Karlsruher Institut für Technologie

Institute for Theoretical Physics (ITP)
Karlsruhe Institute of Technology (KIT)

General Relativity II
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- Handing-in: Monday, 19.06.2017; Discussion: Wednesday, 21.06.2017
- All up-to-date information related to the course can be found under the link: https://www.itp.kit.edu/~slava/relativitaetstheorie_ss_17.html

Name: ____________________ Points: ________

Exercise Sheet 8

Exercise 8.1: Derrick's theorem (4 points)

We consider the 4-dimensional Minkowski spacetime. Show that any static localised solution of

$$\Box\Phi + V'(\Phi) \;=\; 0 \tag{1}$$

cannot be stable.

Remark: The solution of (1) *is said to be static if it does not depend on time. It is localised if it possesses finite energy, i.e.*

$$E \;=\; \int d^3\mathbf{x}\left(\frac{1}{2}(\nabla\Phi)^2 + V(\Phi)\right) \;<\; \infty\,. \tag{2}$$

Exercise 8.2: Leptogenesis and neutrino masses (8 points)

A lepton-number excess can be generated via heavy right-handed Majorana neutrinos. This excess can, in turn, be partially transformed into a baryon-number excess by electroweak sphalerons.

(a) Neutrinos are massless in the Standard Model, but massive in nature. These neutrinos can acquire a gauge-invariant mass through a Yukawa coupling to the Higgs field (like quarks and charged leptons), but only if one introduces singlet right-handed neutrinos. Why is this method, however, less appealing for neutrinos?

Remark: If one stays in the framework of the Standard Model, i.e. its particle content and gauge-interaction structure, one can generate neutrino masses through a non-renormalizable term (Weinberg, 1979 & Klinkhamer, arXiv:1112.2669).

(b) For the sake of simplicity, we consider the first generation of leptons in the following. Suppose that we have a right-handed neutrino ν_R with $\mathcal{L}_{\nu,\text{Majorana}} = -\frac{1}{2}m_\text{M}\bar{\nu}_R^c\nu_R + \text{h.c.}$, where the charge conjugate of ν_R is given by $\nu_R^c \equiv -i\gamma_2\nu_R^*$ in the Weyl representation of the gamma matrices. Show that the lepton number is not conserved due to the Majorana mass term.

(c) Let us now assume that ν_R is coupled to the Standard Model left-handed neutrino ν_L and the Higgs field H as follows: $\mathcal{L}_{\nu,\text{Yukawa}} = -y_\nu\bar{L}\cdot\tilde{H}\nu_R + \text{h.c.}$, where $L = (\nu_L, e_L^-)^\text{T}$ is the lepton doublet and $\tilde{H} \equiv i\sigma_2\cdot H^*$. Show that the left- and right-handed neutrinos acquire a non-vanishing Dirac mass m_D after electroweak symmetry breaking, namely we have that $\mathcal{L}_{\nu,\text{Dirac}} = -m_\text{D}\bar{\nu}_L\nu_R + \text{h.c.}$

(d) Show that the total mass term, i.e. $\mathcal{L}_{\nu,\text{Dirac}} + \mathcal{L}_{\nu,\text{Majorana}}$, can be rewritten as follows:

$$\mathcal{L}_{\nu,\text{mass}} = -\frac{1}{2}\begin{pmatrix}\bar{\nu}_L^c\\ \bar{\nu}_R\end{pmatrix}^\text{T}\cdot M\cdot\begin{pmatrix}\nu_L\\ \nu_R^c\end{pmatrix} + \text{h.c.} \quad \text{with} \quad M \equiv \begin{pmatrix}0 & m_\text{D}\\ m_\text{D} & m_\text{M}\end{pmatrix}. \tag{3}$$

(e) Find the eigenstates and eigenvalues of M for $m_\text{D} \ll m_\text{M}$.

(f) Argue why the lepton number excess can be generated in this model.