A213939, Wolfdieter Lang , Jul 20 2012
Partition array for the number of representative bracelets (dihedral symmetry D_n) with n beads, each available in n colors. Only the color type (signature) matters.

n\k	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**
1	1																					
2	1	1																				
3	1	1	1																			
4	1	1	2	2	3																	
5	1	1	2	2	4	6	12															
6	1	1	3	3	3	6	11	10	16	30	60											
7	1	1	3	4	3	9	10	18	15	30	48	60	90	180	360							
8	1	1	4	5	8	4	12	19	33	38	21	54	70	108	171	105	210	318	420	630	1260	2520
...																						

Row No. 9: [1, 1, 4, 7, 10, 4, 16, 28, 38, 48, 76, 94, 28, 84, 140, 216, 280, 432, 168, 420, 560, 840, 1272, 840, 1680, 2520, 3360, 5040, 10080, 20160]

Row No. 10: [1, 1, 5, 8, 16, 16, 5, 20, 44, 66, 74, 132, 174, 216, 36, 128, 252, 318, 384, 636, 840, 978, 1272, 252, 756, 1260, 1896, 2520, 3792, 5736, 1512, 3780, 5040, 7560, 11352, 7560, 15120, 22680, 30240, 45360, 90720, 181440]

Row No. 11: [1, 1, 5, 10, 20, 26, 5, 25, 60, 110, 126, 100, 220, 330, 420, 540, 45, 180, 420, 630, 640, 1260, 1590, 2100, 1920, 3180, 4200, 360, 1260, 2520, 3150, 3780, 6300, 8400, 9480, 12600, 18960, 2520, 7560, 12600, 18900, 25200, 37800, 56760, 15120, 37800, 50400, 75600, 113400, 75600, 151200, 226800, 302400, 453600, 907200, 1814400]

Row No. 12: [1, 1, 6, 12, 29, 38, 50, 6, 30, 85, 170, 236, 140, 340, 610, 708, 781, 1170, 1493, 55, 250, 660, 1160, 1386, 1000, 2320, 3480, 4620, 5790, 3530, 6960, 8760, 11580, 15402, 495, 1980, 4620, 6930, 6940, 13860, 17340, 23100, 20820, 34680, 46200, 52170, 69360, 3960, 13860, 27720, 34650, 41580, 69300, 92400, 103980, 138600, 207960, 312240, 27720, 83160, 138600, 207900, 277200, 415800, 623760, 166320, 415800, 554400, 831600, 1247400, 831600, 1663200, 2494800, 3326400, 4989600, 9979200, 19958400]

Row No. 13: [1, 1, 6, 14, 35, 57, 76, 6, 36, 110, 255, 396, 472, 180, 510, 1020, 1416, 1320, 2340, 2772, 3510, 66, 330, 990, 1980, 2772, 1500, 3960, 6960, 8316, 9240, 13860, 17370, 6000, 13920, 20880, 27720, 34740, 46200, 660, 2970, 7920, 13860, 16632, 11880, 27720, 41580, 55440, 69300, 41640, 83160, 104040, 138600, 184800, 124920, 208080, 277200, 5940, 23760, 55440, 83160, 83160, 166320, 207900, 277200, 249480, 415800, 554400, 623880, 831600, 1247760, 47520, 166320, 332640, 415800, 498960, 831600, 1108800, 1247400, 1663200, 2494800, 3742560, 332640, 997920, 1663200, 2494800, 3326400, 4989600, 7484400, 1995840, 4989600, 6652800, 9979200, 14968800, 9979200, 19958400, 29937600, 39916800, 59875200, 119750400, 239500800]

