Institute for Theoretical Physics (ITP)
Karlsruhe Institute of Technology (KIT)

General Relativity II

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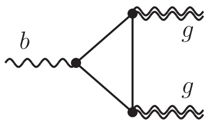
- `Handing-in: Monday, 29.05.2017; Discussion: Wednesday, 31.05.2017`
- `All up-to-date information related to the course can be found under the link: https://www.itp.kit.edu/~slava/relativitaetstheorie_ss_17.html`

Name: ______________________ Points: __________

Exercise Sheet 5

Exercise 5.1: Gravitational anomaly in the Standard Model (8 points)

All matter fields as well as gauge fields are coupled to gravity. Thus, the process $b \to gg$ (where b and g are, respectively, a gauge boson and graviton) can, in principle, occur through the one-loop diagram:



This diagram is proportional to $\langle \hat{J}(x)\hat{T}(y)\hat{T}(z)\rangle$ in symbolic notation, where the current is either

$$J_\alpha(x) \;\propto\; \bar{\psi}(x)\gamma_\alpha\psi(x) \quad \text{or} \quad J^a_\alpha(x) \;\propto\; \mathrm{Tr}\big[\bar{\psi}(x)T^a\gamma_\alpha\psi(x)\big] \tag{1}$$

with T^a being a non-Abelian group generator, and the stress-energy tensor reads

$$T^{\mu\nu}(x) \;\propto\; \bar{\psi}(x)\Big(\gamma^\mu\overset{\leftrightarrow}{\partial^\nu} + \gamma^\nu\overset{\leftrightarrow}{\partial^\mu}\Big)\psi(x)\,. \tag{2}$$

(a) What does this process imply physically?

(b) Argue why this process is certainly forbidden for non-Abelian gauge bosons.

(c) This argument is inapplicable for the Abelian gauge boson associated with $\mathrm{U}(1)_Y$ and, hence, the reaction is possible, but with chiral fermions only. This is known as the gravitational anomaly. Consider chiral fermions $\psi_L(x)$ and $\psi_R(x)$ of hypercharge Y_L and Y_R, respectively. Determine how $\langle \hat{J}_L(x)\hat{T}_L(y)\hat{T}_L(z)\rangle$ and $\langle \hat{J}_R(x)\hat{T}_R(y)\hat{T}_R(z)\rangle$ are related to each other.

(d) Can this process occur for the Abelian gauge boson in the Standard Model?

Exercise 5.2: Baryon asymmetry generation (4 points)

(a) Why does baryogenesis require both C and CP violation?

(b) Are Sakharov's baryogenesis conditions fulfilled in the Standard Model?