Institute for Theoretical Physics (ITP)
Karlsruhe Institute of Technology (KIT)

General Relativity II

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- Handing-in: Monday, 22.05.2017; Discussion: Wednesday, 24.05.2017
- All up-to-date information related to the course can be found under the link: https://www.itp.kit.edu/~slava/relativitaetstheorie_ss_17.html

Name: ________________________________ Points: __________

Exercise Sheet 4

Exercise 4.1: Excess of particles over antiparticles (6 points)

(a) Show that the chemical potential μ_b of bosons cannot exceed their mass, while the chemical potential μ_f of fermions can be arbitrary large.

(b) Employing the results of Exercise 3.2.a, one can obtain that

$$n_b - n_{\bar{b}} \;=\; \frac{g_b T^3}{\pi^2} \int\limits_{m_b/T}^{+\infty} dx x\big[x^2 - (m_b/T)^2\big]^{\frac{1}{2}} \frac{e^x \sinh(\mu_b/T)}{e^{2x} + 1 - 2e^x \cosh(\mu_b/T)} \,. \tag{1}$$

Compute $n_b - n_{\bar{b}}$ at high/low temperatures for bosons with g_b internal degrees of freedom.

(c) The results of Exercise 3.2.a imply that

$$n_f - n_{\bar{f}} \;=\; \frac{g_f T^3}{\pi^2} \int\limits_{m_f/T}^{+\infty} dx x\big[x^2 - (m_f/T)^2\big]^{\frac{1}{2}} \frac{e^x \sinh(\mu_f/T)}{e^{2x} + 1 + 2e^x \cosh(\mu_f/T)} \,. \tag{2}$$

Compute $n_f - n_{\bar{f}}$ at high/low temperatures for fermions with g_f internal degrees of freedom.

Exercise 4.2: Deuterium abundance (6 points)

(a) Consider the reaction $p + n \rightleftarrows D^+ + \gamma$. Compute the deuterium equilibrium abundance, i.e.

$$X_D \;\equiv\; \frac{2n_D}{n_N} \,, \tag{3}$$

where n_N is a total number of nucleons (baryons), as a function of the proton and neutron abundances, i.e. $X_p = n_p/n_N$ and $X_n = n_n/n_N$, respectively.

Hint: The Big Bang Nucleosynthesis took place at the temperature $T_{BBN} \sim 0.7\,MeV$. *This means that protons and neutrons were non-relativistic at that time.*

The number density of deuteriums at temperatures much smaller than $m_D - \mu_D$ *is approximately given by*

$$n_D \approx g_D \Big(\frac{m_D T}{2\pi}\Big)^{\frac{3}{2}} \exp\Big(-\frac{m_D - \mu_D}{T}\Big)\,.$$

The analogous expression holds for protons and neutrons. The number density of photons can be extracted from Exercise 3.2.c.

(b) Estimate the temperature T at which $X_D \sim \mathrm{O}(1)$, employing your result of Exercise 4.2.a and making use of $\eta_{10} \equiv 10^{10}(n_N/n_\gamma) \approx 6.2$, $X_p \approx 0.84$ and $X_n \approx 0.16$ at that epoch.

(c) Why was X_D, actually, much smaller than 1 at that epoch?