Universität Karlsruhe **Institut für Theoretische Physik**
Wolfdieter Lang **April 2006**

Some remarks, also on Girard's notations

• An equation like

$$1\,x^2 \;=\; 6\,x^1 \;+\; 40 \quad \text{is written as} \quad 1②\ \text{ et e⌈gale à } 6①\ +\ 40\ .$$

Scan 1, I. Definition, Explication. Such an equation with more than two terms is called 'composée' or 'me⌈lée' (mêlée). ⌈ is used here for the long s).

• Scan 2, line 2 should read 1④ e⌈gale à 5② + 36. This is an example of an incomplete ('incomplette') equation because not all powers from 0 to 4 appear.

• VIII. Definition. The leading power is called 'maxime' or 'haute extremité'. Its coefficient is usually taken as 1 (monic polynomial). The next lower one 'premier me⌈lée'. Then follow 'second', 'troi⌈ie⌈me' (troisième), 'quatrie⌈me' (quatrième), *etc.*

• Scan 3, XI. Definition. The sum of some given numbers (later coefficients of a given monic polynomial with alternating signs) is called 'premiere faction'. The sum over (ordered) products of two of the given numbers is called 'deuxie⌈me faction', *etc.* For a polynomial these products become the elementary symmetric polynomials of its zeros (solutions).

• Scan 4 shows the top of the 'triangle d'extraction' (*Pascal*'s triangle from 1653, published 1665, according to *W. W. Rouse Ball*: *A short Account of the History of Mathematics, Dover, New York, 1960, p.284*). This triangle was known centuries before in China and Persia.

• Scan 4, II. Theorem. The fundamental theorem of algebra is stated for complete equations. It is just exemplified on scan 5 in the 'Explication' (where the last sign of the considered equation is wrong: it should be +24). This example of a fourth order polynomial with four different solutions $1, 2, -3, 4$ also explains *Vieta*'s theorem on the connection between the coefficients of the polynomial (me⌈lé) $4, -7, -34, -24$ and the elementary symmetric polynomials of the solutions (the 'factions'). Here the convention with alternating signs becomes important.
Incomplete equations ('equations incomplettes') are considered separatly (Scans 5,6). Square roots, also of negative numbers, are used (Scan 6): solutions of $x^2 \,-\, 13\,x \,+\, 2 = 0$ and of $x^4 \,-\, 4\,x \,+\, 3 = 0$. Here *e.g.* $40\frac{1}{4}$ is, of course, $\frac{161}{4}$.

• Scan 8 shows the expressions for the sums of the first four powers of zeros for a monic polynomial of the form

$$p_n(x) \;:=\; 1\,x^n \,-\, A\,x^{n-1} \,+\, B\,x^{n-2} \,-\, C\,x^{n-3} \,+\, D\,x^{n-4} \;+-\ldots$$

A is the first power sum: the sum of the zeros of $p_n(x)$. The sum over the second power of the zeros is called '⌈omme des quarez' and is $A^2 \,-\, 2\,B$, written by *Girard* as $A\,q \,-\, B\,2$. Similarly, the third power sum ('⌈omme des Cubes' is $A^3 - 3\,A\,B + 3\,C$, written as $A\,cub - A\,B\,3 \,+\, C\,3$, and the sum over the fourth power of the zeros ('⌈omme des quaré-quarez') is $A^4 - 4\,A^2\,B + 4\,A\,C + 2\,B^2 - 4\,D$, written as $A\,q\,q \,-\, A\,q\,B\,4 + A\,C\,4 + B\,q\,2 \,-\, D\,4$. These are the equations quoted in *MacMahons* book *Combinatorial Analysis*, Vol. II, p.vii.

A quotation from *W.W. Rouse Ball*'s book (op.cit., p.234-5) on *Girard*:
"Girard's investigations were unknown to most of his contemporaries, and exercised no appreciable influence on the development of mathematics".

http://mathworld.wolfram.com/VietasFormulas.html quotes *Girard* for the general proof of Vieta's formulae.