Row No. 14: [1, 1, 7, 16, 47, 79, 126, 133, 7, 42, 146, 365, 651, 868, 237, 730, 1665, 2604, 3078, 2160, 4320, 6036, 7620, 9054, 78, 432, 1430, 3225, 5148, 6016, 2160, 6450, 12900, 18048, 17160, 30060, 36036, 45090, 9765, 25800, 45270, 54144, 60120, 90180, 112950, 120120, 150240, 858, 4290, 12870, 25740, 36036, 19320, 51480, 90120, 108108, 120120, 180180, 225270, 77280, 180240, 270360, 360360, 450540, 600600, 270720, 540720, 676350, 901080, 1201200, 8580, 38610, 102960, 180180, 216216, 154440, 360360, 540540, 720720, 900900, 540600, 1081080, 1351440, 1801800, 2402400, 1621800, 2702880, 3603600, 4055400, 5405760, 77220, 308880, 720720, 1081080, 1081080, 2162160, 2702700, 3603600, 3243240, 5405400, 7207200, 8108280, 10810800, 16216560, 24327000, 617760, 2162160, 4324320, 5405400, 6486480, 10810800, 14414400, 16216200, 21621600, 32432400, 48648960, 4324320, 12972960, 21621600, 32432400, 43243200, 64864800, 97297200, 25945920, 64864800, 86486400, 129729600, 194594400, 129729600, 259459200, 389188800, 518918400, 778377600, 1556755200, 3113510400]

Row No. 15: [1, 1, 7, 19, 56, 111, 185, 232, 7, 49, 182, 511, 1001, 1519, 1716, 294, 1022, 2555, 4557, 6076, 3338, 7560, 12012, 14086, 15120, 21126, 25226, 91, 546, 2002, 5005, 9009, 12012, 3024, 10010, 22575, 36036, 42112, 30030, 60060, 84084, 105210, 126126, 15120, 45150, 90300, 126336, 120120, 210420, 252252, 315630, 280284, 420420, 525840, 1092, 6006, 20020, 45045, 72072, 84084, 30030, 90090, 180180, 252252, 240240, 420420, 504504, 630630, 135240, 360360, 630840, 756756, 840840, 1261260, 1576890, 1681680, 2102100, 540960, 1261680, 1892520, 2522520, 3153780, 4204200, 5605608, 12012, 60060, 180180, 360360, 504504, 270270, 720720, 1261260, 1513512, 1681680, 2522520, 3153150, 1081080, 2522520, 3783780, 5045040, 6306300, 8408400, 3784200, 7567560, 9460080, 12612600, 16816800, 11352600, 18920160, 25225200, 120120, 540540, 1441440, 2522520, 3027024, 2162160, 5045040, 7567560, 10090080, 12612600, 7567560, 15135120, 18918900, 25225200, 33633600, 22702680, 37837800, 50450400, 56757960, 75675600, 113515920, 1081080, 4324320, 10090080, 15135120, 15135120, 30270240, 37837800, 50450400, 45405360, 75675600, 100900800, 113513400, 151351200, 227026800, 340542720, 8648640, 30270240, 60540480, 75675600, 90810720, 151351200, 201801600, 227026800, 302702400, 454053600, 681080400, 60540480, 181621440, 302702400, 454053600, 605404800, 908107200, 1362160800, 363242880, 908107200, 1210809600, 1816214400, 2724321600, 1816214400, 3632428800, 5448643200, 7264857600, 10897286400, 21794572800, 43589145600]

The length of row No. n is given by the partition numbers A000041(n): [1, 2, 3, 5, 7, 11, 15, 22, 30, 42, 56, 77, 101, 135, 176].
The row sums give A213943: [1, 2, 3, 9, 28, 144, 832, 6012, 48447, 444198, 4469834, 49650464, 597810739, 7809600123, 109524985564,…]

See also the link given in A212359 with comments on representative multiset used for the color multinomials.

The formula for a(n,k), the representative bracelet numbers, is based on Z(D_n,c_n), with the cycle index polynomial of the dihedral group D_n (of order 2n) and the figure counting polynomial c_n:=sum(c[i],i=1..n). The formula for Z(D_n) is given, e.g., in the Harary-Palmer reference on p.37, Corollary, eq. (2.2.11). We use here variables x[j] instead of their s_j. See also p.42, Theorem (PET= P\'olya's enumeration theorem) for the abbreviation Z(D_n,c), which stands for the replacement of the Z(D_n) variables x[j], j=1,2,...,n, by the j-th power sum sum(c[i]^j,i=1..n). See also a link under A212355 for the cycle indices Z(D_n), n=1..15. One has to compute in Z(D_n,c_n) the coefficient of the multinomial corresponding to the multiset representative belonging to a partition of n, specifically to pa(n,m,j) for m=1,2,...,n and j=1,...,p(n,m), with p(n,m) the number of partitions of n with m parts, given in A008284. Call the corresponding multiset representative msr(n,m,j). E.g., the color multinomial in Z(D_n,c_n) corresponding to the 4-multiset representative denoted by {1^2,2,3} = {1,1,2,3} obtained, after `exponentiation', from the 4-partition [2,1^2] = [2,1,1] is c[1]^2*c[2]*c[3].This example should suffice to explain the general prescription.

