On Generalizations of the Stirling Number Triangles *

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Abstract

Sequences of generalized *Stirling* numbers of both kinds are presented. These sequences of triangles of numbers (or infinite dimensional lower triangular matrices) will be denoted by $S2(k;n,m)$ and $S1(k;n,m)$, with $k \in \mathbf{Z}$. The original *Stirling* number triangles of the second and first kind show up for $k = 1$. $S2(2;n,m)$ is identical with the unsigned S1(2;n,m) triangle, called $S1p(2;n,m)$, which represents the triangle of signless *Lah* numbers. Associated number triangles, called $s2(k;n,m)$ and $s1(k;n,m)$, are also defined. $s2(2;n,m)$ as well as $s1(2;n+1,m+1)$ is *Pascal*'s triangle, and $s2(-1,n,m)$ turns out to be *Shapiro*'s *Catalan* triangle.

Generating functions for the columns of these triangles are given. Each $\mathbf{S2}(k)$ and each $\mathbf{S1}(k)$ matrix is an example of a *Jabotinsky*-matrix. Therefore, the generating functions of the rows of these triangular arrays constitute exponential convolution polynomials. The sequences of the row-sums of these triangles are also considered.

The connection to the problem of finding finite transformations from infinitesimal ones generated by $x^k \frac{d}{dx}$, for $k \in \mathbf{Z}$, is made.

*In memory of my mother Else Gertrud Lang.

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