Karlsruher Institut für Technologie

Institute for Theoretical Physics (ITP)
Karlsruhe Institute of Technology (KIT)

General Relativity II
Lecturer: Prof. Dr. Frans R. Klinkhamer
Assistant: Dr. Viacheslav A. Emelyanov

- Handing-in: Monday, 08.05.2017; Discussion: Wednesday, 10.05.2017
- All up-to-date information related to the course can be found under the link: https://www.itp.kit.edu/~slava/relativitaetstheorie_ss_17.html

Name: _____ Points: _____

Exercise Sheet 2

Exercise 2.1: Particle and event horizon (6 points)

For a particular observer O at cosmic time t, the *particle horizon* is the boundary of a space-time volume from which he or she can receive information. The *event horizon* is the boundary between space-time events which will at some time be observable by O, and those that will never be observable by O.

(a) We consider the FRW universe with

$$ds^2 \;=\; a^2(\eta)\big[d\eta^2 - d\chi^2 - \Phi^2(\chi) d\Omega^2\big]\,, \tag{1a}$$

$$\Phi^2(\chi) \;=\; \begin{cases} \sinh^2\chi\,, & k=-1, \\ \chi^2\,, & k=0\,, \\ \sin^2\chi\,, & k=+1\,. \end{cases} \tag{1b}$$

Studying the radial propagation of light, i.e. the light propagation in the $\eta\chi$-plane, derive the physical size of the particle and event horizon at a given moment of the cosmic time t.

(b) Suppose the universe is radiation-dominated (i.e. $P = \frac{1}{3}\rho$) and closed (i.e. $k = +1$). Find $a(\eta)$ and compute then the physical size of the particle and event horizon.

(c) Consider a closed and dust-dominated (i.e. $P = 0$) universe. Determine $a(\eta)$ and the physical size of the particle and event horizon.

State 02. 05. 2017 at 17:23:32

Exercise 2.2: Conformal diagram (6 points)

The causal structure of a spherically symmetric universe is encoded in the temporal-radial part of its metric. Thus, it can be represented by a 2-dimensional *conformal diagram* in which every point corresponds to a 2-dimensional sphere, whereas the η and χ coordinates are bounded.

(a) Draw the conformal diagram of the universe from Exercise 2.1.b. Depict the particle and event horizon for a freely falling observer at $\chi = 0$.

(b) Draw the conformal diagram of the universe from Exercise 2.1.c. Depict the particle and event horizon for a freely falling observer at $\chi = 0$.

(c) Draw the conformal diagram of Minkowski spacetime for an inertial observer.
Hint: One needs to bring the Minkowski metric in the form conformal to the closed Einstein universe, $ds^2 = dt^2 - dr^2 - r^2 d\Omega^2 \propto d\eta^2 - d\chi^2 - \sin^2\chi d\Omega^2$*, for the new radial coordinate* χ *to be bounded. This implies that* $r \propto \sin\chi$*. Taking this into account, as well as the absence of the particle and event horizon in Minkowski spacetime, draw the conformal diagram.*