Z(D_n) starts with Z(C_n)/2 and receives additional contributions. If n is **odd** there is only one extra contribution:

(1/2)*sum(c[i],i=1..n)*sum(c[i]^2,i=1..n)^((n-1)/2).

If n=1 this is just (1/2)*c[1]. For odd n >= 3,one scans the partitions of n in their reversed form (non-increasing parts) by subtracting 1 consecutively from each part and checks whether the resulting partition of n-1 has only even parts

(including 0). A possible 0 part is then removed. Only those subtractions which satisfy this condition will contribute. If the subtraction of 1 from the j-th part survives, all new parts (excluding 0) are divided by 2, ending up with some partition of (n-1)/2. The contribution for this subtraction is then the M_1 multinomial number for the obtained partition of (n-1)/2 multiplied by ½ of the color multinomial for this partition, with all exponents (including 1) doubled, and the extra color c[j] has to be multiplied. This last multiplication picks the missing j-th color from the first factor sum(c[i],i=1..n). The M_1 multinomials for partitions of (n-1)/2 enter here because of the second factor sum(c[i]^2,i=1..n)^((n-1)/2). The M_1 array is found in A036038.
An example will make this clear. Take n=5. Partition [5] leads after subtraction to [4]. Dividing by 2 results in [2], the first partition of n'=(5-1)/2 = 2. The corresponding M_1 number is 1, and the contribution from partition [5] is then ½*1*c[1]^(2*2)*c[1] = ½*c[1]^5. Partition [1,4], reversed as [4,1], leads to two cases: [3,1] and [4,0] → [4]. Only the second case (j=2) survives as [2] with the contribution ½*1*c[1]^(2*2)*c[2] = c[1]^4*c[1]/2. Partition [2,3], in its reversed form [3,2], only contributes from [2,2] (j=1) and not from [3,1] (j=2), leading to [1,1] with M_1 number 2. Therefore the contribution is ½*2*c[1]^(1*2)*c[2]^(1*2)*c[1] = c[1]^3*c[2]^2. In general there is no contribution from partitions which have a part 1 more than once. Therefore, partition [1^2,3], [1^3,2] and [1^5] can be skipped. Partition [1,2^2]=[1,2,2], reversed [2,2,1], leads to [1,1] for j=3 with the contribution ½*c[1]^(1*2)*c[2]^(1*2)*c[3] = c[1]^2*c[2]^2*c[3]. The total contribution for partition [5] is ½ + ½ = 1, because the representative necklace number for [5] is 1 (see A212359). Thus there is one representative bracelet with color multinomial 1*c[1]^5. Similarly, for [4,1] one finds ½ + ½ = 1, because the necklace number for [4,1] is also 1. Partition [3,2] leads to 2/2 + 1 = 2, because of its necklace number 2. Partition [2,2,1] leads to 6/2 + 1 = 4. All other partitions of n=5 have no extra contribution and their representative bracelet numbers are one half of their representative necklace numbers. Thus partitions [3,1,1], [2,1,1,1] and [1,1,1,1,1] (we use their reversed forms here) have representative bracelet numbers 2, 6 and 12, respectively.

The **even** n case has two possible extra contributions:

(i) from (1/4)*sum(c[i]^2,i=1..n)^(n/2), and
(ii) from (1/4)*sum(c[i],i=1..n)^2*sum(c[i]^2,i=1..n)^((n-2)/2).

The case (i) is similar to the necklace case with divisor d=2 (see the discussion in the link given in A212359) and we will skip this here.

The case (ii) leads for n=2, when only the first sum is present, to the two contributions ¼*c[1]^2 and ½*c[1]*c[2]. For even n >= 4, there will also be two possible contributions, (a) from subtracting 2 from the j-th part of a partition of n in reversed form (instead of 1 in the above described odd n case) and (b) from subtracting 1 at two different positions j and k with odd parts.

