Karlsruher Institut für Technologie

Institute for Theoretical Physics (ITP)
Karlsruhe Institute of Technology (KIT)

General Relativity II

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- `Handing-in: Monday, 26.06.2017; Discussion: Wednesday, 28.06.2017`
- `All up-to-date information related to the course can be found under the link: https://www.itp.kit.edu/~slava/relativitaetstheorie_ss_17.html`

Name: _____ Points: _____

Exercise Sheet 9

Exercise 9.1: Scalar field model and inflation (12 points)

Let us consider a scalar field model with the following Lagrangian density:

$$\mathcal{L} \;=\; -\frac{1}{2}\nabla_\mu\Phi\nabla^\mu\Phi + V(\Phi)\,. \tag{1}$$

We assume that the space-time geometry is described by an FRW universe.

(a) Show that the stress tensor of the scalar field can be written in the perfect-fluid-like form:

$$T_{\mu\nu}(\Phi) \;=\; \big(\rho + P\big)u_\mu u_\nu - P g_{\mu\nu}\,. \tag{2}$$

(b) Assume that $\Phi(x) = \Phi(t)$, where t is the cosmic time. Under which condition imposed on the potential $V(\Phi)$ does the universe expand exponentially fast?

(c) Derive the scalar field equation which the homogeneous field $\Phi(t)$ satisfies.

(d) From now on we set $k = 0$, i.e. the universe has a flat spatial section. Show that there exists an inflationary phase if $V(\Phi) = \frac{1}{2}m^2\Phi^2$. Estimate the duration of this phase.

(e) Show that if the slow-roll approximation holds, i.e.

$$G^{-1}\big(V'/V\big)^2 \;\ll\; 1 \quad \text{and} \quad G^{-1}\big|V''/V\big| \;\ll\; 1\,, \tag{3}$$

where the prime stands for the derivative with respect to Φ, then the scale factor is given by

$$a(t) \;\approx\; a_i \exp\left(8\pi G \int_{\Phi(t)}^{\Phi_i} \big(V/V_{,\Phi}\big)d\Phi\right). \tag{4}$$

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(f) Show that the slow-roll approximation can be fulfilled for the potential $V(\Phi) = \frac{1}{2}m^2\Phi^2$ if and only if Φ has a trans-Planckian value. Argue why the semi-classical approximation, i.e. absence of non-perturbative quantum gravity, can still be applicable.