composite b quarks. The light blue points are excluded by direct searches for vector-like fermions at 95% C.L., the dark blue points are not excluded. From M. Gillioz, R. Gröber, A. Kapuvari and M. Mühlleitner, JHEP **1403** (2014) 037; for more details, see there.

Chapter 5

Appendix

5.1 Beispiel: Feldtheorie für ein komplexes Feld

Wir betrachten die Lagrangedichte für ein komplexes Skalarfeld

$$\mathcal{L} = (\partial_\mu \phi)^* (\partial^\mu \phi) - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2 \quad \text{mit dem Potential} \quad V = \mu^2 \phi^* \phi + \lambda (\phi^* \phi)^2 \,. \tag{5.1}$$

(Hinzufügen höherer Potenzen in ϕ führt zu einer nicht-renormierbaren Theorie.) Die Lagrangedichte ist invariant unter einer $U(1)$-Symmetrie,

$$\phi \to \exp(i\alpha)\phi \,. \tag{5.2}$$

Wir betrachten den Grundzustand. Dieser ist gegeben durch das Minimum von V,

$$0 = \frac{\partial V}{\partial \phi^*} = \mu^2 \phi + 2\lambda(\phi^* \phi)\phi \quad \Rightarrow \quad \phi = \begin{cases} 0 & \text{für} \quad \mu^2 > 0 \\ \phi^* \phi = -\frac{\mu^2}{2\lambda} & \text{für} \quad \mu^2 < 0 \end{cases} \tag{5.3}$$

Der Parameter λ muß positiv sein, damit das System nicht instabil wird. Für $\mu^2 < 0$ nimmt das Potential die Form eines Mexikanerhutes an, siehe Fig. 5.1. Bei $\phi = 0$ liegt ein lokales Maximum, bei

$$|\phi| = v = \sqrt{-\frac{\mu^2}{2\lambda}} \tag{5.4}$$

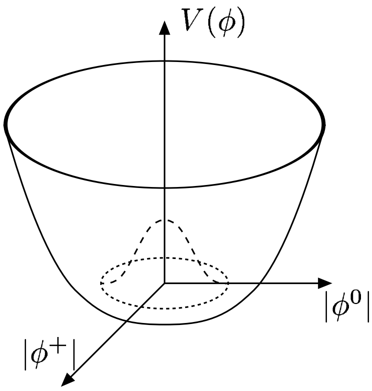

Figure 5.1: Das Higgspotential.

ein globales Minimum. Teilchen entsprechen harmonischen Oszillatoren für die Entwicklung um das Minimum des Potentials. Fluktuationen in Richtung der (unendlich vielen degenerierten) Minima besitzen Steigung null und entsprechen masselosen Teilchen, den Goldstone Bosonen. Fluktuationen senkrecht zu dieser Richtung entsprechen Teilchen mit Masse $m > 0$. Die Entwicklung um das Maximum bei $\phi = 0$ würde zu Teilchen negativer Masse (Tachyonen) führen, da die Krümmung des Potentials hier negativ ist.

Entwicklung um das Minimum bei $\phi = v$ führt zu (wir haben für das komplexe skalare Feld zwei Fluktuationen φ_1 und φ_2)

$$\begin{aligned} \phi &= v + \frac{1}{\sqrt{2}}(\varphi_1 + i\varphi_2) = \left(v + \frac{1}{\sqrt{2}}\varphi_1\right) + i\frac{\varphi_2}{\sqrt{2}} \quad \Rightarrow && (5.5) \\ \phi^*\phi &= v^2 + \sqrt{2}v\varphi_1 + \frac{1}{2}(\varphi_1^2 + \varphi_2^2) \; . && (5.6) \end{aligned}$$

Damit erhalten wir für das Potential

$$\begin{aligned} V &= \lambda(\phi^*\phi - v^2)^2 - \frac{\mu^4}{4\lambda^2} \qquad \text{mit} \qquad v^2 = -\frac{\mu^2}{2\lambda} \quad \Rightarrow && (5.7) \\ V &= \lambda\left(\sqrt{2}v\varphi_1 + \frac{1}{2}(\varphi_1^2 + \varphi_2^2)\right)^2 - \frac{\mu^4}{4\lambda^2} \; . && (5.8) \end{aligned}$$

Vernachlässige den letzten Term in V, da es sich nur um eine konstante Nullpunktsverschiebung handelt. Damit ergibt sich für die Lagrangedichte

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\varphi_1)^2 + \frac{1}{2}(\partial_\mu\varphi_2)^2 - 2\lambda v^2\varphi_1^2 - \sqrt{2}v\lambda\varphi_1(\varphi_1^2 + \varphi_2^2) - \frac{\lambda}{4}(\varphi_1^2 + \varphi_2^2)^2 \; . \tag{5.9}$$

Die in den Feldern quadratischen Terme liefern die Massen, die in den Feldern kubischen und quartischen Terme sind die Wechselwirkungsterme. Es gibt ein massives und ein masseloses Teilchen,

$$m_{\varphi_1} = 2v\sqrt{\lambda} \quad \text{und} \quad m_{\varphi_2} = 0 \; . \tag{5.10}$$

Bei dem masselosen Teilchen handelt es sich um das Goldstone Boson.

Bibliography

[1] J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. **127** (1962) 965; S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264; S.L. Glashow, S. Weinberg, Phys. Rev. Lett. **20** (1968) 224; A. Salam, Proceedings Of The Nobel Symposium, Stockholm 1968, ed. N. Svartholm.