In case (a), the part 2 may appear more than once (in contrast to the odd n case, where the part 1 could not appear more than once). Each subtraction of 2 from these parts 2 has to be taken into account separately. Again one has to test whether the remaining partition has only even parts (possibly arising parts 0 are discarded), arriving, after dividing each part by a factor of 2, at a partition of (n-2)/2. Its corresponding M_1 number is determined and the contribution is then ¼*M_1 with the appropriate color multinomial. Here one has to take care of the positions where 2 was subtracted, especially if the original partition has multiple even parts. Each color exponent pertaining to the partition of (n-2)/2 has to be multiplied by a factor of 2, and a factor c[j]^2 has to be appended, if the 2 was subtracted at position j from the starting partition of n in its reversed form. An example will make this procedure clear. Take n=4. [4] → [2] → [1] with M_1 number 1, leads to ¼*1*c[1]^(1*2)*c[1]^2 (j=1), rewritten as ¼*c[1]^4. The reversed partition [3,1] does not contribute because subtracting 2 at either position does not lead to only even parts. Partition [2,2] leads to [0,2] → [1] (j=1) and [2,0] → [1] (j=2), giving ¼*1*c[2]^(1*2)*c[1]^2 = ¼*c[1]^2*c[2]^2 and ¼*1*c[1]^(1*2)*c[2]^2 = ¼*c[1]^2*c[2]^2, thus the result for [2,2] is ½*c[1]^2*c[2]^2 from both contributions. In general one will always obtain the same result for each of the multiple even parts. Therefore one computes the contribution from one subtraction and multiplies with the multiplicity. Partitions [2,1,1] and [1,1,1,1] do not contribute to this extra piece of type (a).

In case (b) when 1 has to be subtracted twice at different positions of a reversed partition there will be a contribution if precisely two parts are odd besides possibly even parts because after subtraction one has to have only even parts (including 0) for the resulting partition of (n-2)/2. Finally one has to multiply ¼ times the M_1 number times the appropriate color multinomial for the obtained partition of (n-2)/2, after doubling all exponents, with 2*c[j]*c[k] if the two 1's were subtracted at positions j and k. The example n=6 will suffice to explain this procedure. Only the partitions [5,1], [3,3], [4,1,1], [3,2,1] and [2,2,1,1] (all given in their reversed forms) can contribute because all other partitions of 6 do not have precisely two odd parts besides even ones. [5,1] → [4,0] = [4] → [2], the first partition of (6-2)/2 = 2 with M_1 number 1. The contribution is ¼*1*c[1]^(2*2)*2*c[1]*c[2] = ½*c[1]^5*c[2]. Similarly, [3,3] → [2,2] → [1,1], the second partition of 2 with M_1 number 2. The contribution is ¼*2*c[1]^(1*2)*c[2]^(1*2)*2*c[1]*c[2] = 1*c[1]^3*c[2]^3. [4,1,1] → [4,0,0]=[4] → [2], producing ¼*1*c[1]^(2*2)*2*c[2]*c[3] = ½*c[1]^4*c[2]*c[3]. [3,2,1] → [2,2,0]=[2,2] → [1,1], producing ¼*2*c[1]^(1*2)*c[2]^(1*2)*2*c[1]*c[3] = 1*c[1]^3*c[2]^2*c[3], and [2,2,1,1] → [2,2,0,0] = [2,2] → [1,1], producing ¼*2*c[1]^(1*2)*c[2]^(1*2)*2*c[3]*c[4] = 1*c[1]^2*c[2]^2*c[3]*c[4].

As an example for the complete contribution for even n we look at representative bracelets with 6 beads coming in up to 6 colors c[1], c[2], …,c[6]. To the extra term (i) [6] contributes with ¼*1*c[1]^(3*2) = ¼*c[1]^6, [4,2] (reversed form) with ¼*3*c[1]^(2*2)*c[2]^(1*2) = ¾*c[1]^4*c[2]^2, and [2,2,2] contributes ¼*6*c[1]^(1*2)*c[2]^(1*2)*c[3]^(1*2) = (3/2)*c[1]^2*c[2]^2*c[3]^2. All other partitions of 6 do not appear in the extra contribution of type (i). The two pieces (a) and (b) from the extra contribution of type (ii) are for partition [6] ¼*c[1]^(2*2)*c[1]^2 + 0 = 1/4*c[1]^6, for [5,1] one obtains 0 + ½*c[1]^5*c[2], [4,2] yields (½ + ¼)*c[1]^4*c[2]^2 + 0, [3,3] yields 0 + 1*c[1]^3*c[2]^3 , [4,1,1] yields (0 + ½)*c[1]^4*c[2]*c[3], [3,2,1] yields (0 + 1)*c[1]^3*c[2]^2*c[3], [2,2,2] yields (¼*3*2 + 0)*c[1]^2*c[2]^2*c[3]^2, and finally [2,2,1,1] yields (0 + 1) *c[1]^2*c[2]^2*c[3]*c[4]. Thus the partitions of 6 which receive additional contributions to ½ of their necklace numbers are: [6] with 1/4 + 1/4 = 1/2, [5,1] with 1/2, [4,2] with 3/4 + 3/4 = 3/2, [3,3] with 1 and [2,2,2] with 3/2 + 3/2 = 3. All-together the number of representative bracelets with 6 beads and up to 6 colors are therefore: 1/2 + 1/2 = 1, 1/2 + 1/2 = 1, 3/2 + 3/2 = 3, 2 + 1 = 3, 5/2 + 1/2 = 3, 5 + 1 = 6, 8 + (3/2 + 3/2) = 11, 10 + 0 = 10, 15 + 1 = 16, 30 + 0 = 30 and 60 + 0 = 60 for the eleven partitions of n=6 in A-St order.

Each representative n-bracelet color multinomal stands for a whole equivalence class of n-bracelets with different color choices from c[1], c[2], …, c[n] but the same color exponents or signature. The order of these equivalence classes are shown in the partition array A035206(n,k), k=1,2,...,p(n)=A000041(n) (in A-St order). E.g., the multiset representative {1,1,2,3}, obtained from the reversed partition [2,1,1] of n=4, encodes the color monomial c[1]^2*c[2]*c[3] which represents an equivalence class of 12 members, namely (in short notation c[j]-> j) 112, 113, 114, 221, 223, 224, 331, 332, 334, 441, 442 and 443. This order 12 = A035206(4,k=4), because the signature (the set of exponents) is, after reversion and exponentiation, provided by the partition [1,1,2], the 4-th in the A-St ordered list of the n=4 partitions.
Therefore, the weighted row sum sum(A03506(n,k)*a(n,k),k=1..p(n)=A000041(n)) is the

total number of bracelets with n beads, the colors chosen from a repertoire of n distinct colors. These numbers are [1, 3, 10, 55, 377, 4291, 60028, 1058058, 21552969, 500280022, 12969598086, 371514016094, 11649073935505, ...]. This is the sequence A081721. The partition array with the total bracelets numbers with given color signature corresponding to the reversed partitions, namely A03506(n,k)* a(n,k), can be found as A213941(n,k). The related triangle T(n,m) where all entries corresponding to the partitions of n with the same number of parts m are summed is A214306(n,m).

The color representative polynomials for n=1..10 are (note that these are not the partition polynomials: the partitions in A-St order appear here as exponents):

n=1: c[1].

n=2: c[1]^2 + c[1]*c[2].

n=3: c[1]^3 + c[1]^2*c[2] + 1*c[1]*c[2]*c[3].

n=4: c[1]^4 + c[1]^3*c[2] + 2*c[1]^2*c[2]^2 + 2*c[1]^2*c[2]*c[3] + 3*c[1]*c[2]*c[3]*c[4].

n=5: c[1]^5 + c[1]^4*c[2] + 2*c[1]^3*c[2]^2 + 2*c[1]^3*c[2]*c[3] + 4*c[1]^2*c[2]^2*c[3] + 6*c[1]^2*c[2]*c[3]*c[4] + 12*c[1]*c[2]*c[3]*c[4]*c[5].