[2] P.W. Higgs, Phys. Lett. **12** (1964) 132; Phys. Rev. Lett. **13** (1964) 508 and Phys. Rev. **145** (1966) 1156; F. Englert and R. Brout, Phys. Rev. Lett. **13** (1964) 321; G.S. Guralnik, C.R. Hagen and T.W. Kibble, Phys. Rev. Lett. **13** (1964) 585.

[3] S. Kanemura, Y. Okada, E. Senaha and C.-P. Yuan, Phys. Rev. D **70** (2004) 115002 [hep-ph/0408364].

[4] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher and J. P. Silva, Phys. Rept. **516** (2012) 1 [arXiv:1106.0034 [hep-ph]].

[5] J.F. Gunion, H. Haber, G. Kane and S. Dawson, *"The Higgs Hunter's Guide"*, Perseus Books, 1990.

[6] S. L. Glashow and S. Weinberg, Phys. Rev. D **15** (1977) 1958.

[7] S. L. Glashow and S. Weinberg, Phys. Rev. D **15** (1977) 1958; E. A. Paschos, Phys. Rev. D **15** (1977) 1966.

[8] M. Aoki, S. Kanemura, K. Tsumura and K. Yagyu, Phys. Rev. D **80** (2009) 015017 [arXiv:0902.4665 [hep-ph]].

[9] S. Davidson and H.E. Haber, Phys. Rev. **D72**, 035004 (2005); Erratum-ibid. **D72**, 099902 (2005); I.F. Ginzburg and M. Krawczyk, Phys. Rev. **D72**, 115013 (2005). F.J. Botella and J.P. Silva, Phys. Rev. **D51**, 3870 (1995).

[10] Y. L. Wu and L. Wolfenstein, Phys. Rev. Lett. **73** (1994) 1762 [hep-ph/9409421].

[11] F. J. Botella and J. P. Silva, Phys. Rev. D **51** (1995) 3870 [hep-ph/9411288].

[12] C. C. Nishi, Phys. Rev. D **74** (2006) 036003 [Erratum-ibid. D **76** (2007) 119901] [hep-ph/0605153].

[13] P. M. Ferreira, R. Santos and A. Barroso, Phys. Lett. B **603** (2004) 219 [Erratum-ibid. B **629** (2005) 114] [hep-ph/0406231].

[14] A. Barroso, P. M. Ferreira and R. Santos, Phys. Lett. B **632** (2006) 684 [hep-ph/0507224].

[15] A. Barroso, P. M. Ferreira, R. Santos and J. P. Silva, Phys. Rev. D **74** (2006) 085016 [hep-ph/0608282].

[16] I. P. Ivanov, Phys. Rev. D **77** (2008) 015017 [arXiv:0710.3490 [hep-ph]].

[17] P. Bechtle, O. Brein, S. Heinemeyer, G. Weiglein and K. E. Williams, Comput. Phys. Commun. **181** (2010) 138 [arXiv:0811.4169 [hep-ph]].

[18] P. Bechtle, O. Brein, S. Heinemeyer, G. Weiglein and K. E. Williams, Comput. Phys. Commun. **182** (2011) 182 [arXiv:1102.1898 [hep-ph]].

[19] P. Bechtle, O, Brein, S. Heinemeyer, O. Stål, T. Stefaniak et al., Eur. Phys. J **C** 74 (2014) 2693 [arXiv:1311.0055 [hep-ph]].

[20] P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak and G. Weiglein, Eur. Phys. J **C** 74 (2014) 2711 [arXiv:1305.1933 [hep-ph]].

[21] R. V. Harlander, S. Liebler and H. Mantler, Comp. Phys. Commun. **184** (2013) 1605 [arXiv:1212.3249 [hep-ph]].

[22] A. Djouadi, M. Spira and P.M. Zerwas, Phys. Lett. B **264** (1991) 440 and Z. Phys. C **70** (1996) 427; M. Spira *et al.*, Nucl. Phys. B **453** (1995) 17; A. Djouadi, J. Kalinowski and M. Spira, Comput. Phys. Commun. **108** (1998) 56; J. M. Butterworth, A. Arbey, L. Basso, S. Belov, A. Bharucha, F. Braam, A. Buckley and M. Campanelli *et al.*, arXiv:1003.1643 [hep-ph].

[23] F. Mahmoudi, Comput. Phys. Commun. **178** (2008) 745 [arXiv:0710.2067 [hep-ph]]; F. Mahmoudi, Comput. Phys. Commun. **180** (2009) 1579 [arXiv:0808.3144 [hep-ph]].

[24] R. Coimbra, M. O. P. Sampaio and R. Santos, Eur. Phys. J. C **73** (2013) 2428 [arXiv:1301.2599].

[25] P. M. Ferreira, J. F. Gunion, H. E. Haber and R. Santos, Phys. Rev. D **89** (2014) 115003 [arXiv:1403.4736 [hep-ph]]; P. M. Ferreira, R. Guedes, M. O. P. Sampaio and R. Santos, arXiv:1409.6723 [hep-ph].

[26] R. Contino, Y. Nomura and A. Pomarol, Nucl. Phys. B **671** (2003) 148 [hep-ph/0306259].

[27] K. Agashe, R. Contino and A. Pomarol, Nucl. Phys. B **719** (2005) 165 [hep-ph/0412089].

[28] R. Contino, L. Da Rold and A. Pomarol, Phys. Rev. D **75** (2007) 055014 [hep-ph/0612048].