n=6: c[1]^6 + c[1]^5*c[2] + 3*c[1]^4*c[2]^2 + 3*c[1]^3*c[2]^3 + 3*c[1]^4*c[2]*c[3] + 6*c[1]^3*c[2]^2*c[3] + 11*c[1]^2*c[2]^2*c[3]^2 + 10*c[1]^3*c[2]*c[3]*c[4] + 16*c[1]^2*c[2]^2*c[3]*c[4] + 30*c[1]^2*c[2]*c[3]*c[4]*c[5] + 60*c[1]*c[2]^2*c[3]*c[4]*c[5]*c[6].

n=7: c[1]^7 + c[1]^6*c[2] + 3*c[1]^5*c[2]^2 + 4*c[1]^4*c[2]^3 + 3*c[1]^5*c[2]*c[3] + 9*c[1]^4*c[2]^2*c[3] + 10*c[1]^3*c[2]^3*c[3] + 18*c[1]^3*c[2]^2*c[3]^2, 15*c[1]^4*c[2]*c[3]*c[4] + 30*c[1]^3*c[2]^2*c[3]*c[4] + 48*c[1]^2*c[2]^2*c[3]^2*c[4] + 60*c[1]^3*c[2]*c[3]*c[4]*c[5] + 90*c[1]^2*c[2]^2*c[3]*c[4]*c[5] + 180*c[1]^2*c[2]*c[3]*c[4]*c[5]*c[6] + 360*c[1]*c[2]*c[3]*c[4]*c[5]*c[6]*c[7].

n=8: c[1]^8 + c[1]^7*c[2] + 4*c[1]^6*c[2]^2 + 5*c[1]^5*c[2]^3 + 8*c[1]^4*c[2]^4 + 4*c[1]^6*c[2]*c[3] + 12*c[1]^5*c[2]^2*c[3] + 19*c[1]^4*c[2]^3*c[3] + 33*c[1]^4*c[2]^2*c[3]^2 + 38*c[1]^3*c[2]^3*c[3]^2 + 21*c[1]^5*c[2]*c[3]*c[4] + 54*c[1]^4*c[2]^2*c[3]*c[4] + 70*c[1]^3*c[2]^3*c[3]*c[4] + 108*c[1]^3*c[2]^2*c[3]^2*c[4] + 171*c[1]^2*c[2]^2*c[3]^2*c[4]^2 + 105*c[1]^4*c[2]*c[3]*c[4]*c[5] + 210*c[1]^3*c[2]^2*c[3]*c[4]*c[5] + 318*c[1]^2*c[2]^2*c[3]^2*c[4]*c[5] + 420*c[1]^3*c[2]*c[3]*c[4]*c[5]*c[6] + 630*c[1]^2*c[2]^2*c[3]*c[4]*c[5]*c[6] + 1260*c[1]^2*c[2]*c[3]*c[4]*c[5]*c[6]*c[7] + 2520*c[1]*c[2]*c[3]*c[4]*c[5]*c[6]*c[7]*c[8].

n=9: c[1]^9 + c[1]^8*c[2] + 4*c[1]^7*c[2]^2 + 7*c[1]^6*c[2]^3 + 10*c[1]^5*c[2]^4 + 4*c[1]^7*c[2]*c[3] + 16*c[1]^6*c[2]^2*c[3] + 28*c[1]^5*c[2]^3*c[3] + 38*c[1]^4*c[2]^4*c[3] + 48*c[1]^5*c[2]^2*c[3]^2 + 76*c[1]^4*c[2]^3*c[3]^2 + 94*c[1]^3*c[2]^3*c[3]^3 + 28*c[1]^6*c[2]*c[3]*c[4] + 84*c[1]^5*c[2]^2*c[3]*c[4] + 140*c[1]^4*c[2]^3*c[3]*c[4] + 210*c[1]^4*c[2]^2*c[3]^2*c[4] + 280*c[1]^3*c[2]^3*c[3]^2*c[4] + 432*c[1]^3*c[2]^2*c[3]^2*c[4]^2 + 168*c[1]^5*c[2]*c[3]*c[4]*c[5] + 420*c[1]^4*c[2]^2*c[3]*c[4]*c[5] + 560*c[1]^3*c[2]^3*c[3]*c[4]*c[5] + 840*c[1]^3*c[2]^2*c[3]^2*c[4]*c[5] + 1272*c[1]^2*c[2]^2*c[3]^2*c[4]^2*c[5] + 840*c[1]^4*c[2]*c[3]*c[4]*c[5]*c[6] + 1680*c[1]^3*c[2]^2*c[3]*c[4]*c[5]*c[6] + 2520*c[1]^2*c[2]^2*c[3]^2*c[4]*c[5]*c[6] + 3360*c[1]^3*c[2]*c[3]*c[4]*c[5]*c[6]*c[7] + 5040*c[1]^2*c[2]^2*c[3]*c[4]*c[5]*c[6]*c[7] +

10080*c[1]^2*c[2]*c[3]*c[4]*c[5]*c[6]*c[7]*c[8] +
20160*c[1]*c[2]*c[3]*c[4]*c[5]*c[6]*c[7]*c[8]*c[9].

n=10: c[1]^10 + c[1]^9*c[2] + 5*c[1]^8*c[2]^2 + 8*c[1]^7*c[2]^3 + 16*c[1]^6*c[2]^4
+ 16*c[1]^5*c[2]^5 + 5*c[1]^8*c[2]*c[3] + 20*c[1]^7*c[2]^2*c[3] +
44*c[1]^6*c[2]^3*c[3] + 66*c[1]^5*c[2]^4*c[3] + 74*c[1]^6*c[2]^2*c[3]^2 +
132*c[1]^5*c[2]^3*c[3]^2 + 174*c[1]^4*c[2]^4*c[3]^2 + 216*c[1]^4*c[2]^3*c[3]^3 +
36*c[1]^7*c[2]*c[3]*c[4] + 128*c[1]^6*c[2]^2*c[3]*c[4] +
252*c[1]^5*c[2]^3*c[3]*c[4] + 318*c[1]^4*c[2]^4*c[3]*c[4] +
384*c[1]^5*c[2]^2*c[3]^2*c[4] + 636*c[1]^4*c[2]^2*c[3]^2*c[4]^2 +
840*c[1]^3*c[2]^3*c[3]^3*c[4] + 978*c[1]^4*c[2]^2*c[3]^2*c[4]^2 +
1272*c[1]^3*c[2]^3*c[3]^2*c[4]^2 + 252*c[1]^6*c[2]*c[3]*c[4]*c[5] +
756*c[1]^5*c[2]^2*c[3]*c[4]*c[5] + 1260*c[1]^4*c[2]^3*c[3]*c[4]*c[5] +
1896*c[1]^4*c[2]^2*c[3]^2*c[4]*c[5] + 2520*c[1]^3*c[2]^3*c[3]^2*c[4]*c[5] +
3792*c[1]^3*c[2]^2*c[3]^2*c[4]^2*c[5] + 5736*c[1]^2*c[2]^2*c[3]^2*c[4]^2*c[5]^2 +
1512*c[1]^5*c[2]*c[3]*c[4]*c[5]*c[6] + 3780*c[1]^4*c[2]^2*c[3]*c[4]*c[5]*c[6] +
5040*c[1]^3*c[2]^3*c[3]*c[4]*c[5]*c[6] + 7560*c[1]^3*c[2]^2*c[3]^2*c[4]*c[5]*c[6] +
11352*c[1]^2*c[2]^2*c[3]^2*c[4]^2*c[5]*c[6] +
7560*c[1]^4*c[2]*c[3]*c[4]*c[5]*c[6]*c[7] +
15120*c[1]^3*c[2]^2*c[3]*c[4]*c[5]*c[6]*c[7] +
22680*c[1]^2*c[2]^2*c[3]^2*c[4]*c[5]*c[6]*c[7] +
30240*c[1]^3*c[2]*c[3]*c[4]*c[5]*c[6]*c[7]*c[8] +
45360*c[1]^2*c[2]^2*c[3]*c[4]*c[5]*c[6]*c[7]*c[8] +
90720*c[1]^2*c[2]*c[3]*c[4]*c[5]*c[6]*c[7]*c[8]*c[9] +
181440*c[1]*c[2]*c[3]*c[4]*c[5]*c[6]*c[7]*c[8]*c[9]*c[10].

-- e.o.f. --